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汇率期权定价及波动分析*

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摘 要: 为了使得人们对外汇期权价格从理论上有进一步的了解, 本文基于一个具体的汇率模型讨论了汇率的期权定价问题, 综合考虑了利率和购买力以及交割价对汇率的影响, 探讨了外汇期权价格波动及其均衡价格在一定置信水平下的置信下限.

关键词: 期权; 汇率; 置信区间

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Option Pricing and Fluctuating

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Abstract: In order to have the further understanding of foreign exchange option price theoretically, we discuss the problem of exchange option pricing according to a concrete exchange rate model and think over its effect on exchange rate through interest rate, purchasing power and account pricing. The flucuating state of foreign exchange option price and the belief bottom line of foreign exchange option balanced price under some belief level is also studited.

Key words: option; exchange rates; belief interval

0 引 言

大家知道, 每种货币都有其内在价值 (intrinsic value), 这可以通过它的购买力反映出来. 令 $B(t)$ 为某种货币的结算价格 (accounting price), $H(t)$ 为本国货币的结算价格(二者均可用黄金做标准), 那么外币对本国货币的名义汇率可表示为 $B(t)/H(t)$[1]. 假定在一段时间内这两种货币的利率均为常数, 这里用 r_f 和 r 分别表示外币和本国货币的汇率, 则用本国货币兑换外币的投资者应获得的收益(率)为 (r_f-r), 于是外币对本国货币的变化率应为 (r_f-r), 即 $\mathrm{d}(\frac{B}{H})/(\frac{B}{H})=(r_f-r)\mathrm{d}t$.

1 期权定价及波动

用 $P(t)$ 表示实际汇率, 如果一种货币的名义汇率高(低)于其相对购买力 P^*, 那么 $P(t)$ 呈下降(上升)趋势, 这种现象称为均值回复 (reversion to the mean). 它是许多理论模型的重要组成部分, 通常用来导出所谓的均衡值 (equilibrium value). 另一方面, 受利益的驱动, 当 $P(t)$ 高于名义汇率时(即 $P(t)>\frac{B(t)}{H(t)}=\exp[(r_f-r)t]$ 或 $\ln P(t)>(r_f-r)t$ 时), 投资者将抛售外币, 从而使 $P(t)$ 呈下降趋势; 反之,

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$P(t)$ 呈上升趋势. 同时汇率的波动还受到其他随机因素(技术性因素、心理因素等)的影响. 基于上述考虑, 肖庆宪和茆诗松给出了以下汇率模型[3]

$$\mathrm{d}\ln P(t) = [-\alpha(\ln P(t) - \ln P^*) - \beta(\ln P(t) - (r_f - r)t)]\mathrm{d}t + \sigma \mathrm{d}W(t), \tag{1}$$

式中 α, β, σ 均为正数; $W(t)$ 为标准布朗运动.

引理 1 $\ln P(t) \sim N(\mu(t), F(t))$, 式中

$$\mu(t) = -bt - c + \mathrm{e}^{-at}(\ln P(0) + c), \quad F(t) = \frac{\sigma^2}{2a}(1 - \mathrm{e}^{-2at}),$$

$$a = \alpha + \beta, \ b = \frac{-\beta(r_f - r)}{\alpha + \beta}, \ c = -\frac{\alpha \ln P^*}{\alpha + \beta} + \frac{\beta(r_f - r)}{(\alpha + \beta)^2}.$$

文献[2]讨论了该模型参数的极大似然估计.

对于交割价为 Q 的欧式看涨期权来说, 当 $P(t) > Q$ 时, 其价值为 $P(t) - Q$; 当 $P(t) \leq Q$ 时, 其价值为 0. 因而其价格为随机变量 $(P(t) - Q)^+$, 通常以 $E(P(t) - Q)^+$ 作为其均衡价格.

引理 2

$$E(P(t) - Q)^+ = \exp[\frac{F(t) + 2\mu(t)}{2}]N[\frac{F(t) + \mu(t) - \ln Q}{\sqrt{F(t)}}] - QN[\frac{\mu(t) - \ln Q}{\sqrt{F(t)}}].$$

为了方便使用, 将证明过程一并引入(见文献[1]).

证明

$$\begin{aligned}
E(P(t) - Q)^+ = {} & \int_{-\infty}^{\infty}(P - Q)^+ \frac{1}{\sqrt{2\pi F(t)}}\exp[-\frac{(\ln P - \mu(t))^2}{2F(t)}]\mathrm{d}\ln P = \\
& \int_Q^{\infty}(P - Q)\frac{1}{P\sqrt{2\pi F(t)}}\exp[-\frac{(\ln P - \mu(t))^2}{2F(t)}]\mathrm{d}P = \\
& \int_Q^{\infty}\frac{1}{\sqrt{2\pi F(t)}}\exp[-\frac{(\ln P - \mu(t))^2}{2F(t)}]\mathrm{d}P - \\
& Q\int_Q^{\infty}\frac{1}{P\sqrt{2\pi F(t)}}\exp[-\frac{(\ln P - \mu(t))^2}{2F(t)}]\mathrm{d}P,
\end{aligned} \tag{2}$$

