Study on cooperative behaviors in a supply chain : based on supermodular game theory*

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Abstract

We consider a supply chain with a manufacturer and a retailer, the manufacturer helps his supplier to invest in cost-reducing innovation. Based on the theory of supermodular games,we analyze three different co-operative behaviors .The existence of pure strategy Nash equilibrium was proved by construction of potential function. We also find that if some one makes an effort decision earlier followed by the others , every one can benefit.Coordination in effort decision will lead to a higher effort and higher gross profit for every player.

Keywords: supply chain, compete, cooperate, cooperative behaviors, supermodular game

1 Introduction

1.1 Motivation

More and more, businesses are counting on their suppliers to lower costs, improve quality, and develop innovations faster than their competitors' suppliers can (Jeffrey and Thomas (2004) [14]).Automakers Toyota and Honda have struck successful partnerships with some of the same suppliers and help their vendors continually improve their processes. (Helper, 1991 [6]; Lincoln, 1998 [9]; McLaren, 1999 [10]) . Harabi (1998) [5]found 84% of innovative enterprise contain their customers or suppliers through an empirical study . Licht (1994) [8] By studying the research cooperation of six European country,also find the most common form of cooperation contain customers or suppliers.Related research can see Von Hippel (1986) [12]VanderWerf (1992) [11].

On the other hand, the theory of supermodular games, introduced by Topkis (1978, 1979) [15], [16]and further developed by Vives (1990) [17], Milgrom and Roberts (1990) [20] . Milgrom and Shannon (1991) [21] provides an elegant unifying approach based on lattice-theoretic arguments, to characterize games with strategic complementarities. See Topkis (1998) [22]Vives (1999) [18],and Vives (2005) [19] for detailed accounts of the theory and application.

Main objective of this paper is to analyze the effects of various behaviors between the supplier and the manufacturer(eg:simultaneous effort decision;non simultaneous effort decision;coordinate effort decision) on the optimal solution and channel profit based on supermodular games theory .

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1.2 Literature review

There are three streams of research related to our paper.First,most of the literatures are focused on horizontal cooperation.Namely cooperative enterprises are compete in the same product market.D'Aspremont and Jacquemin (1988) [3], in their model, the two firms cooperate in the first stage (R&D collaboration) while compete in the second stage (market competition).They find larger spillover effect will promote the larger R&D spending .Kamien (1992) [7] comparing the different collaborative R&D scenariosfind the cartelized RJV (research joint venture) is the best R&D scenario.Thena great deal of progress has been made toward understanding horizontal cooperation

Several authors have considered how specific contractual forms between buyers and sellers in a supply chain affect their incentives to invest in either demand enhancement, cost reduction, or capacity .Kim(2000) [29] studies that the manufacturer coordinates its supplier's innovation that can eventually lead to supply cost reduction. Gilbert and Cvsa (2003) [4] discuss how to use strategic commitment to price to stimulate downstream innovation in a supply chain. Tian and Pu (2004) [31] study the problem whether the upstream and downstream firms can sign a contract to increase the upstream cost -reducing expenditure. Banerjee and Lin (2001) [1] examine the incentives of firms to form vertical research joint ventures. Banerjee and Lin (2003) [2] analyze the incentives for cost-reducing R&D by downstream firms in a two-tier market structure. Plambeck and Taylor(2006) [28] investigate how supply contracts can be designed under different breach remedies to create incentives for the buyer and the suppliers to make first best investments in R&D and capacity respectively. Krishnan et al. (2004) [30] examine coordinating contracts of a decentralized supply chain in which the retailer can choose promotional effort. For a channel rebate contract, Taylor (2002) [26] employs a simple model in which a retailer makes quantity and effort decisions and then observes the demand. Furthermore, Taylor (2006) [27] investigates the impact of the retailer's sales effort on the manufacturer's sale-timing decision, in which the payment to the manufacturer depends on the retailer's order quantity.

