Journal of Financial Economics 129 (2018) 69–86

Contents lists available at ScienceDirect

Journal of Financial Economics

journal homepage: www.elsevier.com/locate/jfec

Downside risks and the cross-section of asset returns[☆]

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ARTICLE INFO

Article history:
Received 7 September 2015
Revised 17 January 2017
Accepted 14 February 2017
Available online 26 March 2018

JEL classification:
G12
C12
C31
C32

Keywords:
Generalized disappointment aversion
Downside risks
Cross-section

ABSTRACT

In an intertemporal equilibrium asset pricing model featuring disappointment aversion and changing macroeconomic uncertainty, we show that besides the market return and market volatility, three disappointment-related factors are also priced: a downstate factor, a market downside factor, and a volatility downside factor. We find that expected returns on various asset classes reflect premiums for bearing undesirable exposures to these factors. The signs of estimated risk premiums are consistent with the theoretical predictions. Our most general, five-factor model is very successful in jointly pricing stock, option, and currency portfolios, and provides considerable improvement over nested specifications previously discussed in the literature.

1. Introduction

Downside risk refers to the risk of an asset or portfolio in case of an adverse economic scenario. Upside uncertainty is the analogue if the scenario is favorable. The asymmetric treatment of downside risk versus upside uncertainty by investors has long been accepted among practitioners and academic researchers (see, e.g., Roy, 1952; Markowitz, 1959), and has led to the development of new concepts in asset pricing and risk management, like the value-at-risk and the expected shortfall. Theories of rational behavior have been developed, where investors place greater weights on adverse market conditions in their utility functions. These include the lower-partial moment framework of Bawa and Lindenberg (1977), the loss aversion of Kahneman and Tversky (1979) in their prospect theory, and the disappointment aversion of Gul (1991), which has been generalized by Routledge and Zin (2010). These theories suggest priced downside risks in the capital market equilibrium.

We derive and test the cross-sectional predictions of a consumption-based asset pricing model where the representative investor has generalized disappointment aversion (GDA) preferences and macroeconomic uncertainty is time-varying. In a setting without disappointment aversion, two factors are priced in the cross-section: the market return (r_W) and changes in market volatility

☆ An earlier version of this paper has been circulated under the title "Volatility Downside Risk". For helpful comments and discussions, we thank Alexander Barinov, Joost Driessen, Andras Fulop, Michael Halling, Bryan Kelly, Frederik Lundtofte, Steve Ohana, Artur Rodrigues; seminar participants at Georgia State University, BI Norwegian Business School, the Banque de France, York University, Maastricht University and the Swedish House of Finance; and participants at the 30th Annual Congress of the European Economic Association, 2015 BI-SHoF Conference, 2nd Empirical Finance Workshop at ESSEC, 2013 Annual Meetings of the European Financial Management Association, 4th World Finance Conference, International Risk Management Conference 2013, 20th Annual Conference of the Multinational Finance Society, 3rd International Conference of the Financial Engineering and Banking Society, and ABG Sundal Collier Nordic Quant Seminar.

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https://doi.org/10.1016/j.jfineco.2018.03.010
0304-405X/

($\Delta\sigma_W^2$). That is, investors require two premiums to invest in an asset. The first one is a compensation for covariation with the market return, $Cov(R_i^e, r_W)$, which is line with the prediction of the CAPM. The second premium is a compensation for covariation with changes in market volatility, $Cov(R_i^e, \Delta\sigma_W^2)$. It has been shown by previous empirical studies that volatility risk is priced in the cross-section (see, e.g., Ang et al., 2006b; Adrian and Rosenberg, 2008).

Our main theoretical contribution is to show that when disappointment aversion is added to the framework, investors require three additional premiums as compensation for exposures to disappointment-related risk factors. The first premium is a compensation for the covariance with the downstate factor, $Cov(R_i^e, I(\mathcal{D}))$. The downstate factor, $I(\mathcal{D})$, takes the value one if disappointment sets in and zero otherwise. The model suggests that disappointment ($\mathcal{D}$) may set in due to two reasons: a sufficiently large fall in market return or rise in market volatility. The second premium is a compensation for the covariance with the interaction of the market return and the downstate factor, $Cov(R_i^e, r_W I(\mathcal{D}))$. This factor represents movements of the market return in the downstate and we refer to it as the market downside factor. The third premium is a compensation for the covariance with the interaction of changes in market volatility and the downstate factor, $Cov(R_i^e, \Delta\sigma_W^2 I(\mathcal{D}))$. This factor represents changes in market volatility in the downstate and we refer to it as the volatility downside factor.

In the general case, our setting thus leads to a five-factor model. Although there are five factors in the model, only two time series, the market return (r_W) and changes in market volatility ($\Delta\sigma_W^2$), are needed to construct these factors: the downstate factor is constructed as a function of these two series, and the two downside factors are simply interactions with the downstate factor. We also show that if the representative investor has infinite elasticity of intertemporal substitution, then market volatility has no role in the model, and the disappointing event reduces to a fall of the market return below a reference threshold. This special case corresponds to a three-factor model with the market, the downstate, and the market downside factors.

The cross-sectional implications of downside risk have already been studied, most notably, by Ang et al. (2006a) and Lettau et al. (2014). Our three-factor model nests the models from both of these studies, with different restrictions on the premium corresponding to the downstate factor. We explicitly derive these restrictions and confront them with the data. Our results suggest that the restrictions imposed by the downside risk models of Ang et al. (2006a) and Lettau et al. (2014) are not supported empirically. Therefore, our three-factor model provides considerable improvement in explaining the cross-section of different asset returns, even though all three models use exactly the same information.

The more general five-factor model emphasizes the role of volatility in understanding downside risks. To our knowledge, little or no attention has been paid to volatility downside risk in the literature. We argue that volatility downside risk is also an important factor in explaining the cross-section of asset returns, as the five-factor model provides further improvement compared to the three-factor model.

We use the generalized method of moments (GMM) to empirically investigate the performance of our three- and five-factor models. Our benchmark test assets are various portfolios formed from US stocks, index option portfolios sorted on type, maturity, and moneyness, and currency portfolios sorted on their respective interest rates. These portfolios exhibit large heterogeneity in their average returns, and thus are ideal for cross-sectional asset pricing tests. The main empirical results of the paper relate to the pricing of the disappointment-related risk factors.

All the disappointment-related factors have significant risk premiums and the signs on the risk prices are in line with the theoretical predictions. In terms of pricing errors, when tested on all asset classes jointly, our three-factor model with a root-mean-squared-pricing error (RMSPE) of 20 basis points (bps) per month provides a significant improvement over the CAPM with a RMSPE of 50bps. The corresponding pricing errors of the downside risk models of Ang et al. (2006a) (28bps) and Lettau et al. (2014) (33bps) are considerably higher than that of the three-factor model. Our five-factor model, with a RMSPE of 17bps, largely outperforms a two-factor model with market return and changes in market volatility with a RMSPE of 27bps. Moreover, the five-factor GDA model also outperforms the four-factor model of Carhart (1997) on all asset classes except for stock portfolios. Also, the GDA model has the benefit of being motivated by dynamic consumption-based equilibrium asset pricing and behavioral decision theories, rather than being motivated by asset pricing anomalies themselves. These findings suggest the importance of disappointment-related risk in the cross-section of asset returns. Our results are robust to using additional asset classes and test portfolios, to alternative specifications of the disappointing event, and to alternative measures of market volatility.

This paper contributes to the developing literature that attempts to provide empirical support for the recent generalization by Routledge and Zin (2010) of the axiomatic disappointment aversion framework of Gul (1991). In the literature, GDA preferences have appeared in consumption-based equilibrium models mainly with the goal of explaining the time series behavior of the aggregate stock market, and rarely in cross-sectional asset pricing studies.[1] One exception is Delikouras (2017), who also studies the cross-sectional implications of a consumption-based model with disappointment aversion preferences. There are several differences between our study and that of Delikouras (2017). First, he uses annual and quarterly consumption data. In contrast, we substitute out consumption in a way similar to Campbell (1993), and rely on the market return.

[1] For instance, Bonomo et al. (2011) show that persistent shocks to consumption volatility are sufficient when coupled with GDA preferences to produce moments of asset prices and predictability patterns that are in line with the data. Schreindorfer (2014) aims at explaining properties of index option prices, equity returns, variance, and the risk-free rate using the GDA model and a heteroskedastic random walk for consumption with the multifractal process of Calvet and Fisher (2007). Delikouras (2014) uses the GDA model to explain the credit spread puzzle.

We can then avoid potential measurement problems in consumption data advocated by Wilcox (1992), or delayed responses of consumption to financial market news as discussed by Parker and Julliard (2005), and test the model at the monthly frequency using market return data. Second, he uses the original version of disappointment aversion as introduced by Gul (1991), while we use the generalized version of Routledge and Zin (2010). Our results, when considering different disappointment thresholds, suggest that the generalized version is more appropriate in a representative agent setting. Third, Delikouras (2017) assumes constant volatility of aggregate consumption, while our setting also allows for time-varying macroeconomic uncertainty. This feature is supported empirically (see, for example, Bansal et al., 2005) and it gives rise to the volatility-related premiums in our cross-sectional model. Finally, since we derive the cross-sectional implications in the form of a factor model and rely on market return rather than consumption, our results are directly comparable to recent cross-sectional studies on downside risks such as Ang et al. (2006a) and Lettau et al. (2014).

