

Analysis of Channel of Sales Promotion under Consignment Contract with Revenue Sharing

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Abstract: The manufacturer sells the product through a retailer with the consignment sales scheme in international trade, meanwhile many demand enhancing promotion, eg, after sales services, advertisement etc. can be provided either by the manufacturer or by the retailer. We study the preferences of the supply chain members to invest on the sales promotion when the trade scheme is the consignment sales. We find (1) the manufacturer always prefer providing the sales promotion, (2) but, there exists a threshold level of the percentage allocation of revenue in model IC, the retailer wishes the manufacturer's sales promotion if the level of the percentage allocation of revenue is more than the threshold, otherwise wishes to invest on the sales promotion by herself.

Key Words: Supply Chain, Consignment Sales, Sales Promotion, Channel Management

I. INTRODUCTION

Consignment sales is an important international trade scheme. Under the consignment sales, two firms relate with a revenue sharing contract. The manufacturer also produces and sells a product through an independent retailer, the manufacturer decides on the retail price and retains ownership of the goods, For each item sold, the retailer will deduct an agree-upon percentage from the selling price and no money changes hands until the item is sold. Such a business arrangement is well documented in Bolen (1988)^[1] and is getting prevalent in video rental industry (Cachon and Lariviere (2001)^[2] and Van der Veen and Venugopal (2005)^[3]) and internet commerce (see, for examples, Wang et al. (2004)^[4] and <http://www.amazon.com>). Several authors have considered the supply chain management with consignment contract with revenue sharing and retail fixed markup (RFM). Wang et al (2004)^[4] considered a supply chain with one-manufacturer and one-retailer, where the retailer decides allocation of sales revenue between herself and the manufacturer, and the manufacturer then decides production quantity and retail price. They analyzed performance of the supply chain and of individual firms. Gerchak and Wang(2004)^[5], considering assembly systems, investigated two very distinct types of arrangements between an retailer and its suppliers. One arrangement is a vendor-managed inventory with revenue sharing (is more closely related to the setting of consignment with revenue sharing), and the other a wholesale-price contract where the retail price of the final product is assumed to be a constant. Some papers studied a retail fixed markup (RFM) policy where an ex-ante commitment is made to the retail price markup before wholesale price have been set in decentralized supply chains facing price-dependent demand. The RFM policy is chosen by the retailer, by the manufacturer and through negotiation. If the RFM (the markup of the retailer

under RFM, is first decided by the retailer facing price-dependent deterministic demand, then the manufacturer decides wholesale price which is equivalent that the manufacturer decides retail price. We can find the RFM is equivalent to the consignment contract with revenue sharing. Liu et al (2006)^[6] considered RFM policies and compared the relative performance of RFM to a price-only contract and find that RFM results in higher profit for the supply chain than the price-only contract in a variety of scenarios under linear additive demand models. Liu et al. (2009)^[7] expanded extend the study of Liu et al. (2006) by considering both linear additive and multiplicative demand functions and show that the form of the demand function greatly affects the performance of RFM. In practice, firms usually invest to promote their products. Sales promotion may include advertisement, post-sale service, and other sales efforts. They may increase perceived customer value, furthermore, improve product demand. The first stream related to our research is the supply chain coordination with sales promotion. Most papers in the literature concentrate on designing a contractual mechanism to improve the coordination in the channel with sales promotion. By using Hotelling Model Iyer (1998)^[8], studied how the manufacturer should coordinate distribution channels when two retailers compete in retail prices and important nonprice factor such as after-sales service and advertisement. Tsay and Aggrawal (2000)^[9] studied a supply chain with one-manufacturer and two retailers who compete both in price and service. They found that the retailers are benefited from adding the service dimension. Xiao et al(2005)^[10] considered a price-subsidy rate contract to coordinate investments of competing retailers on sales promotion. In the above research, the demand is assumed to be deterministic. The following two papers studied coordination of the supply chain with sales promotion and stochastic demand. For a channel rebate contract, Taylor (2002)^[11] employed a simple model in which a retailer makes quantity and effort decisions and then observes demand. Krishnan et al. (2004)^[12] examined coordinating contracts of a supply chain in which the retailer can choose promotional effort.

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Furthermore, Taylor (2006) [13] investigated impact of the retailer's sales effort on the manufacturer's sale-timing decision, in which payment to the manufacturer depends on the retailer's order quantity.