Our work is also related to the literature on some topics about co-invest of members of supply chain in either demand enhancement(advertising),or cost reduction. Compared with horizontal cooperation , there are few studies on the vertical cooperation. Huang and Li (2001) [23] , Huang et al. (2002) [24] , Li et al. (2002) [25] , and Yue et al. (2006) [13] have studied the coordination of cooperative advertising in a two-level supply chain under the price independent/dependent demand.

Most of the literatures considering investment in either demand enhancement, cost reduction, or capacity in a supply chain ,they have focuse on the Stackelberg structure: a upstream firm acts as a Stackelberg leader and a downstream firm is a Stackelberg follower. They all considered how specific contractual forms between buyers and sellers to coordinate their incentives to invest. In this paper,the manufacturer helps the supplier to reduce cost in the first stage, while in the second stage, the supplier offers a wholesale price at first. As a response, the manufacturer decides retail price.we consider special demand function :linear and exponential demand .We use a new tool of supermodular game theory to analysis the cooperative behaviors: simultaneous effort decision ;non simultaneous effort decision ;coordinate effort decision .

Based on the theory of supermodular games by construction of potential function, Nash equilibrium is shown to exist. we can identify the supermodularity of the players profit function in their effort decision. We show that the competition with a leader can benefit every one. Finally ,we prove the coordination in effort investment will lead to a higher effort and higher profit for every player. Our results can be easily extended to the case with stochastic demand if the stochastic factor is realized before price decision. However, if the manufacturer faces a news vendor problem with a price decision, although the existence of Nash equilibrium still exists, the property of system behavior is then hard to analyze. This would be an interesting open question for future research.

The paper is organized as follows: Section 2 describes the assumptions and notations in our model. Section 3 shows the pricing game in the stage 2. Section 4 examines the effects of the supplier and the manufacturer different competitive behaviors on the optimal solutions of the models based supermodular game. Section 5 provides a summary of the results.

2 Model and notations

The supply chain considered here consists of a single manufacturer who sells the final product that consists of one component provided by a single component producer in a single selling season.

The notations are as follows:

- c_s: the supplier's original unit cost ;
- c_m: the manufacturer's unit cost
- x_s: the supplier's cost reducing effort level, and the corresponding cost is $k_s(x_s)$;
- x_m: the manufacturer's cost reducing effort level, and the corresponding cost is $k_m(x_m)$;
- $\beta(x_s)$: the supplier's absorptive capacity which is a function of its accumulated investment $x_s, \beta(x_s) : [0, +\infty) \to [0, 1]$ and $\beta(0) = 0, \beta(\infty) = 1, \beta'(x_s) \geq 0, \beta''(x_s) \leq 0$;
- w: the wholesale price determined by the supplier;
- p: the manufacture's retail price.

Demand $D(p)$ is a function of the market price p set by the manufacturer.

stage1: The manufacturer will help the supplier to reduce cost in the first stage.The manufacturer and the supplier select cost reducing level $x_i, i = s, m.$ and the cost functions of supplier and manufacturer is $k_s(x_s)$ and $k_m(x_m)$ respectively.When the innovation succeeds, the supplier's unit cost changes into $(c_s - x_s - \beta(x_s)x_m)$.In addition, it is obvious that the ex post unit cost cannot exceed c_s.

stage2: After the investments in cost -reducing effort are observed by all the players, the component producer chooses a wholesale price w to maximize his expected profit.Based on the wholesale price of the component and production cost, the manufacturer selects the production quantity q so as to maximize his own expected profit.

Keeping price-competition in the second stage, we consider three different scenarios for the first stage decision based on the sequence of the decisions and the objective of the decision makers:

Case 1: The manufacturer and the component supplier make their cost reducing level decision simultaneously to maximize their own individual profit.

Case 2: Any one of the firms announces his cost reducing level first and then the other follows to maximize their own individual profit.

Case 3: The manufacturer and the component supplier decide on their joint cost reducing level strategy together to maximize the total system profit.