The remainder of this paper is organized as follows. In Section 2, we present the theoretical setup from which we derive the implied cross-sectional model. Section 3 contains the empirical analysis with several robustness checks. Section 4 concludes, while the Appendix contains the description of the data sources and some technical derivations. An Online Appendix contains additional details that are omitted from the main text for brevity.

2. Theoretical motivation

We consider an economy where the representative investor has recursive utility as in Epstein and Zin (1989) and Weil (1989)

$$V_{t-1} = \begin{cases} \left[(1-\delta)C_{t-1}^{1-\frac{1}{\psi}} + \delta[\mathcal{R}_{t-1}(V_t)]^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}} & \text{if } \psi > 0 \text{ and } \psi \neq 1 \\ C_{t-1}^{1-\delta}[\mathcal{R}_{t-1}(V_t)]^{\delta} & \text{if } \psi = 1 \end{cases}, \tag{1}$$

with $t = 1, 2, \ldots,$ and where $0 < \delta < 1$ is the parameter of time preference and $\psi > 0$ is the elasticity of intertemporal substitution. The lifetime utility at $t-1$, V_{t-1}, is a function of the period's consumption, C_{t-1}, and the certainty equivalent of next period's lifetime utility, $\mathcal{R}_{t-1}(V_t)$. Routledge and Zin (2010) embed generalized disappointment aversion (GDA) into this framework by assuming that the certainty equivalent $\mathcal{R}_{t-1}$ is implicitly defined by

$$\begin{aligned} U(\mathcal{R}_{t-1}) &= E_{t-1}[U(V_t)] \\ &\quad -\ell E_{t-1}[(U(\kappa\mathcal{R}_{t-1}) - U(V_t))I(V_t < \kappa\mathcal{R}_{t-1})], \end{aligned} \tag{2}$$

where $E_t[\cdot]$ denotes the expectation conditional on information up to time t. The utility function, U, is defined as

$$U(x) = \begin{cases} \dfrac{x^{1-\gamma}-1}{1-\gamma} & \text{if } \gamma \geq 0 \text{ and } \gamma \neq 1 \\ \ln x & \text{if } \gamma = 1 \end{cases}, \tag{3}$$

where the parameter $\gamma \geq 0$ is the coefficient of relative risk aversion. When ℓ is equal to zero, $\mathcal{R}_{t-1}$ reduces to expected utility (EU) preferences and V_{t-1} represents the Epstein and Zin (1989) recursive utility. GDA preferences are a two-parameter extension of the EU framework. When $\ell > 0$, outcomes lower than $\kappa\mathcal{R}_{t-1}$ receive an extra weight, decreasing the certainty equivalent. The larger weight given to these bad outcomes implies an aversion to losses. The parameter $\ell \geq 0$ is interpreted as the degree of disappointment aversion, while the parameter $0 < \kappa \leq 1$ is the percentage of the certainty equivalent such that outcomes below it are considered disappointing. The special case $\kappa = 1$ corresponds to the original disappointment aversion preferences of Gul (1991).

The investor maximizes the lifetime utility subject to the budget constraint

$$W_t = (W_{t-1} - C_{t-1})R_{Wt}, \tag{4}$$

where W_{t-1} is the wealth in period $t-1$ and R_{Wt} is the simple gross return on wealth, which we refer to as the market return. Following Hansen et al. (2007), Routledge and Zin (2010), and Bonomo et al. (2011), the stochastic discount factor (SDF) between periods $t-1$ and t in the model with generalized disappointment aversion is

$$M_{t-1,t}^{GDA} = M_{t-1,t}\left(\frac{1+\ell I(\mathcal{D}_t)}{1+\kappa^{1-\gamma}\ell E_{t-1}[I(\mathcal{D}_t)]}\right), \tag{5}$$

where $I(\cdot)$ denotes the indicator function taking the value one if the condition is met and zero otherwise, and

$$\begin{aligned} M_{t-1,t} &= \delta\left(\frac{C_t}{C_{t-1}}\right)^{-\frac{1}{\psi}}\left(\frac{V_t}{\mathcal{R}_{t-1}(V_t)}\right)^{\frac{1}{\psi}-\gamma} \quad \text{and} \\ \mathcal{D}_t &= \{V_t < \kappa\mathcal{R}_{t-1}(V_t)\}, \end{aligned} \tag{6}$$

where $M_{t-1,t}$ is the SDF without disappointment aversion ($\ell = 0$), and $\mathcal{D}_t$ denotes the disappointing event. The logarithm of $M_{t-1,t}$ and $\mathcal{D}_t$ may be written as

$$\begin{aligned} \ln M_{t-1,t} &= \ln\delta - \gamma\Delta c_t - \left(\gamma - \frac{1}{\psi}\right)\Delta z_{Vt} \quad \text{and} \\ \mathcal{D}_t &= \{\Delta c_t + \Delta z_{Vt} < \ln\kappa\}, \end{aligned} \tag{7}$$

where $\Delta c_t \equiv \ln(\frac{C_t}{C_{t-1}})$ and $\Delta z_{Vt} \equiv \ln(\frac{V_t}{C_t}) - \ln(\frac{\mathcal{R}_{t-1}(V_t)}{C_{t-1}})$ represent the change in the log consumption level (or consumption growth) and the change in the log welfare valuation ratio (or welfare valuation ratio growth), respectively.

For every asset i, optimal consumption and portfolio choice by the representative investor induces a restriction on the simple excess return R_{it}^e that is implied by the Euler condition:

$$E_{t-1}\left[M_{t-1,t}^{GDA}R_{it}^e\right] = 0. \tag{8}$$

In the special case when $\ell = 0$ and $\gamma = 1/\psi$, the moment condition (8) is readily testable by GMM using actual data on aggregate consumption growth and asset returns. Earlier results for this test of the standard model are presented in Hansen and Singleton (1982,1983). In the general case, however, the moment condition (8) is not directly testable by GMM since the continuation value is not observable from the data. Following the long-run risks asset pricing literature pioneered by Bansal and Yaron (2004), an assumed endowment dynamics can be exploited, together with the utility recursion (1) and the certainty equivalent definition (2), to express welfare valuation ratios in terms of economic state variables such as aggregate volatility, which may be measured or estimated from the data.

2.1. Cross-sectional implications

In order to obtain the cross-sectional implications that form the basis of our empirical investigation, we make two substitutions in the expressions for $\ln M_{t-1,t}$ and $\mathcal{D}_t$ in (7). First, we substitute out consumption growth following Epstein and Zin (1989), Hansen et al. (2007), and Routledge and Zin (2010) who show that in equilibrium, the market return is related to consumption growth and the welfare valuation ratio growth through[2]

$$r_{Wt} = -\ln\delta + \Delta c_t + \left(1 - \frac{1}{\psi}\right)\Delta z_{Vt} \ . \tag{9}$$

Second, assuming that aggregate consumption growth is heteroskedastic and unpredictable as in Bollerslev et al. (2009), Tauchen (2011), and Bonomo et al. (2011), and consistent with the empirical evidence presented in Beeler and Campbell (2012) among many others, we can solve for the welfare valuation ratio endogenously and express the welfare valuation ratio growth, $\Delta z_{V,\,t}$, in terms of changes in the volatility of the market return, which we refer to as market volatility.

Making these substitutions, and after some algebraic manipulation, the Euler equation in (8) may be written as a cross-sectional linear factor model

$$\begin{aligned} E\left[R^e_{it}\right] &= p_W\sigma_{iW} + p_{\mathcal{D}}\sigma_{i\mathcal{D}} + p_{W\mathcal{D}}\sigma_{iW\mathcal{D}} \\ &\quad + p_X\sigma_{iX} + p_{X\mathcal{D}}\sigma_{iX\mathcal{D}} \ , \end{aligned} \tag{10}$$

with

$$\begin{aligned} \sigma_{iW} &\equiv Cov\left(R^e_{it}, r_{Wt}\right) \\ \sigma_{i\mathcal{D}} &\equiv Cov\left(R^e_{it}, I(\mathcal{D}_t)\right) \\ \sigma_{iW\mathcal{D}} &\equiv Cov\left(R^e_{it}, r_{Wt}I(\mathcal{D}_t)\right) \\ \sigma_{iX} &\equiv Cov\left(R^e_{it}, \Delta\sigma^2_{Wt}\right) \\ \sigma_{iX\mathcal{D}} &\equiv Cov\left(R^e_{it}, \Delta\sigma^2_{Wt}I(\mathcal{D}_t)\right) \ , \end{aligned} \tag{11}$$

where r_{Wt} is the log-return on the market and $\Delta\sigma^2_{Wt}$ is the change in the variance of the market return (we are going to refer to this as the volatility factor).[3] Eq. (10) corresponds to a linear multifactor representation of expected excess returns in the cross-section. This is a five-factor model which we refer to as GDA5 throughout the rest of the paper. It states that in addition to the market (r_{Wt}) and volatility ($\Delta\sigma^2_{Wt}$) factors, three additional factors command a risk premium: the *downstate factor* $I(\mathcal{D}_t)$, the *market downside factor* $r_{Wt}I(\mathcal{D}_t)$, and the *volatility downside factor* $\Delta\sigma^2_{Wt}I(\mathcal{D}_t)$.