However, the sales promotion can be provided by either the manufacturer or by the retailer. The manufacturer offering the sales promotion, such as post-sale service is common, for example, HP, DELL, and the software industry, etc. Because the retailer directly faces the consumers, the sales promotion is provided by her, such as many of the automotive industry, Wal-Mart, etc. In this paper, we investigate who should invest on the sales promotion when the trade scheme is the consignment sales with revenue sharing contract.

II. MODEL FORMULATION AND SOLUTION

The supply chain considered here consists of two risk neutral firms: a manufacturer and a retailer. The manufacturer produces a product at a constant marginal cost, and then sells the product through the retailer with the consignment sales. In order to have a higher market share, firms usually promote their products by advertisement, post-sale service, or other sales promotion investment. The sales promotion is performed by only either the manufacturer or only the retailer. We denote by I that the manufacturer provides sales pro-motion ; by O that the retailer provides sales promotion ,that is, the manufacturer outsources it to the retailer; and by C that the consignment sales with revenue sharing contract is signed between them. We then have two possible combinations of the supply chain. (1) the manufacturer takes over sales promotion and the consignment sales arrangement(IC), (2) the retailer takes over sales promotion and the consignment sales arrangement (OC). Throughout this paper we will refer to the manufacturer as he and the retailer as she. Our objective is to characterize and compare equilibriums of the two types of supply chain and provide managerial insights for the manufacturer and the retailer. Under the consignment sales trade scheme, who provide the sales promotion is optimal for them?

The notations are as follows:

c : the manufacturer's unit cost;

s : the manufacturer's or retailer's investment on sales promotion, a decision variable;

r : the revenue share, $0 \leq \gamma \leq 1$, allocated by the retailer, a decision variable;

p : the retail price determined by the retailer or the manufacturer, a decision variable;

In general, demand in each model depends on both the retail price and the sales promotion provision. Then, we assume that the demand function is $D(p, s) = D_0 p^{-\alpha} s^\beta$. Here, D_0 is the size of the potential market and α is the price-elasticity index of (expected) demand. The larger the value of α , the more sensitive the demand is in price. A

product is said to be price elastic, if the price-elasticity index is greater than 1; Otherwise, the product is said to be inelastic. β is the investment-elasticity for the incremental demand. The

total market demand increases with the investment in sales promotion. Further, we assume $0 < \beta < 1$, representing the marginal decrease of demand in the sales promotion.

In the next subsection, we will analyze the optimal price and investment in the two decentralized models, in which the manufacturer and the retailer make decisions according to their own profits, respectively.

A. The Manufacturer's Sales Promotion and the Consignment Sales

In this scenario, the manufacturer also provides the sales Promotion, but the retailer sells products with the consignment sales. Under the consignment sales, the retailer, acting as the leader, offers the manufacturer a revenue-sharing contract under which for each unit of the product sold, she keeps γ share (percent) of the revenue for herself and remits the rest, $1 - \gamma$, to the manufacturer. The manufacturer, acting as a follower, has the right to decide the retail price p for her product and s the investment in sales promotion. This is shown in Figure 1. We denote this model by IC.

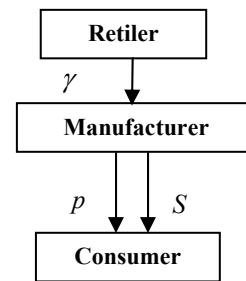


FIGURE 1 MODEL IC

Both the manufacturer and the retailer want to maximize her own profit. The game between the retailer and the manufacturer is a Stackelberg game given as follows:

$$stage1 : \max_{\gamma} \pi_r^{IC}(p, s, \gamma) = \gamma p D(p, s),$$

$$stage2 : \max_{p, s} \pi_m^{IC}(p, s, \gamma) = [(1 - \gamma)p - c]D(p, s) - s$$

Because the retailer is the Stackelberg leader, we begin by solving the best-response function of the manufacturer. We find the optimal price p^{IC} and optimal investment s^{IC} on the sales promotion for any given γ as follows:

$$\begin{cases} p^{IC}(\gamma) = \frac{\alpha c}{(1 - \gamma)(\alpha - 1)} \\ s^{IC}(\gamma) = \left(\frac{\alpha - 1}{\beta D_0 c} \right)^{\frac{1}{\beta - 1}} \left(\frac{\alpha c}{(1 - \gamma)(\alpha - 1)} \right)^{\frac{\alpha}{\beta - 1}} \end{cases} \quad (1)$$

Then, the retailer's problem becomes:

$$\max_{\gamma} \pi_r^{IC}(\gamma) = \frac{\gamma \alpha}{(1 - \gamma) \beta} s^{IC}(\gamma) \quad (2)$$