3 Price competition in the second stage

We should first solve the manufacturer's problem in stage 2 for any given x_s, x_m and w. The manufacturer's problem is to choose p to maximize his profit as follows,

$$\max_p \pi_M(p) = (p - c_m - w)D(p) - k_m(x_m) \tag{1}$$

The first order condition for maximizing $\pi_m(p)$ is as follows,

$$\pi'_m(p) = D(p) + (p - c_m - w)D'_p(p) = 0. \tag{2}$$

We denote its solution by $p^*(w|x_s, x_m)$. Since

$$\pi''_m(p) = 2D'_p(p) + (p - c_m - w)D''_p(p). \tag{3}$$

In order to ensure the optimality of $p^*(w|x_s, x_m)$, we make the following assumption.

Assumption 1 *(1) $D(p)$ is twice differentiable, decreasing and concave in p for each given x_s, x_m.*
(2) The domain of retail price p is $[0, \infty)$ and $\lim_{p\to\infty} pD(p) = 0$.

In the above assumption, Part (1) is usual in the literature. For Part (2), the former is for convenience. In fact, if p is limited in a finite region then we can extend its domain to $[0, \infty)$ by letting $D(p) = 0$ for all other p; While the later is just to ensure that we have a finite optimal retail price for the manufacturer. Hereafter, we assume that Assumption 1 is true. With this assumption, we have obviously the following result.

Lemma 3.1 *For each given cost reducing level x_s, x_m and wholesale price w, $p^*(w|x_s, x_m)$, as the solution of equation (2), is the optimal retail price in stage 2 for the manufacturer.*

Proof. For any given x_s, x_m, w, the manufacturer will receive negative profit if he chooses his retail price $p < c_m - w$. Moreover, due to Assumption 1, $p = \infty$ would not be optimal. So, the optimal retail price must be an interior. Since $\pi_m(p)$ is concave due to Assumption 1. This completes the proof.

With the lemma above, we let

$$D^*(w|x_s, x_m) := D(p^*(w|x_s, x_m))$$

be the demand faced by the manufacturer when he chooses $p^*(w|x_s, x_m)$ as his retail price, provided the given cost reducing level x_s, x_m and wholesale price w. Thus, by substituting $D^*(w|x_s, x_m)$ into the profit function of the supplier in Stage 2, the supplier's problem to choose w to maximize his profit, is as follows,

$$\max_w \pi_s(w) = (w - c_s + x_s + \beta(x_s)x_m))D^*(w|x_s, x_m) - k_s(x_s). \tag{4}$$

For this problem, we have the following result.

Lemma 3.2 *Suppose $\lim_{w\to\infty}(w - c_s)D^*(w|x_s, x_m) = 0$. Then, the optimal solution w^* of $\max_w \pi_s(w)$ is finite, and we define the gross profit for each player i by:*

$$\Phi_s(w) = (w - c_s + x_s + \beta(x_s)x_m)D^*(w|x_s, x_m)$$

and

$$\Phi_m(w) = (p^*(w|x_s, x_m) - c_m - w)D^*(w|x_s, x_m)$$

further have

$$\Phi_m(p^*(w^*|x_s, x_m)) = \gamma(w^*)\Phi_s(w^*)$$

$$\gamma(w^*) = \frac{\partial p^*(w^*|x_s, x_m)}{\partial w}.$$

Proof. First, we say that $D^*(w|x_s, x_m)$ is differentiable in w. In fact, since that $p^*(w|x_s, x_m)$ is the solution of the first order condition (2) and that $D(p)$ is twice differentiable due to Assumption 1, we know that $p^*(w|x_s, x_m)$ is differentiable in w. Thus, $D^*(w|x_s, x_m)$ is differentiable in w.