The covariance risk prices $p_W \geq 0$, $p_{\mathcal{D}} \leq 0$, $p_{W\mathcal{D}} \geq 0$, $p_X \leq 0$, and $p_{X\mathcal{D}} \leq 0$ are functions of the preference parameters δ, γ, ψ, ℓ, and κ, as well as functions of the parameters governing the endowment dynamics. Let us have a detailed look at the signs of the covariance risk prices. First, as $p_W \geq 0$, investors require a premium for a security that has positive covariance with the market return. This is in line with the CAPM theory of Sharpe (1964) and Lintner (1965). Second, as $p_X \leq 0$, investors are willing to pay a premium for a security that has positive covariance with $\Delta\sigma^2_{Wt}$. This is consistent with the existing empirical literature (see, e.g., Ang et al., 2006b; Adrian and Rosenberg, 2008). The third factor in (10), $I(\mathcal{D}_t)$, indicates periods when the economy is in the disappointing state. We refer to it as the downstate factor throughout the paper. The associated risk price is $p_{\mathcal{D}} \leq 0$, showing that disappointment-averse investors are willing to pay a premium for a security that has a positive covariance with the downstate indicator. Note that $\sigma_{i\mathcal{D}} = Prob(\mathcal{D}_t)(E[R^e_{it} \mid \mathcal{D}_t] - E[R^e_{it}])$, i.e., assets with $\sigma_{i\mathcal{D}} > 0$ are desirable because they have a higher expected return in the downstate. The fourth factor is $r_{Wt}I(\mathcal{D}_t)$, and it represents changes in the market index when the economy is in the downstate. We refer to it as the market downside factor throughout the paper. The associated risk price is non-negative, $p_{W\mathcal{D}} \geq 0$. Investors require a premium for a security that has positive covariance with $r_{Wt}I(\mathcal{D}_t)$, since such an asset tends to have a negative return when there is a low market return in the downstate. The fifth and final factor is $\Delta\sigma^2_{Wt}I(\mathcal{D}_t)$, representing changes in market volatility when the economy is in the downstate. We subsequently refer to it as the volatility downside factor. The associated risk price is non-positive, $p_{X\mathcal{D}} \leq 0$. Investors are willing to pay a premium for a security that has positive covariance with the volatility downside factor. Such an asset tends to have positive returns when market volatility increases in a downstate.

We also show in the Online Appendix that the disappointing event may be written as

$$\mathcal{D}_t = \left\{ r_{Wt} - a\frac{\sigma_W}{\sigma_X}\Delta\sigma^2_{Wt} < b \right\}, \tag{12}$$

where $\sigma_W = Std[r_{Wt}]$ and $\sigma_X = Std[\Delta\sigma^2_{Wt}]$ are the standard deviations of market return and changes in market volatility, respectively. Similar to the covariance risk prices, the coefficients $a > 0$ and b are also functions of the preference parameters and the parameters governing the endowment dynamics. The term $(\sigma_W/\sigma_X)\Delta\sigma^2_{Wt}$ may be viewed as the return on a volatility index that has the same standard deviation as the market return. Disappointment occurs if the return on a portfolio consisting of a long position in the market index and a times a short position in the volatility index falls below a constant threshold b. In particular, if the coefficient a is equal to one, the long position in the market index is exactly balanced by the short position in the volatility index in determining disappointment. As a decreases from one towards zero, disappointment is more likely to occur due to a fall in the market index rather than an increase in the volatility index. Note also that the following two nonlinear restrictions apply to the GDA5

[2] The true market return is unobservable and an empirical proxy is later used in asset pricing tests, consistent with the literature. The usual proxy is the return on a stock market index which shall be more volatile than the true market return since stock market dividends are at least five times more volatile than consumption. Therefore, properties of a consumption proxy backed out through Eq. (9) using the stock index return may be different from those of the observed consumption.

[3] Details of the derivation are outlined in the Online Appendix, where we also derive sign restrictions on the covariance risk prices, the definition of disappointment in (12), and the cross-price restrictions in (13). We use "W" in subscript to refer to quantities (e.g., risk measures or risk prices) related to the factor r_{Wt}. Similarly, we use "$\mathcal{D}$" to refer to the factor $I(\mathcal{D})$, "$W\mathcal{D}$" for the factor $r_W I(\mathcal{D})$, "X" for the factor $\Delta\sigma^2_W$, and "$X\mathcal{D}$" for the factor $\Delta\sigma^2_W I(\mathcal{D})$, respectively.

model:

$$\frac{p_{W\mathcal{D}}}{p_W} = \frac{p_{X\mathcal{D}}}{p_X}$$
$$p_{X\mathcal{D}} = -a\frac{\sigma_W}{\sigma_X} p_{W\mathcal{D}} \ . \quad (13)$$

There are two special cases of the model worth examining. First, if the elasticity of intertemporal substitution is infinite ($\psi = \infty$), then $a = 0$ and $p_X = p_{X\mathcal{D}} = 0$. That is, changes in market volatility disappear from the model. In this case, the cross-sectional model (10) reduces to a three-factor model with the market, the downstate, and the market downside factors, and the disappointing event has the simple form $\mathcal{D}_t = \{r_{Wt} < b\}$. We refer to this restricted model as GDA3 throughout the paper. Second, if the representative investor is not disappointment averse ($\ell = 0$), then $p_\mathcal{D} = p_{W\mathcal{D}} = p_{X\mathcal{D}} = 0$, i.e., all disappointment-related factors disappear from the model. In this case, (10) reduces to a two-factor model where only market risk and volatility risk are priced.

Eq. (10) may ultimately be expressed as a multivariate beta pricing model:

$$E\left[R^e_{it}\right] = \lambda_F^\top \beta_{iF} \ , \quad (14)$$

where β_{iF} is the vector containing the multivariate regression coefficients of asset excess returns onto the factors and λ_F is the vector of factor risk premiums, respectively, given by

$$\beta_{iF} = \Sigma_F^{-1}\sigma_{iF} \quad \text{and} \quad \lambda_F = \Sigma_F p_F \ . \quad (15)$$

The vector σ_{iF} contains the covariances of the asset excess returns with the priced factors as shown in (11), the vector p_F contains the associated factor risk prices, and Σ_F is the factor covariance matrix. Since the risk premiums in λ_F are linear combinations of the risk prices in p_F, the restrictions in (13) can easily be translated into equivalent restrictions on the λ-s. Also note that if the covariance between the market return and changes in market volatility is negative, then the sign restrictions on the elements of p_F discussed earlier in this section imply the same sign restrictions on the corresponding elements of λ_F, i.e., $\lambda_W \geq 0$, $\lambda_\mathcal{D} \leq 0$, $\lambda_{W\mathcal{D}} \geq 0$, $\lambda_X \leq 0$, and $\lambda_{X\mathcal{D}} \leq 0$. The negative covariance, $Cov(r_{Wt}, \Delta\sigma^2_{Wt}) < 0$, is consistent with the leverage effect postulated by Black (1976) and documented by Christie (1982) and others, and it is also empirically supported in our data. The model specification in (14) is the basis of our empirical analysis.

3. Empirical assessment

In this section, we provide an empirical assessment of the GDA3 and GDA5 models. The GDA3 is a three-factor model with the market, the downstate, and the market downside factors:

$$E\left[R^e_{it}\right] = \lambda_W \beta_{iW} + \lambda_\mathcal{D}\beta_{i\mathcal{D}} + \lambda_{W\mathcal{D}}\beta_{iW\mathcal{D}} \ , \quad (16)$$

where the disappointing event has the simple form $\mathcal{D}_t = \{r_{Wt} < b\}$. The GDA5 is a five-factor model containing also the volatility-related factors:

$$E\left[R^e_{it}\right] = \lambda_W \beta_{iW} + \lambda_\mathcal{D}\beta_{i\mathcal{D}} + \lambda_{W\mathcal{D}}\beta_{iW\mathcal{D}} + \lambda_X\beta_{iX} + \lambda_{X\mathcal{D}}\beta_{iX\mathcal{D}} \ . \quad (17)$$

Additionally for the GDA5, volatility enters the definition of the disappointing event as shown in (12), and the two cross-price restrictions in (13) should also be satisfied. The number of freely estimated λ-s decreases to three due to the two cross-price restrictions. For the GDA5, we also estimate the parameter a, which determines the relative importance of the market return and changes in volatility in the definition of disappointment. Altogether, there are four parameters to estimate in case of the GDA5 model. For both models, the disappointment threshold is set to $b = -0.03$ for the empirical analysis, but we also consider other values in the robustness section.

Finally, note that we do not estimate the underlying preference parameters, but instead we estimate the risk premiums, which are functions of both the preference parameters and the parameters governing the endowment dynamics. There are several reasons for focusing on the risk premiums. First, the market return is not observable and we use the return on the aggregate stock index as a proxy. This proxy is much more volatile, since it is a claim on the aggregate stock market dividend, whose growth rate is at least five times more volatile than the aggregate consumption growth rate. So, estimating the underlying preference parameters with this proxy would induce large estimation bias. Second, the preference parameter estimates would be dependent on the dynamics assumed for the aggregate endowment in the economy. By estimating the reduced-form risk premiums in the linear beta representations (16) and (17), the assumed endowment dynamics do not have a direct effect on our results. Third, estimating the reduced-form risk premiums makes our results comparable to existing cross-sectional tests of models with downside risks (e.g., Ang et al., 2006a; Lettau et al., 2014).

