Helper Mode Analysis in a Supply Chain : Based on supermodular game theory

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***Abstract*—Consider a supply chain involving an independent supplier and an independent manufacturer, the manufacturer will help his supplier to invest in cost-reducing innovation. Based on the theory of supermodular games,we analyze two helper modes .The existence of pure strategy Nash equilibrium was proved by construction of potential function.The characteristics of the equilibrium were analyzed for simultaneous competition. We also prove that cooperating in effort decision will lead to a higher effort and higher profit for every player based supermodular games theory.**

***Index Terms*—supply chain, compete, cooperate,helper mode, supermodular game**

I. Introduction

"More and more, businesses are counting on their suppliers to lower costs, improve quality, and develop innovations faster than their competitors' suppliers can." Jeffrey and Thomas (2004) [1].Automakers Toyota and Honda have struck successful partnerships with some of the same suppliers and help their vendors continually to improve their processes. (Lincoln, 1998 [2]; McLaren, 1999 [3]) . Harabi (1998) [4]found 84% of innovative enterprises contain their customers or suppliers through an empirical study . Related research can see Von Hippel (1986) [5]VanderWerf (1992) [6].

Several authors have considered how specific contractual forms between buyers and sellers in a supply chain affect their incentives to invest in either demand enhancement, cost reduction, or capacity .Kim(2000) [7] studies the manufacturer coordinates its supplier's innovation that can eventually lead to supply cost reduction. Gilbert and Cvsa (2003) [8] discuss how to use strategic commitment to price to stimulate downstream innovation in a supply chain. Banerjee and Lin (2001) [9] examine the incentives of firms to form vertical research joint ventures. Banerjee and Lin (2003) [10] analyze the incentives for cost-reducing R&D by downstream firms in a two-tier market structure. Plambeck and Taylor(2006) [11] investigate how supply contracts can be designed under different breach remedies to create incentives for the buyer and the suppliers to make first best investments in R&D and capacity respectively. Krishnan et al. (2004) [12] examine coordinating contracts of a decentralized supply chain in which the retailer can choose promotional effort.

Our work is also related to the literature on some topic about co-invest of members of supply chain in either demand enhancement(advertising),or cost reduction. Huang and Li (2001) [13] , Huang et al. (2002) [14] , Li et al. (2002) [15] , and Yue et al. (2006) [16] have studied the coordination of cooperative advertising in a two-level supply chain under the price independent/dependent demand.

On the other hand, the theory of supermodular optimization/games, introduced by Topkis (1978, 1979) [17], [18]and further developed by Vives (1990) [19], Milgrom and Roberts (1990) [20] and Milgrom and Shannon (1991) [21], provides an elegant unifying approach, based on lattice-theoretic arguments, to characterizing games with strategic complementarities. See Topkis (1998) [22]Vives (1999) [23],and Vives (2005) [24] for detailed accounts of the theory and application. In the paper,we use supermodular game theory to analyze co-invest problem.

Most of the literatures considering investment in either demand enhancement, cost reduction, or capacity in a supply chain,have focused on the following Stackelberg structure: a upstream firm acts as a Stackelberg leader and a downstream firm is a Stackelberg follower. They considered how specific contractual form between buyer and seller affect and coordinate their incentives to invest. In this paper,The manufacturer helps the supplier to reduce cost in the first stage, while in the second stage, the supplier offers a wholesale price at first,then the manufacturer decides retail price.we consider special demand function : exponential demand .We use a new tool of supermodular game theory by construction of potential function to analysis the manufacturer helper modes: simultaneous effort decision with the supplier;cooperative effort decision with the supplier.

II. Model and notations

The supply chain considered here consists of a single manufacturer who sells the final product that consists of one component provided by a single component supplier in a single selling season.

The notations are as follows:

c_s: the supplier's original unit cost ;
c_m: the manufacturer's unit cost

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x_s: the supplier's cost reducing effort level, and the corresponding cost is $k_s(x_s)$;

x_m: the manufacturer's cost reducing effort level for his supplier, and the corresponding cost is $k_m(x_m)$;

$\beta(x_s)$: the supplier's absorptive capacity which is a function of its accumulated investment x_s, $\beta(x_s) : [0, +\infty) \to [0, 1]$ and $\beta(0) = 0, \beta(\infty) = 1, \beta'(x_s) \geq 0, \beta''(x_s) \leq 0$;

w: the wholesale price determined by the supplier;

p: the manufacture's retail price.