We denote by w^* the optimal solution of $\max_w \pi_s(w|x_s, x_m)$. Obviously, $w^* \geq c_s + x_s + \beta(x_s)x_m$, otherwise the supplier will received negative profit. Moreover, due to the given supposition in the lemma1, we know that w^* is finite and thus satisfies the first order condition as follows,,

$$\phi_s'(w^*) = D(p^*(w^*|x_s, x_m)) + (w^* - c_s + x_s + \beta(x_s)x_m)\frac{\partial D(p^*(w^*|x_s, x_m))}{\partial w} = 0. \tag{5}$$

This implies that

$$w^* - c_s + x_s + \beta(x_s)x_m = -\frac{D(p^*(w^*|x_s, x_m))}{\frac{\partial D(p^*(w^*|x_s, x_m))}{\partial w}}. \tag{6}$$

Hence, we have due to (4)

$$\phi_s(w^*) = -\frac{D^2(p^*(w^*|x_s, x_m))}{\frac{\partial D(p^*(w^*|x_s, x_m))}{\partial w}}.. \quad (7)$$

On the other hand, we know from Lemma 3.1 that $p^*(w^*|x_s, x_m)$ is the solution of the corresponding first order condition (2):

$$D(p^*(w^*|x_s, x_m)) + (p^*(w^*|x_s, x_m) - w^* - c_m D'_p(p^*(w|x_s, x_m)). \quad (8)$$

So,

$$\phi_m(p^*(w^*|x_s, x_m)) = -\frac{D^2(p^*(w^*|x_s, x_m))}{D'_p(p^*(w|x_s, x_m))}. \quad (9)$$

Now,

$$\frac{\partial D(p^*(w^*|x_s, x_m))}{\partial w} = D'_p(p^*(w^*|x_s, x_m))\frac{\partial p^*(w^*|x_s, x_m)}{\partial w}. \quad (10)$$

Therefore,

$$\Phi_m(p^*(w^*|x_s, x_m)) = \frac{\partial p^*(w^*|x_s, x_m)}{\partial w}\Phi_s(w^*) = \gamma(w^*)\pi_s(w^*).$$

This completes the proof.

3.1 Special demand functions

Assumption 1 is true for most usual demand functions, such as the linear demand and the exponential demand shown in the following.

Linear demand. This demand is given by

$$D(p) = D_0 - kp$$

Here, $D_0 > 0$ is the total market.

Exponential demand: This demand is given by

$$D(p) = D_0 e^{-\lambda p}$$

where, D_0 is the total market size.

Proposition 1 *Suppose* $D_0 \geq k(c_s + c_m)$.

1. *For linear demand function, we have*

$$\pi_m(x_s, x_m) = \frac{[D_0 - k(c_s + c_m + x_s + \beta(x_s)x_m)]^2}{16} - k_m(x_m),$$

$$\pi_s(x_s, x_m) = \frac{[D_0 - k(c_s + c_m + x_s + \beta(x_s)x_m)]^2}{8} - k_s(x_s),$$

where $\phi_m(x_s, x_m) = \frac{1}{2}\phi_s(x_s, x_m)$.

2. *For the exponential demand function, we have*

$$\Pi_m(x_s,x_m) = \frac{D_0}{\lambda}\exp(-2-\lambda(c_s+c_m-x_s-\beta(x_s)x_m)-k_m(x_m).$$

$$\Pi_s(x_s,x_m) = \frac{D_0}{\lambda}\exp(-2-\lambda(c_s+c_m-x_s-\beta(x_s)x_m)-k_s(x_s)$$

where $\phi_m(x_s,x_m) = \phi_s(x_s,x_m)$

The proof of Proposition 1 is so easy that we don't give.The above proposition suggests that in the linear demand function case, the manufacture gets half of the supplier's profit, while in the exponential demand case, the manufacture gets the same profit as the supplier.

4 First Stage Effort Decision

4.1 Simultaneous effort decision

In this section, we discuss the existence of Nash equilibriums of the first case:the manufacturer and the component supplier make simultaneous cost reducing level decision.