3.1. Data and estimation method

Following Lewellen et al. (2010), we do not restrict our attention to pricing size/book-to-market portfolios. Instead, we estimate our models using various sets of stock portfolios, and also include additional asset classes like index options and currencies. Monthly returns on several sets of US stock portfolios are from Kenneth French's data library. Index option returns are from Constantinides et al. (2013), who construct a panel of leverage-adjusted (that is, with a targeted market beta of one) S&P 500 index option portfolios. Currency returns are from Lettau et al. (2014), who use monthly data on 53 currencies to create six portfolios by sorting them based on their respective interest rates. The detailed description of the data and sample periods can be found in Appendix A.

The risk-free rate is the one-month US Treasury bill rate from Ibbotson Associates, while the market return is the value-weighted average return on all CRSP stocks.[4] Both series were obtained from Kenneth French's data library. Empirical tests of the GDA5 model require a measure of market volatility. Several approaches have been used for measuring market volatility in cross-sectional asset pricing

[4] We closely follow the predictions of the theoretical model by using the log-return on the market (r_{Wt}) as the market factor and using simple excess returns on the portfolios (R^e_{it}) as the dependent variable.

studies. Ang et al. (2006b) use the VIX, Adrian and Rosenberg (2008) estimate volatility from a GARCH-type model, while Bandi et al., (2006) use realized volatility computed from high-frequency index returns. In our main analysis, we measure monthly volatility as the realized volatility of the daily market returns during the month. The main advantage of this latter measure is that it is very easy to construct as it requires only daily market return data. Therefore, it allows us to use a longer sample period. Nevertheless, we also use alternative measures in our robustness checks, including the VIX, realized volatility calculated from intra-daily market returns, and GARCH volatility.

Portfolio betas and factor premiums from (14) are estimated jointly using GMM with moment conditions as in Cochrane (2000):

$$\begin{cases} E\left[R^e_{it} - \alpha_i - F_t\beta_{iF}\right] = 0 & i = 1, \ldots, N \\ E\left[\left[R^e_{it} - \alpha_i - F_t\beta_{iF}\right]f_{jt}\right] = 0 & i = 1, \ldots, N\ , \quad j = 1, \ldots, K \\ E\left[R^e_{it} - \beta_{iF}\lambda_F\right] = 0 & i = 1, \ldots, N \end{cases}, \tag{18}$$

where R^e_{it} is the excess return on portfolio i, f_{jt} denotes factor j, F_t is the row vector of all factors in the model, β_{iF} is the vector of factor betas for portfolio i, and λ_F is the vector of factor risk premiums. The first two sets of moment conditions from (18) directly correspond to the formula for estimating the β-s in (15), while the last set of moment conditions represents the model in (14). The advantage of using the GMM is that it allows us to impose the cross-price restrictions in the GDA5 model and that the standard errors account for the "generated regressors" problem, i.e., the fact that the β-s are also estimated.[5]

When estimating the factor risk premiums, we always apply the additional restriction that the market portfolio should be perfectly priced. This additional restriction reduces the number of free parameters in all the models by one. As a consequence, there are two free parameters to estimate for the GDA3, and three free parameters to estimate for the GDA5, which makes the models more parsimonious. As it can be seen from (10) and (11), the return to be explained in our cross-sectional models is in the form of simple excess return (R^e_{it}), while the market factor is the log-return on the market (r_{Wt}).[6] Thus, when the test asset is the market portfolio, the return to be explained and the market factor are not exactly the same. Therefore, imposing the restriction that the market is priced perfectly is not equivalent to setting the market premium equal to the expected excess return on the market, but it imposes a linear restriction on the λ-s. This restriction is discussed in detail in the Online Appendix. Essentially, we have to pick one of the premiums, whose value is implied by the other risk premiums through the market restriction. We pick the downstate premium ($\lambda_{\mathcal{D}}$) to be imposed for the GDA models, but the risk premium estimates would be exactly the same if we chose another one instead (e.g., λ_W or $\lambda_{W\mathcal{D}}$).

3.2. Results

Table 1 presents risk premium estimates for the CAPM, GDA3, and GDA5 models using several sets of US stock portfolios: (i) 25 (5×5) portfolios formed on size and book-to-market, (ii) 25 (5×5) portfolios formed on size and momentum, (iii) 30 portfolios consisting of ten size, ten book-to-market, and ten momentum portfolios, (iv) 25 (5×5) portfolios formed on size and operating profitability, and (v) 25 (5×5) portfolios formed on size and investment.

Panel A corresponds to the CAPM, which also arises as a restricted version of the GDA3 if the representative agent is not disappointment averse. The value of the market risk premium is not estimated, but imposed by the restriction that the market portfolio should be perfectly priced by the model. To make it clear that certain λ values are imposed instead of estimated, we report these values with the superscript i and do not report their standard errors in Table 1 and in subsequent tables throughout the paper.

Panel B presents the results for the GDA3. In the three middle columns, all risk premiums have the expected signs: the market (λ_W) and market downside ($\lambda_{W\mathcal{D}}$) factors have a positive premium, while the downstate factor ($\lambda_{\mathcal{D}}$) has a negative premium. Also, the estimated premiums are statistically significant.[7] For the size/book-to-market and size/investment portfolios however, the downstate premium is positive and the market downside premium is not statistically significant.

Panel C shows the results for the GDA5. Recall that the GDA5 involves two cross-price restrictions. We substitute out the volatility-related premiums using these restrictions and estimate the premiums on the rest of the factors. Additionally, the value of $\lambda_{\mathcal{D}}$ is imposed by the restriction that the market portfolio is perfectly priced, similar to the GDA3. In all the columns, the signs are as expected both on the estimated and on the implied premiums: the market (λ_W) and market downside ($\lambda_{W\mathcal{D}}$) factors have a positive premium, while the premiums on the downstate ($\lambda_{\mathcal{D}}$), the volatility (λ_X), and the volatility downside ($\lambda_{X\mathcal{D}}$) factors are negative. The only exception is $\lambda_{\mathcal{D}}$ for the size-investment portfolios. All estimated risk premiums are statistically significant. In the case of the GDA5, the parameter a in the definition of the disappointing event (12) is also estimated. The value of a is less than one in four of the five cases and the typical value is close to 0.5. Recall that an a value less than one means that the market return has a bigger weight in determining disappointing states than changes in market volatility.

Table 2 presents risk premium estimates for the same models when index option and currency portfolios are

[5] It is shown by Cochrane (2000), for example, that the correction due to Shanken (1992) can be recovered as a special case of the GMM standard errors. During the GMM estimation we use the identity weighting matrix, and we use the Newey–West estimator with three lags for the covariance matrix of the moment conditions. Delikouras (2017) shows that the GMM estimators are consistent and asymptotically normal even when the GMM moment conditions include indicator functions as in the case of the GDA models.

[6] It is shown in the Online Appendix that deviating from the theoretical predictions and using R^e_{Wt} instead of r_{Wt} as the market factor does not change our empirical results considerably.

[7] Note again, that the value of $\lambda_{\mathcal{D}}$ is not directly estimated, but is imposed by the restriction that the market portfolio should be perfectly priced.

Table 1
Risk premiums for the CAPM and GDA models using stock portfolios. The table shows risk premium estimates for the CAPM and GDA models using five different sets of US stock portfolios as test assets: (i) 25 (5 × 5) portfolios formed on size and book-to-market, (ii) 25 (5 × 5) portfolios formed on size and momentum, (iii) 30 portfolios consisting of 10 size, 10 book-to-market, 10 momentum portfolios, (iv) 25 (5 × 5) portfolios formed on size and operating profitability, and (v) 25 (5 × 5) portfolios formed on size and investment. We use monthly data and the sample period is from July 1964 to December 2016. The premiums are estimated using GMM. Standard errors are in parentheses, and *, **, and *** denote significance at the 10%, 5%, and 1% levels. Values with the superscript *i* are imposed by the restriction that the market portfolio should be correctly priced (and by cross-price restrictions for the GDA5). RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