Demand $D(p)$ is a function of the market price p set by the manufacturer.

stage1: The manufacturer will help the supplier to reduce cost in the first stage.The manufacturer and the supplier select cost reducing level $x_i, i = s, m$. and the cost functions of supplier and manufacturer is $k_s(x_s)$ and $k_m(x_m)$ respectively.When the innovation succeeds, the supplier's unit cost changes into $(c_s - x_s - \beta(x_s)x_m)$.In addition, it is obvious that the ex post unit cost cannot exceed c_s.

stage2: After the cost reducing level is observed by all the players, the component producer choose a wholesale price w to maximize his expected profit.Based on the wholesale price of the component and production cost, the manufacturer select the retail price p so as to maximize his own expected profit.

Keeping price competition in the second stage, we consider two different scenarios for the first stage decision based on the sequence of the decisions and the objective of the decision makers:

Case 1: The manufacturer and the component supplier make their cost reducing level decision simultaneously to maximize their own individual profit.

Case 2: The manufacturer and the component supplier decide on their joint cost reducing level strategy together to maximize the total system profit.

III. PRICE COMPETITION IN THE SECOND STAGE

We should first solve the manufacturer's problem in stage 2 for any given x_s, x_m and w. The manufacturer's problem is to choose p to maximize his profit as follows,

$$\max_p \pi_M(p) = (p - c_m - w)D(p) - k_m(x_m) \tag{1}$$

The first order condition for maximizing $\pi_m(p)$ is as follows,

$$\pi'_m(p) = D(p) + (p - c_m - w)D'_p(p) = 0. \tag{2}$$

We denote its solution by $p^*(w|x_s, x_m)$. Since

$$\pi''_m(p) = 2D'_p(p) + (p - c_m - w)D''_p(p).$$

In order to ensure the optimality of $p^*(w|x_s, x_m)$, we make the following assumption.

Assumption 1: (1) $D(p)$ is twice differentiable, decreasing and concave in p for each given x_s, x_m.

(2) The domain of retail price p is $[0, \infty)$ and $\lim_{p\to\infty} pD(p) = 0$.

In the above assumption, Part (1) is usual in the literature. For Part (2), the former is for convenience. In fact, if p is limited in a finite region then we can extend its domain to $[0, \infty)$ by letting $D(p) = 0$ for all other p; While the later is just to ensure that we have a finite optimal retail price for the manufacturer. Hereafter, we assume that Assumption 1 is true. With this assumption, we have obviously the following result.

Lemma 1: For each given cost reducing level x_s, x_m and wholesale price w, $p^*(w|x_s, x_m)$, as the solution of equation (2), is the optimal retail price in stage 2 for the manufacturer.

Proof: For any given x_s, x_m, w, the manufacturer will receive negative profit if he chooses his retail price $p < c_m - w$. Moreover, due to Assumption 1, $p = \infty$ would not be optimal. So, the optimal retail price must be an interior. Since $\pi_m(p)$ is concave due to Assumption 1, we then complete the proof. ■

With the lemma above, we let

$$D^*(w|x_s, x_m) := D(p^*(w|x_s, x_m))$$

be the demand faced by the manufacturer when he chooses $p^*(w|x_s, x_m)$ as his retail price, provided the given cost reducing level x_s, x_m and wholesale price w. Thus, by substituting $D^*(w|x_s, x_m)$ into the profit function of the supplier in Stage 2, the supplier's problem to choose w to maximize his profit, is as follows,

$$\max_w \pi_s(w) = (w - c_s + x_s + \beta(x_s)x_m)D^*(w|x_s, x_m) - k_s(x_s)$$

For this problem, we have the following result.

Lemma 2: Suppose $\lim_{w\to\infty}(w - c_s)D^*(w|x_s, x_m) = 0$. Then, the optimal solution w^* of $\max_w \pi_s(w)$ is finite, and we define the gross profit for each player i by:

$$\Phi_s(w) = (w - c_s + x_s + \beta(x_s)x_m)D^*(w|x_s, x_m)$$

and

$$\Phi_m(w) = (p^*(w|x_s, x_m) - c_m - w)D^*(w|x_s, x_m)$$

further have

$$\Phi_m(p^*(w^*|x_s, x_m)) = \gamma(w^*)\Phi_s(w^*)$$

$$\gamma(w^*) = \frac{\partial p^*(w^*|x_s, x_m)}{\partial w}.$$

Proof: First, we say that $D^*(w|x_s, x_m)$ is differentiable in w. In fact, since that $p^*(w|x_s, x_m)$ is the solution of the first order condition (2) and that $D(p)$ is twice differentiable due to Assumption 1, we know that $p^*(w|x_s, x_m)$ is differentiable in w. Thus, $D^*(w|x_s, x_m)$ is differentiable in w.