Assumption 2 *Given any first stage manufacturer and component producer'investment effort chosen from a compact space* $x_i \in \prod[\underline{x_i},\bar{x_i}] = E, i = s, m$*.and in the second stage,the component producers' pricing game results in the gross profit* $\phi_s(x_s,x_m)$*,Also, the manufacturer's gross profit* $\phi_m(x_s,x_m) = \rho\phi_s(x_s,x_m)$*,where* ρ *is a constant for any given* x_s,x_m*. Suppose that* $k_i(x_i)$ *and* $\beta(x_s)$ *are continuous functions in a compact space E,Then,* $\phi_s(x_s,x_m)$*,*$\phi_m(x_s,x_m)$ *are continuous function.*

With the assumption we have the following lemma:

Lemma 4.1 *The simultaneous investment effort game among the supplier and the manufacturer has a Nash Equilibrium in a compact space* E*.*

Proof. The utility function for the component producer and the manufacturer are given by:

$$\pi_s(x_s,x_m) = \phi_s(x_s,x_m) - k_s(x_s)$$

$$\pi_m(x_s,x_m) = \rho\phi_s(x_s,x_m) - k_m(x_m)$$

Let $\gamma_s = 1$ and $\gamma_m = \frac{1}{\rho}$.Consider the potential function:

$$\Psi(x_s,x_m) = \phi_s(x_s,x_m) - \frac{k_m(x_m)}{\gamma_m} - \frac{k_s(x_s)}{\gamma_s}$$

For any firm of the component producer and the manufacturer i,for instance $i = s$, the other firm'(the manufacturer')strategy be given $x_{-i} = x_m$.For all $(x_s,x_m)$and $(x'_s,x_m) \in E$.such that $x_s \neq x'_s$.We have :

$$\begin{aligned}\Psi(x_s,x_m) \geq \Psi(x'_s,x_m) \quad &\Leftrightarrow \quad \phi_s(x_s,x_m) - \phi_s(x'_s,x_m) - (\frac{k_s(x_s)}{\gamma_s} - \frac{k_s(x'_s)}{\gamma_s}) \geq 0 \\ &\Leftrightarrow \quad \pi_s(x_s,x_m) \geq \pi_s(x'_s,x_m).\end{aligned}$$

Therefore, the maximizer of function $\Psi(x_s, x_m)$ is a Nash Equilibrium for the simultaneous effort game among the component producer and the manufacturer . The maximizer exists since $\Psi(x_s, x_m)$ is continuous and the strategy space E is compact. This completes the proof.

Although we know the equilibrium exists from above lemma , we do not have any further information about the system behavior under the equilibrium. Next, we gain some insight into the property of the equilibrium point.

Proposition 2 *For the linear demand case and the exponential demand case, and any subset $S \subseteq [s, m]$, function $f(x_s, x_m) = \Sigma_{i \in S} \pi_i(x_s, x_m)$ is supermodular in x.*

Proof. To show that f is supermodular in x, we have to check that:

For linear demand function :

$$f(x^1 \vee x^2) + f(x^1 1 \wedge x^2) - f(x^1) - f(x^2) = \frac{1}{2}[\phi_s(x^1 \vee x^2) + \phi_s(x^1 1 \wedge x^2) - \phi_s(x^1) - \phi_s(x^2)] \geq 0$$

For exponential demand function:

$$f(x^1 \vee x^2) + f(x^1 1 \wedge x^2) - f(x^1) - f(x^2) = \phi_s(x^1 \vee x^2) + \phi_s(x^1 1 \wedge x^2) - \phi_s(x^1) - \phi_s(x^2) \geq 0$$

For linear demand function ,we have

$$\frac{\partial'' \phi_s(x_s, x_m)}{\partial x_s \partial x_m} = \frac{k\beta(x_s)(1 + \beta'(x_s)x_m) + \beta'(x_s)[D_0 - k(c_s + c_m - x_s - \beta(x_s)x_m)]}{4} \geq 0$$

For exponential demand function:

$$\frac{\partial'' \phi_s(x_s, x_m)}{\partial x_s \partial x_m} = D_0 \exp(-2 - \lambda(c_s + c_m - x_s - \beta(x_s)x_m)[\lambda\beta(x_s)(1 + \beta'(x_s)x_m) + \beta'(x_s)] \geq 0$$

Therefore, $\phi_s(x_s, x_m)$ is supermodular in x for linear, and exponential demand functions.So $f(x)$ is supermodular in x. This completes the proof.