		25 S × BM	25 S × Mom	10 S,B,M	25 S × OP	25 S × INV
Panel A: CAPM						
	λ_W	0.0050^i	0.0050^i	0.0050^i	0.0050^i	0.0050^i
	RMSPE	30.6 [0.40]	39.4 [0.52]	24.9 [0.39]	23.9 [0.33]	29.0 [0.38]
Panel B: GDA3						
	λ_W	0.0065***	0.0070***	0.0065***	0.0066***	0.0064***
		(0.0016)	(0.0008)	(0.0008)	(0.0017)	(0.0013)
	$\lambda_{\mathcal{D}}$	0.0367^i	-0.2449^i	-0.2022^i	-0.0917^i	0.1895^i
	$\lambda_{W\mathcal{D}}$	0.0125	0.0256***	0.0197***	0.0173*	0.0060
		(0.0091)	(0.0070)	(0.0061)	(0.0103)	(0.0062)
	RMSPE	25.7 [0.34]	23.6 [0.31]	19.8 [0.31]	18.8 [0.26]	22.6 [0.30]
Panel C: GDA5						
	λ_W	0.0078***	0.0068***	0.0064***	0.0069***	0.0072***
		(0.0015)	(0.0008)	(0.0007)	(0.0017)	(0.0009)
	$\lambda_{\mathcal{D}}$	-0.1825^i	-0.1673^i	-0.1717^i	-0.0778^i	0.1363^i
	$\lambda_{W\mathcal{D}}$	0.0261**	0.0202**	0.0171***	0.0181*	0.0135*
		(0.0114)	(0.0094)	(0.0063)	(0.0105)	(0.0070)
	λ_X	-0.0021^i	-0.0018^i	-0.0012^i	-0.0024^i	-0.0035^i
	$\lambda_{X\mathcal{D}}$	-0.0036^i	-0.0023^i	-0.0017^i	-0.0026^i	-0.0040^i
	a	0.8462	0.4692	0.5335	0.3275	1.2827
		(0.5485)	(1.0453)	(0.7009)	(0.5046)	(1.1586)
	RMSPE	21.4 [0.28]	20.7 [0.27]	18.7 [0.29]	17.5 [0.24]	16.9 [0.22]

also used as test assets. Note that if multiple asset classes are included, each asset class is represented with the same number of portfolios, so that they have similar importance in the estimation. Panel A of Table 2 presents the CAPM. The market risk premium, λ_W, is positive for all five sets of portfolios. Panel B corresponds to the GDA3. All risk premiums have the expected signs, and all the estimated risk premiums are statistically significant. Panel C presents result for the GDA5. Similar to the GDA3, all risk premiums have the expected signs, and all the estimated risk premiums are statistically significant.[8]

To facilitate model comparison, both Tables 1 and 2 report the root-mean-squared-pricing error (RMSPE) of the models, expressed in basis points (bps) per month, and the ratio of RMSPE to root-mean-squared returns (in brackets after the RMSPE values). The GDA3 provides a better fit than the CAPM for all sets of test assets, and the improvement is considerable in several cases. For example, the RMSPE reduces from 39 to 24 bps in case of the size-momentum stock portfolios, reduces from 44 to 12 bps for the option portfolios, and reduces from 50 to 20 bps when all three asset classes are included in the estimation. The GDA5 provides further improvement compared to GDA3 for all the ten sets of portfolios presented in Tables 1 and 2.

Fig. 1 shows scatter plots of actual versus predicted returns, corresponding to the case when all three asset classes are included in the estimation.[9] Panel A highlights the failure of the CAPM to price our test portfolios. Within each asset class, the actual returns vary considerably, but the CAPM predicts similar returns for all portfolios.

[8] To put the magnitudes of the risk premium estimates into perspective, we compare them to the corresponding values implied by the asset pricing model of Section 2, calibrated as in Bonomo et al. (2011). In summary, despite the correct signs, estimates of market downside risk premium, volatility risk premium, and volatility downside risk premium of Tables 1 and 2 are larger than what the calibrated model can actually replicate. We argue that this is due to potential estimation biases that may come from different sources, in particular when portfolios are used as test assets, as discussed in Ang et al. (2017) and Gagliardini et al. (2016). To verify our assertion, we also estimate the factor risk premiums using a large cross-section of individual stocks as test assets. We find that the risk premium estimates obtained with individual stocks are close to the calibration-implied values. A detailed description and discussion of these findings can be found in Section A.8 of the Online Appendix.

[9] In particular, the scatter plots in the top row of Fig. 1 correspond to the last column of Table 2. The Online Appendix contains scatter plots similar to the ones in Fig. 1 for several other sets of portfolios.

Table 2

Risk premiums for the CAPM and GDA models using further asset classes. The table shows risk premium estimates for the CAPM and GDA models using various sets of test assets: (i) 54 index option portfolios from Constantinides et al. (2013); (ii) 25 (5 × 5) size/book-to-market and 24 index option portfolios; (iii) 25 (5 × 5) size/momentum and 24 index option portfolios; (iv) 6 size/book-to-market, 6 option, and 6 currency (from Lettau et al., 2014) portfolios; and (v) 6 size/momentum, 6 option, and 6 currency portfolios. We use monthly data and the sample period varies across the different sets of portfolios. The sample periods and data sources are described in Appendix A. The premiums are estimated using GMM. Standard errors are in parentheses, and *, **, and *** denote significance at the 10%, 5%, and 1% levels. Values with the superscript *i* are imposed by the restriction that the market portfolio should be correctly priced (and by cross-price restrictions for the GDA5). RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

	Stocks		25 S × BM	25 S × Mom	6 S × BM	6 S × Mom
	Options	54	24	24	6	6
	Currencies				6	6
Panel A: CAPM						
	λ_W	0.0053^i	0.0053^i	0.0053^i	0.0051^i	0.0051^i
	RMSPE	43.8 [0.71]	39.8 [0.57]	42.4 [0.60]	49.4 [0.70]	50.3 [0.71]
Panel B: GDA3						
	λ_W	0.0068***	0.0067***	0.0067***	0.0066***	0.0064***
		(0.0006)	(0.0004)	(0.0004)	(0.0005)	(0.0005)
	$\lambda_{\mathcal{D}}$	-0.1672^i	-0.1442^i	-0.1863^i	-0.2294^i	-0.2884^i
	$\lambda_{W\mathcal{D}}$	0.0179***	0.0166***	0.0178***	0.0205***	0.0210***
		(0.0059)	(0.0056)	(0.0041)	(0.0053)	(0.0044)
	RMSPE	11.9 [0.19]	21.5 [0.31]	21.1 [0.30]	20.5 [0.29]	19.8 [0.28]
Panel C: GDA5						
	λ_W	0.0070***	0.0070***	0.0067***	0.0069***	0.0065***
		(0.0008)	(0.0009)	(0.0005)	(0.0007)	(0.0008)
	$\lambda_{\mathcal{D}}$	-0.2927^i	-0.2250^i	-0.1974^i	-0.3753^i	-0.3418^i
	$\lambda_{W\mathcal{D}}$	0.0228***	0.0201***	0.0179***	0.0265***	0.0222***
		(0.0035)	(0.0057)	(0.0047)	(0.0060)	(0.0066)
	λ_X	-0.0006^i	-0.0007^i	-0.0012^i	-0.0001^i	-0.0002^i
	$\lambda_{X\mathcal{D}}$	-0.0017^i	-0.0020^i	-0.0016^i	-0.0014^i	-0.0007^i
	a	0.5820	0.7026	0.3816	0.5259	0.3170
		(0.6764)	(0.8943)	(0.9693)	(0.4416)	(0.5694)
	RMSPE	10.0 [0.16]	19.1 [0.27]	18.8 [0.27]	18.6 [0.26]	17.3 [0.24]

Consequently, portfolios within each asset class line up close to a vertical line. The improvement in fit is evident when we move from the CAPM to the GDA3 in Panel B, where the portfolios lie much closer to the 45-degree line. Finally, the portfolios line up almost perfectly along the 45-degree line in Panel C, which corresponds to the GDA5.

In Section A.7 of the Online Appendix, we provide a detailed discussion on why the GDA model is successful in pricing the option portfolios of Constantinides et al. (2013). We rely on option Greeks to study how the sensitivity of the option price to the underlying price and to volatility varies with option moneyness when disappointment sets in. Portfolios containing out-of-the-money (OTM) calls have the lowest sensitivity to the price of the underlying, conditional on disappointment. They are followed by portfolios with in-the-money (ITM) calls, then ITM puts, and portfolios with OTM puts have the highest sensitivity. When considering the sensitivity to volatility conditional on disappointment, the ordering is reversed: portfolios containing OTM puts have the lowest sensitivity and portfolios with OTM calls have the highest. Since market downside risk carries a positive premium and volatility downside risk carries a negative premium, these imply that the GDA model predicts the lowest return for the OTM call portfolios and the highest return for the OTM put portfolios, which is in line with the data.

Daniel and Moskowitz (2016) argue that momentum profits are linked to the option-like behavior of the momentum strategy. Clarida et al. (2009) show that currency carry trade strategies resemble the payoff and risk characteristics of currency option strategies. These results, together with our previous discussion on option portfolios, may explain why the GDA model is also successful in pricing the momentum equity and currency portfolios.