Its optimal solution is:

$$\gamma^{IC} = \frac{1-\beta}{\alpha} \quad (3)$$

By substituting γ^{IC} above into (1), we get the retail price, the investment in sales promotion and profits in equilibrium.

$$\left\{ \begin{array}{l} p^{IC} = \frac{\alpha^2 c}{(\alpha + \beta - 1)(\alpha - 1)} \\ s^{IC} = \left(\frac{\alpha - 1}{\beta D_0 c} \right)^{\frac{1}{\beta - 1}} \left(\frac{\alpha^2 c}{(\alpha + \beta - 1)(\alpha - 1)} \right)^{\frac{\alpha}{\beta - 1}} \\ \pi_r^{IC} = \frac{(1 - \beta)\alpha}{(\alpha + \beta - 1)} s^{IC} \\ \pi_m^{IC} = \frac{1 - \beta}{\beta} s^{IC} \end{array} \right. \quad (4)$$

B. The Retailer's Sales Promotion and the Consignment Sales

In this case, the retailer offers the manufacturer a consignment sales contract, which specifies the percentage allocation of sales revenue between the retailer and the manufacturer. She keeps γ share (percent) of the revenue for herself and remits the rest, $1 - \gamma$, to the manufacturer. Meanwhile, the retailer invests on the sales promotion. The manufacturer then chooses the retail price p . This is shown in Figure II. We denote this model by OC.

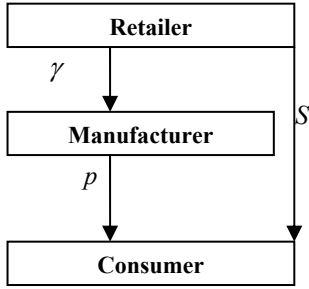


FIGURE II MODEL OC

The manufacturer accepts the contract as long as he can earn a positive profit. (That is, we normalize the manufacturer's reservation profit to zero.) The game between the retailer and the manufacturer is a Stackelberg game given as follows,

$$\text{stage1: } \max_{\gamma, s} \pi_r^{OC}(p, \gamma, s) = \gamma p D(p, s) - s$$

$$\text{stage2: } \max_p \pi_m^{OC}(p, s, \gamma) = ((1 - \gamma)p - c)D(p, s)$$

The best response of the manufacturer in stage 2 is given by,

$$p^{OC}(\gamma, s) = \frac{\alpha c}{(1 - \gamma)(\alpha - 1)} \quad (5)$$

Then, the retailer's problem in stage 1 becomes

$$\max_{\gamma, s} \pi_r^{OC}(\gamma, s)$$

$$= \frac{\gamma \alpha c}{(1 - \gamma)(\alpha - 1)} D_0 \left(\frac{\alpha c}{(1 - \gamma)(\alpha - 1)} \right)^{-\alpha} s^{\beta} - s \quad (6)$$

Its optimal solution can be gotten as

$$\gamma^{OC} = \frac{1}{\alpha}, \quad s^{OC} = \left(\frac{(\alpha - 1)^2}{\beta D_0 c \alpha} \right)^{\frac{1}{\beta - 1}} \left(\frac{\alpha^2 c}{(\alpha - 1)^2} \right)^{\frac{\alpha}{\beta - 1}} \quad (7)$$

Hence, the retail price, the investment on sales promotion and profits in equilibrium can easily be computed.

$$\left\{ \begin{array}{l} p^{OC} = \frac{\alpha^2 c}{(\alpha - 1)^2} \\ s^{OC} = \left(\frac{(\alpha - 1)^2}{\beta D_0 c \alpha} \right)^{\frac{1}{\beta - 1}} \left(\frac{\alpha^2 c}{(\alpha - 1)^2} \right)^{\frac{\alpha}{\beta - 1}} \\ \pi_r^{OC} = \frac{(1 - \beta)}{\beta} s^{OC} \\ \pi_m^{OC} = \frac{(\alpha - 1)}{\beta \alpha} s^{OC} \end{array} \right. \quad (8)$$

III. CHANNEL CONFIGURATION ANALYSIS

By comparing the results in Tables 1, we want to study who should invest on the sales promotion when the trade scheme is consignment sales.

Theorem 1: $\pi_m^{IC} > \pi_m^{OC}$. Hence, given the consignment sales scheme, the manufacturer has a larger profit by providing the sales promotion by himself.