We denote by w^* the optimal solution of $\max_w \pi_s(w|x_s, x_m)$. Obviously, $w^* \geq c_s + x_s + \beta(x_s)x_m$, otherwise the supplier will received negative profit. Moreover, due to the given supposition in the lemma1, we know that w^* is finite and thus satisfies the first order condition as follows,

$$\phi'_s(w^*) = D(p^*(w^*|x_s, x_m)) + (w^* - c_s + x_s + \beta(x_s)x_m)\frac{\partial D(p^*(w^*|x_s, x_m))}{\partial w} = 0. \tag{3}$$

This implies that

$$w^* - c_s + x_s + \beta(x_s)x_m = -\frac{D(p^*(w^*|x_s, x_m))}{\frac{\partial D(p^*(w^*|x_s, x_m))}{\partial w}}. \tag{4}$$

Hence, we have

$$\phi_s(w^*) = -\frac{D^2(p^*(w^*|x_s,x_m))}{\frac{\partial D(p^*(w^*|x_s,x_m))}{\partial w}}. \quad (5)$$

On the other hand, we know that $p^*(w^*|x_s,x_m)$ is the solution of the corresponding first order condition (2):

$$D(p^*(w^*|x_s,x_m)) + p^*(w^*|x_s,x_m) - w^* - c_m D'_p(p^*(w|x_s,x_m)) = 0. \quad (6)$$

So,

$$\phi_m(p^*(w^*|x_s,x_m)) = -\frac{D^2(p^*(w^*|x_s,x_m))}{D'_p(p^*(w|x_s,x_m))}. \quad (7)$$

Now,

$$\frac{\partial D(p^*(w^*|x_s,x_m))}{\partial w} = D'_p(p^*(w^*|x_s,x_m))\frac{\partial p^*(w^*|x_s,x_m)}{\partial w}. \quad (8)$$

Therefore,

$$\Phi_m(p^*(w^*|x_s,x_m)) = \frac{\partial p^*(w^*|x_s,x_m)}{\partial w}\Phi_s(w^*) = \gamma(w^*)\pi_s(w^*).$$

This completes the proof. ■

A. Special demand functions

Assumption 1 is true for most usual demand functions, such as the the exponential demand function shown in the following.

Exponential demand: This demand is given by

$$D(p) = D_0 e^{-\lambda p}$$

where, D_0 is the total market size.

Proposition 1: For the exponential demand function, we have

$$\Pi_m(x) = -k_m(x_m) + \frac{D_0}{\lambda}\exp(-2-\lambda(c_s+c_m-x_s-\beta(x_s)x_m))$$

$$\Pi_s(x) = -k_s(x_s) + \frac{D_0}{\lambda}\exp(-2-\lambda(c_s+c_m-x_s-\beta(x_s)x_m))$$

where $\phi_m(x_s,x_m) = \phi_s(x_s,x_m)$

The proof of Proposition 1 is so easy that we don't give.The above proposition suggests that in the exponential demand case, the manufacture gets the same gross profit as the supplier.

IV. First Stage Effort Decision

A. First stage simultaneous effort decision

In this section, we discuss the existence of Nash equilibriums of the first case:The manufacturer and the component supplier make simultaneous cost reducing level decision.

Assumption 2: Given any first stage manufacturer and component producer'investment effort chosen from a compact space $x_i \in \prod[\underline{x_i}, \bar{x_i}] = E, (i = s, m)$.and in the second stage,the component producers' pricing game results in the gross profit $\phi_s(x_s,x_m)$,Also, the manufacturer's gross profit $\phi_m(x_s,x_m) = \rho\phi_s(x_s,x_m)$,where ρ is a constant for any given x_s, x_m. Suppose that $k_i(x_i)$ and $\beta(x_s)$ are continuous functions in a compact space E,Then, $\phi_s(x_s,x_m)$,$\phi_m(x_s,x_m)$ are continuous function.

With the assumption above the following lemma:

Lemma 3: The simultaneous investment effort game among the supplier and the manufacturer has a Nash Equilibrium in a compact space E.