3.2.1. Disappointing states

Panel A of Fig. 2 plots the market return and the NBER recession periods for our longest sample starting in July 1964 and ending in December 2016. The horizontal line indicates a 3% drop in the market index. According to the simple definition $\mathcal{D}_{At} \equiv \{r_{W,t} < -0.03\}$, disappointing months are those when the market return is below this line. Out of 630 months in the sample, 102 are classified as disappointing, giving a 16.2% unconditional probability of disappointment. There are 90 NBER recession

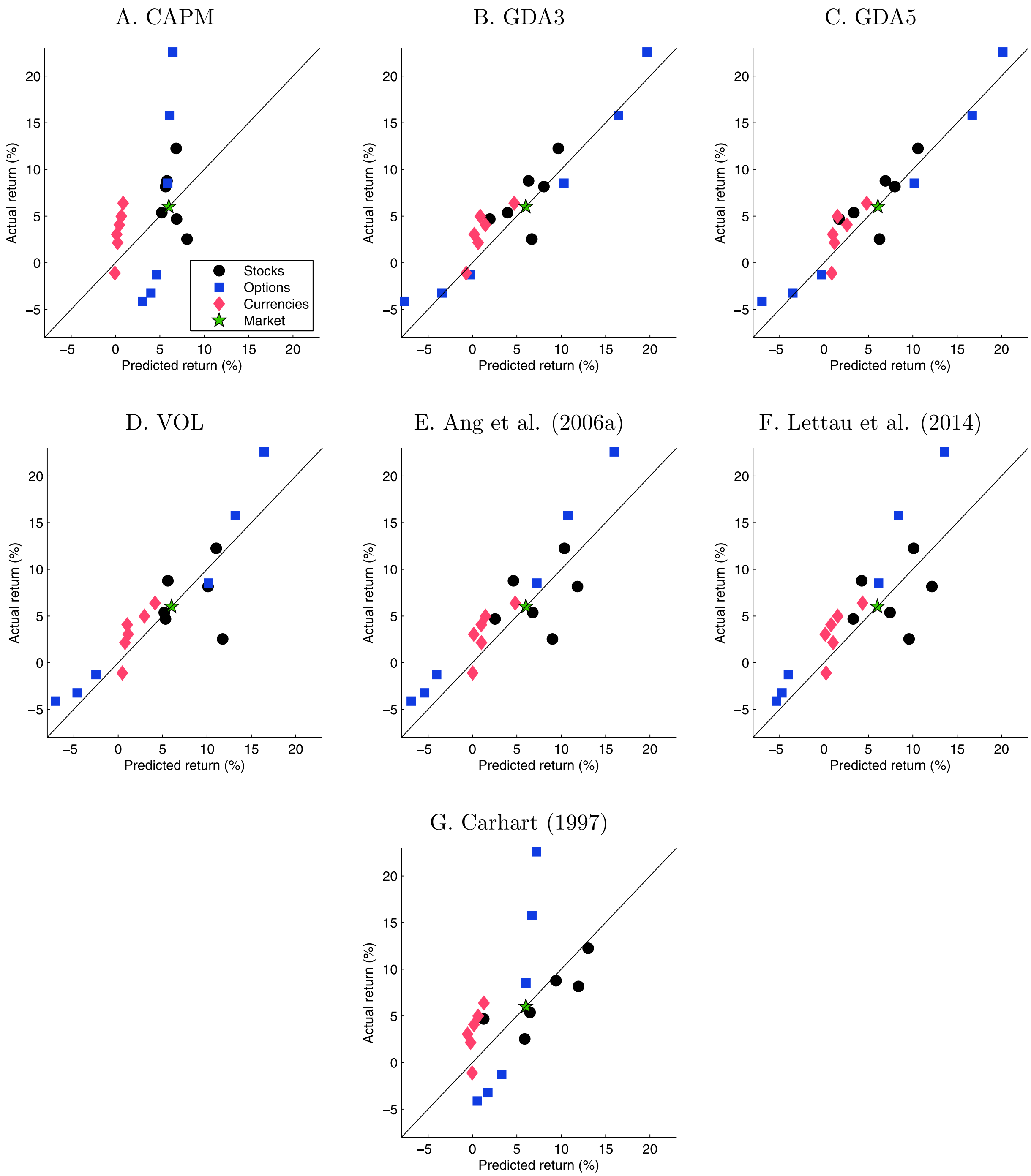

Fig. 1. Actual versus predicted returns. The scatter plots show the realized average excess returns against the predicted excess returns from various models for six size/momentum, six option, and six currency portfolios, together with the market return (see the legend in Panel A). The portfolios and the corresponding data sources are described in Appendix A. The sample period is from April 1986 to March 2010. The corresponding risk premium estimates are reported in the last column of Table 2 (CAPM; GDA3; and GDA5) and Table 4 (VOL; Ang et al., 2006a; Lettau et al., 2014; Carhart, 1997).

months during this period, out of which 28 are classified as disappointing. This implies a 31.1% probability of disappointment conditional on being in recession, and a 13.7% probability of disappointment conditional on being outside of recession. There is a clear positive relationship between recessions and disappointing states as the conditional probability of disappointment more than doubles in recession periods.

Panel B of Fig. 2 shows the value of $r_{Wt} - a\frac{\sigma_W}{\sigma_X}\Delta\sigma^2_{Wt}$ with $a = 0.5$ for the same period. We use $a = 0.5$ since

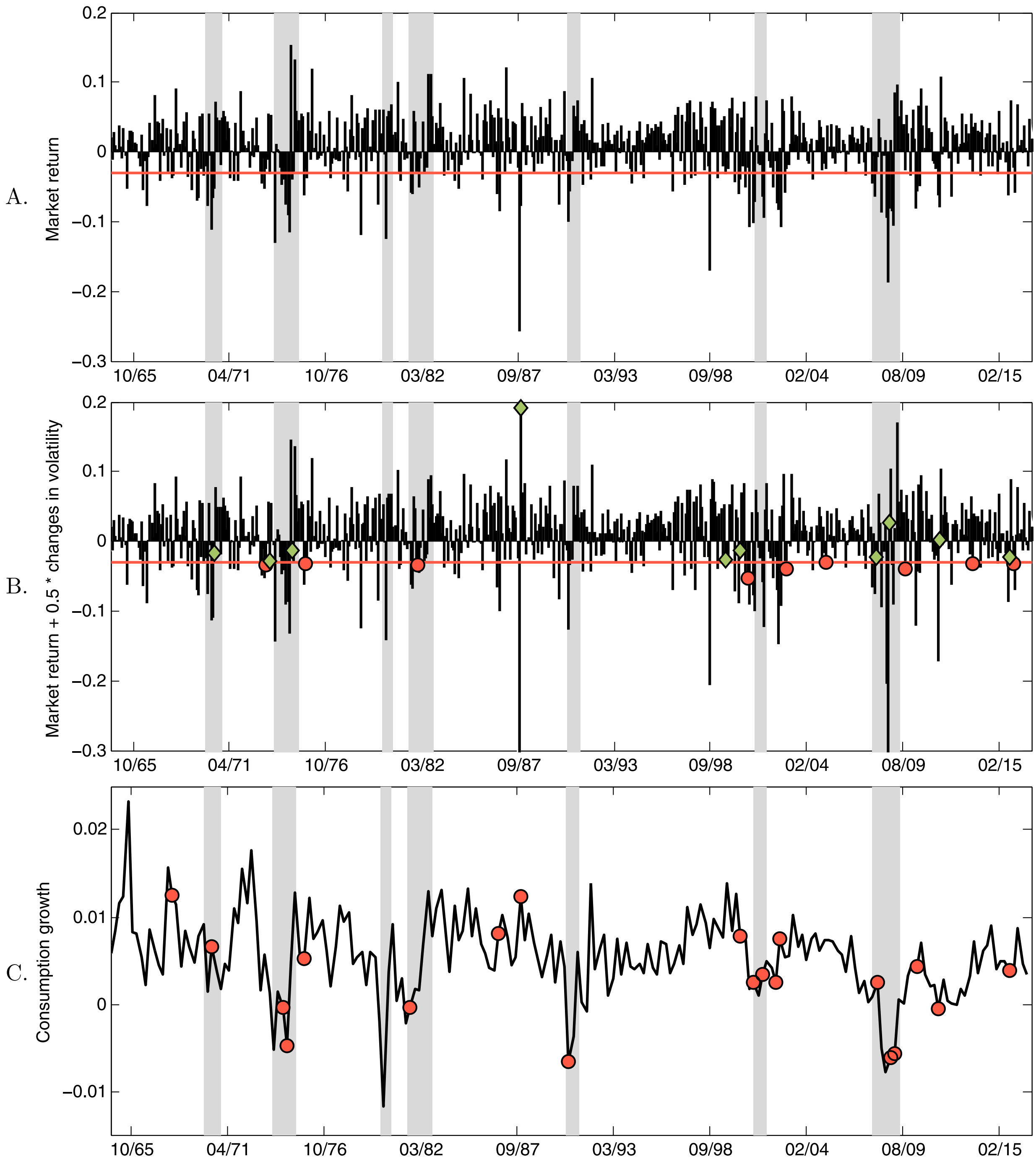

Fig. 2. Disappointing states. Panel A shows the monthly market return ($r_{W,t}$). Disappointing months defined by $\mathcal{D}_{At} \equiv \{r_{W,t} < -0.03\}$ are those when the market return is below the horizontal line indicating the −3% level. Panel B shows the monthly value of $r_{Wt} - 0.5\frac{\sigma_W}{\sigma_X}\Delta\sigma^2_{Wt}$. Disappointing months defined by $\mathcal{D}_{Bt} \equiv \{r_{Wt} - 0.5\frac{\sigma_W}{\sigma_X}\Delta\sigma^2_{Wt} < -0.03\}$ are those when the value of this linear combination falls below the horizontal line in Panel B. The diamond markers in Panel B indicate months that are disappointing according to $\mathcal{D}_{At}$ but not disappointing according to $\mathcal{D}_{Bt}$. The round markers in Panel B indicate months that are disappointing according to $\mathcal{D}_{Bt}$ but not disappointing according to $\mathcal{D}_{At}$. Panel C shows quarterly consumption growth and the round markers indicate quarters with at least two disappointing months. The sample period in all panels is from July 1964 to December 2016, and the shaded intervals correspond to the NBER recessions.

most of the estimated values in Tables 1 and 2 are around this value. The horizontal line is at −0.03, and disappointing states are defined $\mathcal{D}_{Bt} \equiv \left\{ r_{Wt} - 0.5\frac{\sigma_W}{\sigma_X}\Delta\sigma^2_{Wt} < -0.03 \right\}$. The disappointment definitions in Panel A and Panel B are empirically close to each other. Out of 630 months, 101 are classified as disappointing according to $\mathcal{D}_{Bt}$, giving a 16.0% unconditional probability of disappointment. There are only ten months in the sample that are disappointing according to $\mathcal{D}_{At}$ but not disappointing according to $\mathcal{D}_{Bt}$; these are highlighted with the diamond markers in Panel B. At the same time, there are nine months that are disappointing according to $\mathcal{D}_{Bt}$ but not disappointing according to $\mathcal{D}_{At}$; these are highlighted with the round markers in Panel B.