因为

$$\begin{aligned}
& \int_Q^{\infty}\frac{1}{\sqrt{2\pi F(t)}}\exp[-\frac{(\ln P - \mu(t))^2}{2F(t)}]\mathrm{d}P = \\
& \frac{1}{\sqrt{2\pi F(t)}}\int_{\ln Q}^{\infty}\exp(g) \cdot \exp[-\frac{(g - \mu(t))^2}{2F(t)}]\mathrm{d}g = \\
& \frac{1}{\sqrt{2\pi F(t)}}\int_{\ln Q}^{\infty}\exp[-\frac{-2F(t)g + g^2 - 2g\mu(t) + \mu^2(t)}{2F(t)}]\mathrm{d}g = \\
& \frac{1}{\sqrt{2\pi F(t)}}\int_{\ln Q}^{\infty}\exp[-\frac{g^2 - 2g(F(t) + \mu(t)) + \mu^2(t)}{2F(t)}]\mathrm{d}g = \\
& \frac{1}{\sqrt{2\pi F(t)}}\exp[-\frac{\mu^2(t)}{2F(t)}]\int_{\ln Q}^{\infty}\exp\{-\frac{[g - (F(t) + \mu(t))]^2 - ((F(t) + \mu(t))^2}{2F(t)}\}\mathrm{d}g = \\
& \exp[\frac{F^2(t) + 2F(t)\mu(t)}{2F(t)}]\frac{1}{\sqrt{2\pi F(t)}}\int_{\ln Q}^{\infty}\exp\{-\frac{[g - (F(t) + \mu(t))]^2}{2F(t)}\}\mathrm{d}g = \\
& \exp[\frac{F(t) + 2\mu(t)}{2}]N[\frac{F(t) + \mu(t) - \ln Q}{\sqrt{F(t)}}],
\end{aligned} \tag{3}$$

$$\begin{aligned}
& Q\int_Q^{\infty}\frac{1}{P\sqrt{2\pi F(t)}}\exp[-\frac{(\ln P - \mu(t))^2}{2F(t)}]\mathrm{d}P = \\
& \frac{Q}{\sqrt{2\pi F(t)}}\int_Q^{\infty}\exp[-\frac{(\ln P - \mu(t))^2}{2F(t)}]\mathrm{d}\ln P = \\
& \frac{Q}{\sqrt{2\pi F(t)}}\int_{\ln Q}^{\infty}\exp[-\frac{(x - \mu(t))^2}{2F(t)}]\mathrm{d}x = QN[\frac{\mu(t) - \ln Q}{\sqrt{F(t)}}],
\end{aligned} \tag{4}$$

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所以 $E(P(t)-Q)^{+}=\exp[\frac{F(t)+2\mu(t)}{2}]N[\frac{F(t)+\mu(t)-\ln Q}{\sqrt{F(t)}}]-QN[\frac{\mu(t)-\ln Q}{\sqrt{F(t)}}]$.

上述引理给出了汇率期权的均衡价格，可以使用 $D(P(t)-Q)^{+}$ 表示其价格波动.

定理 1

$$D(P(t)-Q)^{+}=\exp[2(\mu(t)+F(t))]N[\frac{\mu(t)+2F(t)-\ln Q}{\sqrt{F(t)}}]-$$
$$2Q\exp[\frac{F(t)+2\mu(t)}{2}]N[\frac{F(t)+\mu(t)-\ln Q}{\sqrt{F(t)}}]+Q^{2}N[\frac{\mu(t)-\ln Q}{\sqrt{F(t)}}]-$$
$$\{\exp[\frac{F(t)+2\mu(t)}{2}]N[\frac{F(t)+\mu(t)-\ln Q}{\sqrt{F(t)}}]-QN[\frac{\mu(t)-\ln Q}{\sqrt{F(t)}}]\}^{2}.$$

证明 因为

$$D(P(t)-Q)^{+}=E[(P(t)-Q)^{+}]^{2}-[E(P(t)-Q)^{+}]^{2},$$

而

$$E(P(t)-Q)^{+2}=\int_{-\infty}^{\infty}(P-Q)^{+2}\frac{1}{\sqrt{2\pi F(t)}}\exp[-\frac{(\ln P-\mu(t))^{2}}{2F(t)}]\mathrm{d}\ln P=$$
$$\int_{Q}^{\infty}(P-Q)^{2}\frac{1}{P\sqrt{2\pi F(t)}}\exp[-\frac{(\ln P-\mu(t))^{2}}{2F(t)}]\mathrm{d}P=$$
$$\int_{Q}^{\infty}\frac{P}{\sqrt{2\pi F(t)}}\exp[-\frac{(\ln P-\mu(t))^{2}}{2F(t)}]\mathrm{d}P-$$
$$2Q\int_{Q}^{\infty}\frac{1}{\sqrt{2\pi F(t)}}\exp[-\frac{(\ln P-\mu(t))^{2}}{2F(t)}]\mathrm{d}P+$$
$$Q^{2}\int_{Q}^{\infty}\frac{1}{P\sqrt{2\pi F(t)}}\exp[-\frac{(\ln P-\mu(t))^{2}}{2F(t)}]\mathrm{d}P. \quad (5)$$