Proof: Proof of lemma3. The utility function for the component producer and the manufacturer are given by:

$$\pi_s(x_s,x_m) = \phi_s(x_s,x_m) - k_s(x_s)$$

$$\pi_m(x_s,x_m) = \rho\phi_s(x_s,x_m) - k_m(x_m)$$

Let $\gamma_s = 1$ and $\gamma_m = \frac{1}{\rho}$.Consider the potential function:

$$\Psi(x_s,x_m) = \phi_s(x_s,x_m) - \frac{k_m(x_m)}{\gamma_m} - \frac{k_s(x_s)}{\gamma_s}$$

For any firm of the component producer and the manufacturer i,for instance $i = s$, let the other firm'(the manufacturer')strategy be given $x_{-i} = x_m$.For all $(x_s,x_m)$and $(x'_s,x_m) \in E$.such that $x_s \neq x'_s$.We have :

$$\Psi(x_s,x_m) \geq \Psi(x'_s,x_m) \quad (9)$$

$$\Leftrightarrow \quad \phi_s(x_s,x_m) - \phi_s(x'_s,x_m) - \left(\frac{k_s(x_s)}{\gamma_s} - \frac{k_s(x'_s)}{\gamma_s}\right) \geq 0 \quad (10)$$

$$\Leftrightarrow \quad \pi_s(x_s,x_m) \geq \pi_s(x'_s,x_m) \quad (11)$$

$$\Psi(x_s,x_m) \geq \Psi(x'_s,x_m) \quad (12)$$

$$\Leftrightarrow \quad \phi_s(x_s,x_m) - \phi_s(x'_s,x_m) - \left(\frac{k_s(x_s)}{\gamma_s} - \frac{k_s(x'_s)}{\gamma_s}\right) \geq 0 \quad (13)$$

$$\Leftrightarrow \quad \pi_s(x_s,x_m) \geq \pi_s(x'_s,x_m) \quad (14)$$

Therefore, the maximizer of function $\Psi(x_s,x_m)$is a Nash Equilibrium for the simultaneous effort game among the component producer and the manufacturer . The maximizer exists since $\Psi(x_s,x_m)$ is continuous and the strategy space$E$ is compact. This completes the proof. ■

Although we know the equilibrium exists from above lemma , we do not have any further information about the system behavior under the equilibrium. Next, we gain some insight into the property of the equilibrium point.

Proposition 2: For the exponential demand case, and any subset $S \subseteq [s,m]$, function $f(x_s,x_m) = \Sigma_{i\in S}\pi_i(x_s,x_m)$ is supermodular in x.

Proof: To show that f is supermodular in x, we have to check that:

$$\frac{\partial''\phi_s(x_s,x_m)}{\partial x_s \partial x_m} = R[\lambda\beta(x_s)(1+\beta'(x_s)x_m) + \beta'(x_s)] \geq 0 \quad (15)$$

Where

$$R = D_0\exp(-2-\lambda(c_s+c_m-x_s-\beta(x_s)x_m) > 0$$

Therefore, $\phi_s(x_s,x_m)$ is supermodular in x for exponential demand functions.So $f(x)$ is supermodular in x.

This completes the proof. ∎

From above proposition ,we can know the game of the supplier and manufacturer is a a supermodular game, because $x_i$is a compact subset E, and π_i has increasing differences in (s_i, s_{-i}).

We also have the follow proposition:

Proposition 3: For the exponential demand case, π_i is non-decreasing in x_j , for $\forall j \neq i$.

Proof: For exponential demand function, the result comes directly from

$$\frac{\partial \phi_s(x_s, x_m)}{\partial x_s} \geq 0$$

. This completes the proof. ∎

Since the simultaneous game of the supplier and manufacturer is a supermodular game, so we have the follow proposition.

Proposition 4: The supplier and the manufacturer make simultaneous decisions on x_i.We have:

1) The Nash Equilibrium in E forms a compact sublattice: if x_E^1 and $x_E^2$are equilibrium points, $x_E^1 \wedge x_E^2$ and $x_E^1 \vee x_E^2$ are also Nash equilibrium points.
2) There exists a smallest and largest Nash equilibrium points x_E^L and x_E^U, such that $x_E^L \leq x_E^U$ for all $\in s, m$.
3) Among all the Nash equilibriums, every player prefer the largest Nash equilibrium x_E^U.

Proof:

1) and 2) follows directly from the fact that $\pi_i(x_s, x_m)$ is a supermodular function on $(x_s, x_m)$and E is a compact sublattice.