Proof:

According to (4) and (8),

$$\begin{aligned} \pi_m^{IC} / \pi_m^{OC} &= (1 - \beta) \left(\frac{\alpha - 1}{\alpha + \beta - 1} \right)^{\frac{\alpha}{\beta - 1}} \left(\frac{\alpha}{\alpha - 1} \right)^{\frac{\beta}{\beta - 1}} \\ &= (1 - \beta) \left(\frac{\alpha + \beta - 1}{\alpha} \right)^{\frac{\alpha}{1 - \beta}} \left(\frac{\alpha}{\alpha - 1} \right)^{\frac{\alpha - \beta}{1 - \beta}} \end{aligned} \quad (9)$$

Then $\pi_m^{IC} > \pi_m^{OC}$ is equivalent to

$$f_1(\beta) = \ln \frac{\pi_m^{IC}}{\pi_m^{OC}} =$$

$$\ln(1 - \beta) + \frac{\alpha}{1 - \beta} \ln \frac{\alpha + \beta - 1}{\alpha} + \frac{\alpha - \beta}{1 - \beta} \ln \frac{\alpha}{\alpha - 1} > 0$$

Since,

$$\begin{aligned} f_1'(\beta) &= \frac{1}{(1 - \beta)^2} \left(\alpha \ln \frac{\alpha + \beta - 1}{\alpha} + (\alpha - 1) \ln \frac{\alpha}{\alpha - 1} + \frac{(1 - \beta)^2}{\alpha + \beta - 1} \right) \\ &h_1(\beta) / (1 - \beta)^2 \end{aligned}$$

Where

$$h_1(\beta) = \alpha \ln \frac{\alpha + \beta - 1}{\alpha} + (\alpha - 1) \ln \frac{\alpha}{\alpha - 1} + \frac{(1 - \beta)^2}{\alpha + \beta - 1}$$

We can get

$$h_1'(\beta) = \frac{\alpha^2 - 3\alpha(1-\beta) + (1-\beta)^2}{(\alpha + \beta - 1)^2} > 0$$

It is easy to see that $\alpha^2 - 3\alpha(1-\beta) + (1-\beta)^2$ increases with $\beta \in [0,1]$, get its maximum α^2 at $\beta=1$, and the minimum $\alpha^2 - 3\alpha + 1$ at $\beta=0$. We show $h_1(\beta) > 0$ in two cases below.

i. When $h_1(\beta) > \alpha \geq \frac{3+\sqrt{5}}{2}$, $\alpha^2 - 3\alpha + 1 \geq 0$. $h_1(\beta)$ increase in β , and $h_1(0) = \ln((\alpha-1)/\alpha) + 1/(\alpha-1)$

We next prove that: $\ln((\alpha-1)/\alpha) + 1/(\alpha-1) > 0$.

Due to

$$[\ln((\alpha-1)/\alpha) + 1/(\alpha-1) > 0] = -\frac{1}{\alpha(\alpha-1)^2} < 0$$

so $\ln((\alpha-1)/\alpha) + 1/(\alpha-1)$ decreases as α increases. Moreover, $\lim_{\alpha \rightarrow \infty} \{\ln((\alpha-1)/\alpha) + 1/(\alpha-1)\} = 0$. So $\ln((\alpha-1)/\alpha) + 1/(\alpha-1) > 0$ with As shown in the following Hence, $h_1(0) > 0$, and $h_1(\beta) > 0$.

ii. When $\alpha < \frac{3+\sqrt{5}}{2}$, $\alpha^2 - 3\alpha + 1 < 0$ $h_2(\beta)$ gets the

optimal minimum at $\beta^* = 1 - \frac{3-\sqrt{5}}{2}\alpha$ and

$$\begin{aligned} h_2(\beta^*) &= \alpha \ln \frac{\alpha - \frac{3-\sqrt{5}}{2}\alpha}{\alpha} + (\alpha-1) \ln \frac{\alpha}{\alpha-1} + \frac{(\frac{3-\sqrt{5}}{2}\alpha)^2}{\alpha - \frac{3-\sqrt{5}}{2}\alpha} \\ &= \alpha(\sqrt{5}-2 + \ln \frac{\sqrt{5}-1}{2} - \frac{\alpha-1}{\alpha} \ln \frac{\alpha-1}{\alpha}) \end{aligned}$$

Due to that $x \ln x$ increases with x , and $(\alpha-1)/\alpha < (\sqrt{5}-1)/2$ when $\alpha < (3+\sqrt{5})/2$

So,

$$\begin{aligned} \sqrt{5}-2 + \ln \frac{\sqrt{5}-1}{2} - \frac{\alpha-1}{\alpha} \ln \frac{\alpha-1}{\alpha} &> \\ \sqrt{5}-2 + \ln \frac{\sqrt{5}-1}{2} - \frac{\sqrt{5}-1}{2} \ln \frac{\sqrt{5}-1}{2} &> 0 \end{aligned}$$

We then conclude $h_1(\beta) \geq h_1(\beta^*) > 0$. From above i and ii, we know $h_1(\beta) > 0$ So $f_1'(\beta) > 0$, This means that $f_1(\beta)$ increases with $\beta \in [0,1]$. Due to $f_1(0) = 0$, we conclude that $f_1(\beta) > 0$ for any $\beta \in [0,1]$. This completes the proof.