由引理 2 的推导可知后两项的结果，现推导第一项，过程如下:

$$\int_{Q}^{\infty}\frac{P}{\sqrt{2\pi F(t)}}\exp[-\frac{(\ln P-\mu(t))^{2}}{2F(t)}]\mathrm{d}P=$$
$$\frac{1}{\sqrt{2\pi F(t)}}\int_{\ln Q}^{\infty}\exp(g)\cdot\exp[-\frac{(g-\mu(t))^{2}}{2F(t)}]\cdot\exp(g)\mathrm{d}g=$$
$$\frac{1}{\sqrt{2\pi F(t)}}\int_{\ln Q}^{\infty}\exp[2g-\frac{(g-\mu(t))^{2}}{2F(t)}]\mathrm{d}g=$$
$$\frac{1}{\sqrt{2\pi F(t)}}\int_{\ln Q}^{\infty}\exp[-\frac{g^{2}-2g\mu(t)-4gF(t)+\mu^{2}(t)}{2F(t)}]\mathrm{d}g=$$
$$\frac{1}{\sqrt{2\pi F(t)}}\int_{\ln Q}^{\infty}\exp\{-\frac{g^{2}-2g[\mu(t)+2F(t)]+\mu^{2}(t)}{2F(t)}\}\mathrm{d}g=$$
$$\frac{1}{\sqrt{2\pi F(t)}}\int_{\ln Q}^{\infty}\exp\{-\frac{[g-(\mu(t)+2F(t))]^{2}-[\mu(t)+2F(t)]^{2}+\mu^{2}(t)}{2F(t)}]\mathrm{d}g=$$
$$\exp[\frac{4\mu(t)F(t)+4F^{2}(t)}{2F(t)}]\frac{1}{\sqrt{2\pi F(t)}}\int_{\ln Q}^{\infty}\exp\{-\frac{[g-(\mu(t)+2F(t))]^{2}}{2F(t)}\}\mathrm{d}g=$$
$$\exp\{2[\mu(t)+F(t)]\}N[\frac{\mu(t)+2F(t)-\ln Q}{\sqrt{F(t)}}], \quad (6)$$

将式 (3), (4), (16) 代入式 (5) 即得

$$E(P(t)-Q)^{+2}=\exp[2(\mu(t)+F(t))]N[\frac{\mu(t)+2F(t)-\ln Q}{\sqrt{F(t)}}]-$$
$$2Q\exp[\frac{F(t)+2\mu(t)}{2}]N[\frac{F(t)+\mu(t)-\ln Q}{\sqrt{F(t)}}]+Q^{2}N[\frac{\mu(t)-\ln Q}{\sqrt{F(t)}}]. \quad (7)$$

由式 (7) 和引理 2 的结论可得定理 1.

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2 期权均衡价格的置信下限

以下给出期权均衡价格的置信度为 $1-\alpha$ 的置信下限.

因为 $\ln P(t) \sim N(\mu(t), F(t))$. 假设 $X_1, X_2, \cdots, X_n$ 是来自总体的一个样本, 记

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i,\quad S^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n} X_i^2 - n\bar{X}^2\right],$$

又

$$\frac{\bar{X} - \mu(t)}{S/\sqrt{n}} \sim t(n-1),$$

其右边的分布 $t(n-1)$ 不依赖于任何参数, 由此可得

$$P\left\{-t_{\alpha/2}(n-1) < \frac{\bar{X} - \mu(t)}{S/\sqrt{n}} < t_{\alpha/2}(n-1)\right\} = 1-\alpha,$$

即

$$P\left\{\bar{X} - \frac{S}{\sqrt{n}} t_{\alpha/2}(n-1) < \mu(t) < \bar{X} + \frac{S}{\sqrt{n}} t_{\alpha/2}(n-1)\right\} = 1-\alpha,$$

所以, $\mu(t)$ 的置信度 $1-\alpha$ 的置信区间为

$$\left(\bar{X} - \frac{S}{\sqrt{n}} t_{\alpha/2}(n-1), \bar{X} + \frac{S}{\sqrt{n}} t_{\alpha/2}(n-1)\right).$$

又因为 $(P(t) - Q)^+ = (\exp[\ln P(t)] - Q)^+$, 再结合 Jensen 不等式 $Eg(X) \geq g(EX)$[5], 故有

$$E(P(t) - Q)^+ \geq \{E\exp[\ln P(t)] - Q\}^+ \geq \{\exp[E\ln P(t)] - Q\}^+ \geq$$
$$\left\{\exp\left[\bar{X} - \frac{S}{\sqrt{n}} t_{\alpha/2}(n-1)\right] - Q\right\}^+,$$

故汇率期权均衡价格的置信度 $1-\alpha$ 的置信下限为

$$\left\{\exp\left[\bar{X} - \frac{S}{\sqrt{n}} t_{\alpha/2}(n-1)\right] - Q\right\}^+.$$

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