The main reason for $\mathcal{D}_{At}$ and $\mathcal{D}_{Bt}$ being empirically close is that decreasing market return and increasing market volatility tend to coincide empirically; which is also known as the leverage effect. That is, even if increasing market volatility is not explicitly included in the definition, disappointment tends to be accompanied with increasing volatility. In the period from July 1964 to December 2016, the unconditional correlation between r_{Wt} and $\Delta\sigma^2_{Wt}$ is -0.25 in our sample. Their conditional correlation (conditional on being in the disappointing state according to $\mathcal{D}_{At}$) is even stronger, -0.46. Extreme volatility increases also happen in disappointing months when disappointment is defined as $r_{Wt} < -0.03$. Nine of the largest ten $\Delta\sigma^2_{Wt}$ values in our stock sample period are realized in disappointing months (and 16 of the largest 20).

Finally, Panel C of Fig. 2 shows quarterly consumption growth throughout the period, and the round markers indicate quarters with two or three disappointing months.[10] Out of the 209 quarters in the sample, there are 20 in which at least two out of three months within the quarter are disappointing. Quarters with multiple disappointing months are associated with a higher probability of declining consumption. There are 16 quarters with negative consumption growth and seven of them have multiple disappointing months. These values imply that the conditional probability of declining consumption is 35.0% if there are two or more disappointing months in a given quarter, and only 4.8% if there is at most one disappointing month.

3.2.2. Risk premium estimates without restrictions

Table 3shows risk premium estimates for selected sets of test portfolios when we do not impose the restriction that the market should be perfectly priced.[11] Thus, the downstate premium is not imposed, but is estimated as a free parameter in Table 3. Panels A and B correspond to the GDA3 and GDA5 models, respectively. The estimated risk premiums have the expected signs and their magnitudes are similar to those reported for our benchmark specifications in Tables 1 and 2. The $\lambda_{\mathcal{D}}$ estimate is statistically significant in all but one of the cases. Also note that the model fit is better if we do not impose the restriction on the market portfolio, as the number of free parameters increases.

The GDA5 model can also be estimated without imposing any cross-price restrictions on the risk premiums and assuming that the disappointing event is of the simple form $\mathcal{D}_t = \{r_{W,t} < -0.03\}$. We refer to this specification as the "unrestricted GDA5." The unrestricted GDA5 has a couple of advantages compared to our main GDA5 specification. First, it is easier to estimate because there are no cross-price restrictions to be imposed and the definition of disappointment is fixed (i.e., the parameter a does not have to be estimated). Second, the GDA3 is nested in the unrestricted GDA5, which facilitates a more direct comparison between the two models. Third, results from the unrestricted GDA5 are also more comparable to previous studies analyzing downside risk, since those studies also define the downstate in terms of the market return only. Panel C of Table 3 provides risk premium estimates for the unrestricted GDA5. All risk premiums are statistically significant and have the expected signs. Magnitudes of the volatility-related risk premiums for the unrestricted GDA5 are somewhat higher than in the case of our benchmark GDA5 specification, but the magnitudes of the other premiums are reasonably similar. In terms of model fit, the unrestricted GDA5 delivers lower pricing errors than the other two models in Table 3. Thus, the fourth advantage of the unrestricted GDA5 is that it actually provides a better fit than the other two GDA models.

Despite all its advantages, we do not focus on the unrestricted GDA5 throughout the paper, as it has one major disadvantage compared to the GDA5: it is less related to the theoretical predictions from Section 2.1. Lewellen et al. (2010) suggest that when theory provides predictions for the risk price estimates, these predictions should be taken seriously.

3.2.3. Comparison to alternative models

In this section we compare the fit of the GDA models to alternative models proposed in previous literature. Table 4 presents results corresponding to four alternative models using the same five sets of portfolios as in Table 3. Results for the other sets of portfolios are relegated to the Online Appendix and lead to very similar conclusions to those presented in Table 4.

The model in Panel A, labeled as "VOL," contains only two priced factors: market return and market volatility. The VOL model can be viewed as a restricted version of the GDA5 that arises if the representative agent is not disappointment averse. The results in Panel A show that volatility risk carries a negative premium. Pricing errors decrease compared to the CAPM, but the size of the improvement varies across different asset classes. The RMSPE barely decreases in case of the stock portfolios, but the improvement is considerable in case of the option portfolios (from 44 to 14 bps) and when all three asset classes are

[10] Aggregate consumption growth is calculated using quarterly data on Personal Consumption Expenditures (PCE) from the U.S. Bureau of Economic Analysis. Aggregate consumption is defined on a per capita basis as services plus non-durables. We use seasonally adjusted series and deflate aggregate consumption by the PCE price index (the base year is 2009). The consumption growth data are available until 2016Q3. The round markers indicate quarters with two or three disappointing months according to $\mathcal{D}_{At}$. Note, however, that exactly the same plot arises if $\mathcal{D}_{Bt}$ is used instead.

[11] In order to save space, results for the other five sets of portfolios are presented in the Online Appendix. Those results lead to very similar conclusions to the ones presented in Table 3.

Table 3

Risk premiums when the perfect market pricing restriction is not imposed. The table shows risk premium estimates for GDA models using various sets of test portfolios (the same sets of portfolios as in Table 4) *without* imposing the restriction that the market portfolio is perfectly priced. We use monthly data and the sample periods and data sources are described in Appendix A. The premiums are estimated using GMM. Standard errors are in parentheses, and *, **, and *** denote significance at the 10%, 5%, and 1% levels. Values with the superscript *i* are imposed by cross-price restrictions for the GDA5. RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Stocks	25 S × BM	25 S × Mom		6 S × BM	6 S × Mom
Options			54	6	6
Currencies				6	6
Panel A: GDA3					
λ_W	0.0069***	0.0078***	0.0056*	0.0070**	0.0070**
	(0.0022)	(0.0021)	(0.0032)	(0.0032)	(0.0034)
$\lambda_{\mathcal{D}}$	−0.0473	−0.3519***	−0.2438***	−0.2200**	−0.2783***
	(0.0896)	(0.1247)	(0.0606)	(0.0940)	(0.1015)
$\lambda_{W\mathcal{D}}$	0.0103	0.0272***	0.0185***	0.0204***	0.0210***
	(0.0067)	(0.0067)	(0.0045)	(0.0047)	(0.0044)
RMSPE	24.8 [0.33]	21.8 [0.29]	11.6 [0.19]	20.3 [0.29]	19.4 [0.27]
Panel B: GDA5					
λ_W	0.0081***	0.0080***	0.0059*	0.0079**	0.0078***
	(0.0023)	(0.0020)	(0.0031)	(0.0031)	(0.0030)
$\lambda_{\mathcal{D}}$	−0.2785*	−0.3071***	−0.3384***	−0.3741***	−0.3397**
	(0.1518)	(0.1061)	(0.0632)	(0.1256)	(0.1374)
$\lambda_{W\mathcal{D}}$	0.0262***	0.0238***	0.0229***	0.0273***	0.0241***
	(0.0093)	(0.0065)	(0.0035)	(0.0056)	(0.0058)
λ_X	-0.0012^i	-0.0011^i	-0.0007^i	0.0001^i	-0.0005^i
$\lambda_{X\mathcal{D}}$	-0.0027^i	-0.0016^i	-0.0012^i	-0.0016^i	-0.0013^i
a	0.8483**	0.4193	0.4008*	0.6212**	0.4324
	(0.3863)	(0.3943)	(0.2398)	(0.2573)	(0.3674)
RMSPE	20.6 [0.27]	16.4 [0.22]	9.7 [0.16]	17.8 [0.25]	16.1 [0.23]
Panel C: Unrestricted GDA5					
λ_W	0.0089***	0.0082***	0.0086***	0.0093***	0.0080***
	(0.0023)	(0.0021)	(0.0028)	(0.0032)	(0.0029)
$\lambda_{\mathcal{D}}$	−0.6088***	−0.2242**	−0.3744***	−0.3245***	−0.1945*
	(0.1640)	(0.1060)	(0.0588)	(0.1195)	(0.1165)
$\lambda_{W\mathcal{D}}$	0.0355***	0.0205***	0.0312***	0.0279***	0.0157**
	(0.0094)	(0.0061)	(0.0062)	(0.0065)	(0.0074)
λ_X	−0.0050***	−0.0025***	−0.0022***	−0.0032	−0.0035*
	(0.0011)	(0.0008)	(0.0006)	(0.0020)	(0.0020)
$\lambda_{X\mathcal{D}}$	−0.0063***	−0.0031***	−0.0045***	−0.0053**	−0.0043**
	(0.0013)	(0.0008)	(0.0012)	(0.0026)	(0.0019)
RMSPE	18.7 [0.25]	15.4 [0.20]	9.3 [0.15]	11.1 [0.16]	13.1 [0.18]

included (from 50 to 26 bps). It is more important for the current paper to compare the VOL and GDA3 models. The GDA3 delivers lower pricing errors than the VOL model for all five sets of portfolios, and the improvement in fit can be considerable as in the case of the size/momentum stock portfolios (from 35 to 24 bps) and when all three asset classes are included (from 26 to 20 bps).