From above proposition ,we can know the game of the supplier and manufacturer is a a supermodular game, because x_i is a compact subset e, and π_i has increasing differences in (s_i, s_{-i}).

We also have the follow proposition:

Proposition 3 *For the linear demand case and the exponential demand case, π_i is nondecreasing in x_j , for $\forall j \neq i$.*

Proof. For linear demand function and exponential demand function, the result comes directly from

$$\frac{\partial \phi_s(x_s, x_m)}{\partial x_s} \geq 0$$

. This completes the proof.

Proposition 4.1will be used in the following two subsection.

Since the simultaneous game of the supplier and manufacturer is a supermodular game, so we have the follow proposition.

Proposition 4 *The supplier and the manufacturer make simultaneous decisions on x_i.We have:*

1. *The Nash Equilibrium in E forms a compact sublattice: if x_E^1 and x_E^2 are equilibrium points, $x_E^1 \wedge x_E^2$ and $x_E^1 \vee x_E^2$ are also Nash equilibrium points.*

2. *There exists a smallest and largest Nash equilibrium points x_E^L and x_E^U, such that $x_E^L \le x_E^U$ for all $\in s, m$.*

3. *Among all the Nash equilibriums, every player prefer the largest Nash equilibrium x_E^U.*

Proof. 1) and 2) follows directly from the fact that $\pi_i(x_s, x_m)$ is a supermodular function on $(x_s, x_m)$and E is a compact sublattice and lemma 4.2.2 in [44].

To prove part 3), let (x_i^*, x_i^*) be an equilibrium point,for $i = s, m$.

From the fact π_i is increasing in j,then

$$\pi_i(x_i^*, x_{-i}^*) \le \pi_i(x_i^*, x_{-i}^U)$$

Since x_i^U is the best response givenx_{-i}^U iso

$$\pi_i(x_i^*, x_{-i}^U) \le \pi_i(x_i^U, x_{-i}^U)$$

That is to say

$$\pi_i(x_i^*, x_{-i}^*) \le \pi_i(x_i^U, x_{-i}^U)$$

This completes the proof.

Various cooperation behaviors between the supplier and the manufacturer is our focus,so we don't discuss the uniqueness of equilibrium.

4.2 Non simultaneous effort decision

In this subsection, we analysis the second case:non simultaneous effort decision. There are two possible: the supplier first make decision followed by the manufacturer,or, conversely . In our the following discussion ,We do not differentiate the two scenario.

We have the following two proposition:

Lemma 4.2 *For a compact space $x_i \in \prod[\underline{x_i}, \bar{x}_i] = E, i = s, m \forall k \in [s, m]$. Let $x'_k > x''_k$,and $x'_j = Y_j(x'_k)$,$x''_j = Y_j(x''_k)$, we have $x'_j = x''_j$.Besides $\pi_j(x'_k, x'_j) > \pi_j(x''_k, x''_j)$. where $Y_j(x_k)$ is the player $j, j \ne k$ best response ,when k makes decision first ,and is a function of x_k*

Proof.

Since x'_j is the equilibrium for x_k, we have:

$$\pi_j(x'_k, x'_j) \le \pi_j(x'_k, x''_j)$$

From the fact π_j is nondecreasing with respect to x_k,then

$$\pi_j(x'_k, x''_j) \ge \pi_j(x''_k, x''_j)$$

This completes the proof.