Theorem 1 tells us that provided the trade scheme is the consignment sales, the manufacturer is always benefited from providing the sales promotion. The manufacturer has full power to control the market demand in model IC, in Model OC while has only partial power.

Theorem 2: For each given α , there is unique

$\beta^* \in (1-(e-1)\alpha/e, 1)$ such that $\pi_r^{IC} > \pi_r^{OC}$. This is always true, when $\alpha > e/e-1$ or $\beta > 1/e$. Hence, given the consignment sales scheme, the retailer wishes the manufacturer's sales promotion if $\beta \in (\beta^*, 1)$, otherwise wishes to invest on the sales promotion by herself.

Proof:

According to (4) and (8),

$$\pi_r^{IC} / \pi_r^{OC} = \left(\frac{\alpha}{\alpha + \beta - 1} \right)^{\frac{\beta}{\beta-1}} \left(\frac{\alpha-1}{\alpha + \beta - 1} \right)^{\frac{\alpha-1}{\beta-1}} \quad (10)$$

It is apparent that $\pi_r^{IC} > \pi_r^{OC}$ if and only if $f_2(\beta) = (\alpha + \beta - 1) \ln(\alpha + \beta - 1) - \beta \ln \alpha - (\alpha - 1) \ln(\alpha - 1) > 0$

$$\text{Now, } f_2'(\beta) = \ln(\alpha + \beta - 1) + \ln \frac{e}{\alpha}, f_2''(\beta) = \frac{1}{\alpha + \beta - 1}$$

So, $f_2(\beta)$ is convex.

Moreover $f_2(0) = 0, f_2(1) = (\alpha - 1) \ln \frac{\alpha}{\alpha - 1} > 0$, and the unique solution of $f_2(\beta) = 0$ is $\beta_0 = 1 - \frac{e-1}{e}\alpha < 1$. So $f_2'(\beta) > 0$ if

and only if $\beta > \beta_0$, and there is unique $\beta^* \in (\beta_0, 1)$ such that $f_2(\beta^*) = 0$. Thus, $f_2(\beta) > 0$ for $\beta > \beta^*$, and $f_2(\beta) < 0$ for $\beta < \beta^*$. Hence, $\pi_r^{IC} > \pi_r^{OC}$ if and only if $\beta > \beta^*$. Certainly, when $\alpha > e/e-1$ or $\beta > 1/e$. $f_2(\beta) > 0$ for all $\beta \in [0,1]$ and so $\pi_r^{IC} > \pi_r^{OC}$ is always true. This completes the proof.

Theorem 2 tells us that provided the trade scheme is the consignment sales, the retailer is benefited only when $\beta > \beta^*$, otherwise wishes to invest on the sales promotion by herself. We can also find from Table 1, that $\gamma^{IC} = (1-\beta)/\alpha$ and $\gamma^{OC} = 1/\alpha$. Thus, under the consignment sales scheme, the impact of sales promotion has no effect on the retailer's revenue sharing decision when the retailer provide the sales promotion, but the impact of sales promotion has effect on the retailer's revenue sharing decision when the manufacturer provide the sales promotion. So the retailer need to consider the manufacturer's sales promotion how to affect the allocation of sales revenue. When the allocation percent $\gamma^{IC} < (e-1)/e$ the retailer wishes to invest on the sales promotion by the manufacturer, otherwise wishes to invest on the sales promotion by herself.

IV. Conclusion

In this paper, we consider who should invest on the sales promotion when the trade scheme is consignment sales? The results are (1) the manufacturer always better off providing the sales promotion by himself, (2) but, the retailer wishes the manufacturer's sales promotion if $\gamma^{IC} < (e-1)/e$ otherwise wishes to invest on the sales promotion by herself, when the trade scheme is the consignment sales. There are several directions that this research would be fruitful.

One is to consider stochastic demand. in our model, the supply

chain considered consists of one manufacturer and one retailer.

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