To prove part 3), let (x_i^*, x_i^*) be an equilibrium point,for $i = sorm$.

From the fact π_i is increasing in j,then

$$\pi_i(x_i^*, x_{-i}^*) \leq \pi_i(x_i^*, x_{-i}^U)$$

Since x_i^U is the best response givenx_{-i}^U iso

$$\pi_i(x_i^*, x_{-i}^U) \leq \pi_i(x_i^U, x_{-i}^U)$$

That is to say

$$\pi_i(x_i^*, x_{-i}^*) \leq \pi_i(x_i^U, x_{-i}^U)$$

This completes the proof. ∎

Various cooperation behaviors between the supplier and the manufacturer is our focus,so we don't discuss the uniqueness of equilibrium.

B. First stage cooperative effort decision

In this subsection ,we suppose the supplier and manufacturer cooperate together in the first stage effort decision while keeping the second stage in price competition,That is different from integrated supply chain which cooperate together effort and pricing decision.

Lemma 4: For exponential demand $F(t, x) = t\phi_s(x) - k_s(x_s) - k_m(x_m)$ has increasing difference in (t, x) when $t > 0$.

Proof: To show that F has increasing difference in (t, x), it is enough to show that $F$is supermodular in (t, x), we have to check that:

$$\frac{\partial'' t\phi_s(x_s, x_m)}{\partial x_s \partial x_m} = tR[\lambda\beta(x_s)(1 + \beta'(x_s)x_m) + \beta'(x_s)] \geq 0$$

Therefore, $\phi_s(x_s, x_m)$ is supermodular in x for exponential demand functions.So $F(x)$ is supermodular in x.

$$\frac{\partial'' t\phi_s(x_s, x_m)}{\partial x_s \partial t} = tR[\lambda\beta(x_s)(1 + \beta'(x_s)x_m) + \beta'(x_s)] \geq 0$$

So F has increasing difference in (t, x)

This completes the proof. ∎

The above lemma will be used in the proof of the following proposition.

Proposition 5: For exponential demand , Let $x^c = \max \arg\max_{x_s, x_m}(\pi_s(x_s, x_m) + \pi_m(x_s, x_m))$ .and χ^* be the set of all the Nash Equilibria of investment effort game among all the firms. Then$\exists x^* \in \chi^*$ such that $x^c \succ x^*$ and $\phi_i(x^c) > \phi_i(x^*)$ for $i = s, m$

Proof: Let $x^t = \max \arg\max_x t\pi_s(x) - k_s(x_s) - k_m(x_m)$ .if $t = 1$, $\pi_s(x) - k_s(x_s) - k_m(x_m)$ is a potential function of the investment effort game. Also note that the function $t\pi_s(x) - k_s(x_s) - k_m(x_m)$is a supermodular function in$x$ and has increasing difference in (t, x), hence x^t is increasing with t.On the other hand, when cooperate in the first stage , $t = 2$ and $x^2 = x^c = \max \arg\max_x 2\pi_s(x) - k_s(x_s) - k_m(x_m)$.

Hence $x^c = x^2 \succeq x^1$ and $\pi_s(x^c) \geq \pi_s(x^1)$

This completes the proof. ∎

The above lemma shows that if the demand is exponential and the firms coordinate together in investment effort to maximize the system profit, every firm will set a higher effort and get higher gross profit (with larger demand) compared with competition system.

In the above proposition of the subsection, based on the approach by construction of potential function, and supermodular theory, we find Cooperation in effort investment will lead to a higher effort and higher profit for every player relative simultaneous effort investment.

V. Conclusion

In this paper, a two echelon assembly system involving pricing and effort investment decisions is investigated. Demand in this model is exponential demand functions. In the first stage ,the manufacturer helps the suppler to reduce cost .In the second stage the suppliers sells the component to the manufacturer with a price only contract. The unique equilibrium leads to a constant proportion of the manufacturer's gross profit to the supplier. Taking the unique price competition as a response, we analyze the effort investment decision in the first stage. Based on supermodular game theory,by construction of potential function, we find that in the effort decision stage, Nash equilibrium always exists for simultaneous competition. The players' profit is supermodular in effort and increasing with the other players' effort. Thus we know that the equilibrium set consists of a largest and a smallest equilibrium such that in the largest equilibrium every one get most of the profit. We also find that coordination in effort investment in the first stage will lead to a higher effort and higher profit for every player.

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