Lemma 4.3 *For a compact space $x_i \in \prod[\underline{x_i}, \bar{x}_i] = E, i = s, m$denote the largest equilibrium among in the simultaneous investment effor game among players be x^* .Now player$k (\forall k \in [s, m])$ deviate from x_k^*followed by player$j, (j \ne k)$ .Let $x'_k = \max \arg \max_{x_k} \pi_k(x_k Y_j(x_k))$,and $x'_j = Y_j(x_k)$.we have $x_j^* < x'_j$,$x_k^* < x'_k$.Besides $\pi_j(x'_k, x'_j) > \pi_j(x_k^*, x_j^*)$,and $\pi_k(x'_k, x'_j) > \pi_k(x_k^*, x_j^*)$*

Proof.

Clearly, if $x'_k \geq x^*_k$, then by lemma 4.3, $x'_j \geq x^*_j$.Otherwise, suppose $x'_k < x^*_k$, then $x'_j < x^*_j$.
We assume $x'_k < x^*_k$,we have :

$$\pi_k(x'_k, x'_j) < \pi_k(x^*_k, x^*_j) \tag{11}$$

However, note that:

$$\pi_k(x^*_k, x^*_j) \leq \pi_k(Y(x^*_j), x^*_j) \leq \pi_k(Y(x^*_j), x'_j) \leq \pi_k(Y(x'_j), x'_j) = \pi_k(x'_k, x'_j) \tag{12}$$

Which lead to a contradiction with (11).Therefore ,$x'_k \geq x^*_k$ and by lemma 4.3, $x'_j \geq x^*_j$
by lemma 4.3,

$$\pi_j(x'_k, x'_j) \geq \pi_k(x^*_k, x^*_j).$$

This completes the proof.

Above two propositions suggest that if one firm makes an investment effort decision first followed by the other firm investment decision game, every firm can benefit, and the resulting equilibrium investment effort is higher.

4.3 Coordinate effort decision

In the above two subsections ,simultaneous and Stackelberg structure game are discussed . In this subsection ,we suppose the supplier and manufacturer cooperate together in the first stage effort decision while keeping the second stage in price competition,That is different from integrated supply chain which cooperate together effort and pricing decision.

Lemma 4.4 *For exponential demand $F(t, x) = t\phi_s(x) - k_s(x_s) - k_m(x_m)$ has increasing difference in (t, x) when $t > 0$.*

Proof. To show that F has increasing difference in (t, x), it is enough to show that F is supermodular in (t, x).in x, We have :

$$\frac{\partial'' t\phi_s(x_s, x_m)}{\partial x_s \partial x_m} = tD_0 \exp(-2 - \lambda(c_s + c_m - x_s - \beta(x_s)x_m)[\lambda\beta(x_s)(1 + \beta'(x_s)x_m) + \beta'(x_s)] \geq 0$$

Therefore, $\phi_s(x_s, x_m)$ is supermodular in x for linear, and exponential demand functions.So $f(x)$ is supermodular in x.

$$\frac{\partial'' t\phi_s(x_s, x_m)}{\partial x_s \partial t} = \frac{\partial'' t\phi_s(x_s, x_m)}{\partial x_m \partial t} t = tD_0 \exp(-2 - \lambda(c_s + c_m - x_s - \beta(x_s)x_m)[\lambda\beta(x_s)(1 + \beta'(x_s)x_m) + \beta'(x_s)] \geq 0$$

So F has increasing difference in (t, x)

This completes the proof.

The above lemma will be used in the proof of the following proposition.

Proposition 5 *For exponential demand , Let $x^c = \max \arg \max_{x_s, x_m}(\pi_s(x_s, x_m) + \pi_m(x_s, x_m))$ .and χ^* be the set of all the Nash Equilibria of investment effort game among all the firms. Then $\exists x^* \in \chi^*$ such that $x^c \succ x^*$ and $\phi_i(x^c) > \phi_i(x^*)$ for $i = s, m$*

Proof. Let $x^t = \max \arg \max_x t\pi_s(x) - k_s(x_s) - k_m(x_m)$ .if $t = 1$, $\pi_s(x) - k_s(x_s) - k_m(x_m)$ is a potential function of the investment effort game. Also note that the function $t\pi_s(x) - k_s(x_s) - k_m(x_m)$is a supermodular function in$x$ and has increasing difference in (t, x), hence x^t is increasing with t.On the other hand, when cooperate in the first stage , $t = 2$ and $x^2 = x^c = \max \arg \max_x 2\pi_s(x) - k_s(x_s) - k_m(x_m)$.

Hence $x^c = x^2 \succeq x^1$ and $\pi_s(x^c) \geq \pi_s(x^1)$

This completes the proof.

The above proposition shows that if the demand is exponential and the firms coordinate together in investment effort to maximize the system profit, every firm will set a higher effort and get higher gross profit (with larger demand) compared with competition system.

In the following section , we can analysis the case of linear demand function. First,we have a lemma:

Lemma 4.5 *For linear demand* $F(s, x) = \phi_s(x) - k_s(x_s) - sk_m(x_m)$ *has increasing difference in* $(-s, x)$ *when* $s > 0$.

Proof. Since $s > 0$, $\frac{\partial F(s,x)}{\partial - s} = k_m(x_m)$. Note that $k_m(x_m)$ s only a function of x_m and increasing in x_m, this implies $F(s, x)$ has increasing difference in $(-s, x)$. This completes the proof.

Lemma 4.6 *For linear demand , Let* $x^c = \max \arg \max_{x_s, x_m} (\pi_s(x_s, x_m) + \pi_m(x_s, x_m))$ *.and* χ^* *be the set of all the Nash Equilibria of investment effort game among all the firms. Then* $\exists x^* \in \chi^*$ *such that* $x^c \succ x^*$ *and* $\phi_i(x^c) > \phi_i(x^*)$ *for* $i = s, m$

Proof. Let $x^{t,s} = \max \arg \max_x t\pi_s(x) - k_s(x_s) - sk_m(x_m)$. since the function $t\pi_s(x) - k_s(x_s)$ is a supermodular function in x and has increasing difference in (t, x), $x^{t,1}$ is increasing with t. Therefore $x^{\frac{3}{2},1} \succeq x^{1,1}$, and $\pi_s(x^{\frac{3}{2},1}) \geq \pi_s(x^{1,1})$.the function $\pi_s(x) - k_s(x_s) - sk_m(x_m)$ is supermodular in x and has increasing difference in $(-s; x)$. Hence, $x^{1,s}$ is decreasing with s.Therefore, $x^{1,1} \succeq x^{1,2}$. Note cooperate in the first stage ,$t = 2$,$s = 1$, so$x^* = x^{1,2}$,and, $x^{\frac{3}{2},1} = x^c$. $t\pi_s(x) - k_s(x_s) - sk_m(x_m)$ is a potential function of the investment effort game when $t = 1, s = 2$.

Hence $x^c = x^{\frac{3}{2},1} \succeq x^{1,2}$ and $\phi_i(x^c) > \phi_i(x^*)$

This completes the proof.

In the above proposition of the subsection, based on supermodular theory,through the construction of potential function, we find cooperation in effort investment will lead to a higher effort and higher gross profit for every player relative simultaneous effort investment.

5 Conclusion

In this paper, a two echelon assembly system involving pricing and effort investment decisions is investigated. Demand in this model is deterministic as a function of price.In the first stage ,the manufacturer helps the suppler to reduce cost .In the second stage, the supplier sells the component to the manufacturer with a price only contract. For several common demand functions, the unique Nash equilibrium leads to a constant proportion of the manufacturer's gross profit to the supplier. Taking the unique price competition as a response, we analyze the effort investment decision in the first stage. Based on supermodular game theory,through the construction of potential function, we find that in the effort decision stage, Nash equilibrium always exists for simultaneous competition. The players' profit is supermodular in effort and increasing with the other players' effort. Thus we know that the equilibrium set consists of a largest and a smallest equilibrium such that in the largest equilibrium every one get most of the profit. We also find that if some one makes an effort decision earlier followed by the other, every one can benefit. Coordination in effort investment in the first stage will lead to a higher effort and higher gross profit for every player.

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