The cross-sectional implications of market downside risk have been previously studied by Ang et al. (2006a) and Lettau et al. (2014). These authors propose slightly different models to incorporate the effect of market downside risk. More importantly, it can be shown that our GDA3 specification nests the models from both of these studies, with different restrictions on the value of $\lambda_{\mathcal{D}}$. Ang et al. (2006a) specify the model for expected returns as

$$E\left[R_{it}^e\right] = \lambda^+ \beta_i^+ + \lambda^- \beta_i^- , \quad \text{with}$$

$$\beta_i^+ = \frac{Cov\left(R_{it}^e, r_{Wt} \mid \mathcal{U}_t\right)}{Var\left(r_{Wt} \mid \mathcal{U}_t\right)} \quad \text{and} \quad \beta_i^- = \frac{Cov\left(R_{it}^e, r_{Wt} \mid \mathcal{D}_t\right)}{Var\left(r_{Wt} \mid \mathcal{D}_t\right)}, \tag{19}$$

where $\mathcal{U}$ refers to the upside event, which is the complement of the disappointing event $\mathcal{D}$. The model in (19) is equivalent to the GDA3 in (16) with

$$\lambda_W = \lambda^+ + \lambda^-, \quad \lambda_{\mathcal{D}} = 0, \quad \lambda_{W\mathcal{D}} = \lambda^-. \tag{20}$$

That is, the model proposed by Ang et al. (2006a) imposes the restriction $\lambda_{\mathcal{D}} = 0$. On the other hand, Lettau et al. (2014) propose

$$E\left[R_{it}^e\right] = \lambda \beta_i + \lambda^- \left(\beta_i^- - \beta_i\right), \tag{21}$$

where β_i is the CAPM beta and β_i^- is the same downside beta as in (19). The specification in (21) is equivalent to

Table 4
Risk premiums for alternative models. The table shows risk premium estimates for different models using various sets of test assets: (i) 25 (5 × 5) size/book-to-market portfolios; (ii) 25 (5 × 5) size/momentum portfolios; (iii) 54 index option portfolios from Constantinides et al. (2013); (iv) 6 size/book-to-market, 6 option, and 6 currency (from Lettau et al., 2014) portfolios; and (v) 6 size/momentum, 6 option, and 6 currency portfolios. We use monthly data and the sample periods and data sources are described in Appendix A. The premiums are estimated using GMM. Standard errors are in parentheses, and *, **, and *** denote significance at the 10%, 5%, and 1% levels. Values with the superscript i are imposed by the restriction that the market portfolio should be correctly priced (and by the restriction in (20) and (22) for the models in Panel B and Panel C, respectively). RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

	Stocks	25 S × BM	25 S × Mom		6 S × BM	6 S × Mom
	Options			54	6	6
	Currencies				6	6
Panel A: VOL						
	λ_W	0.0054***	0.0054***	0.0058***	0.0057***	0.0057***
		(0.0005)	(0.0005)	(0.0001)	(0.0002)	(0.0002)
	λ_X	-0.0023^i	-0.0024^i	-0.0030^i	-0.0035^i	-0.0036^i
	RMSPE	25.9 [0.34]	35.4 [0.46]	14.2 [0.23]	26.2 [0.37]	26.8 [0.38]
Panel B: Ang et al. (2006a)						
	λ_W	0.0066***	0.0070***	0.0070***	0.0072***	0.0072***
		(0.0017)	(0.0008)	(0.0005)	(0.0006)	(0.0005)
	$\lambda_{\mathcal{D}}$	0^i	0^i	0^i	0^i	0^i
	$\lambda_{W\mathcal{D}}$	0.0140^i	0.0169^i	0.0138^i	0.0175^i	0.0171^i
	RMSPE	25.8 [0.34]	28.3 [0.37]	19.5 [0.31]	25.9 [0.36]	28.4 [0.40]
Panel C: Lettau et al. (2014)						
	λ_W	0.0065***	0.0066***	0.0069***	0.0073***	0.0071***
		(0.0016)	(0.0009)	(0.0006)	(0.0006)	(0.0005)
	$\lambda_{\mathcal{D}}$	0.0557^i	0.0607^i	0.0531^i	0.0896^i	0.0835^i
	$\lambda_{W\mathcal{D}}$	0.0116^i	0.0122^i	0.0112^i	0.0146^i	0.0140^i
	RMSPE	25.7 [0.34]	31.0 [0.41]	24.0 [0.39]	30.4 [0.43]	33.3 [0.47]
Panel D: Carhart (1997)						
	λ_W	0.0054***	0.0052***	0.0059***	0.0054***	0.0053***
		(0.0002)	(0.0000)	(0.0004)	(0.0004)	(0.0001)
	λ_{SMB}	0.0026^i	0.0020^i	0.0117^i	0.0023^i	0.0023^i
	λ_{HML}	0.0047***	0.0076**	0.0451	0.0041	0.0102
		(0.0014)	(0.0034)	(0.0372)	(0.0026)	(0.0106)
	λ_{WML}	0.0254	0.0073***	0.0018	0.0168	0.0064
		(0.0203)	(0.0021)	(0.0181)	(0.0335)	(0.0039)
	RMSPE	11.2 [0.15]	13.9 [0.18]	19.4 [0.31]	45.6 [0.64]	44.4 [0.63]

the GDA3 in (16) with

$$\lambda_W = \lambda, \quad \lambda_{\mathcal{D}} = \frac{\gamma_2}{1-\gamma_1}(\lambda_W - \lambda_{W\mathcal{D}}),$$
$$\lambda_{W\mathcal{D}} = \gamma_1\lambda + (1-\gamma_1)\lambda^-, \tag{22}$$

where

$$\gamma_1 \equiv \frac{Cov(r_{Wt}I(\mathcal{D}_t), r_{Wt})}{Var(r_{Wt})}, \quad \gamma_2 \equiv \frac{Cov(I(\mathcal{D}_t), r_{Wt})}{Var(r_{Wt})}. \tag{23}$$

That is, the model proposed by Lettau et al. (2014) imposes $\lambda_{\mathcal{D}} = \frac{\gamma_2}{1-\gamma_1}(\lambda_W - \lambda_{W\mathcal{D}})$. The derivation of the above results is shown in Appendix B.

Panels B and C of Table 4 present risk premiums for the models of Ang et al. (2006a) and Lettau et al. (2014), respectively. The models are estimated using GMM, imposing the linear restriction on $\lambda_{\mathcal{D}}$ during the estimation for both models.[12] Note that the restriction imposed by the model of Ang et al. (2006a) ($\lambda_{\mathcal{D}} = 0$) is rejected in four out of five cases for the GDA3 model in Table 3 (where we can assess the statistical significance of $\lambda_{\mathcal{D}}$), where $\lambda_{\mathcal{D}}$ is negative and significantly different from zero. Comparing model fit, the GDA3 model is always associated with lower pricing errors than the model of Ang et al. (2006a), and the difference can be substantial, as in case of the option portfolios (12 bps for the GDA3 in Table 2 and 20 bps

[12] Without the restriction that market portfolio should be priced perfectly, estimating λ^+ and λ^- from (19) using the Fama–MacBeth (1973) procedure and applying the transformations in (20) leads to the same λ values as estimating the GDA3 using GMM with the restriction $\lambda_{\mathcal{D}} = 0$. Similarly, estimating λ and λ^- from (21) using the Fama–MacBeth (1973) procedure and applying the transformations in (22) leads to the same λ values as estimating the GDA3 using GMM with the restriction on $\lambda_{\mathcal{D}}$ from (22). In Panels B and C of Table 4, we also impose the restriction that market portfolio should be correctly priced.

Table 5
Fit of models when the same risk premiums are used. The table shows the root-mean-squared pricing error (reported in basis points per month) of different models on different sets of portfolios when the same risk premiums are used across all sets. The shaded column in all three panels indicates which set is used for estimating the risk premiums. In Panel A, 10 size, 10 book-to-market, and 10 momentum portfolios are used to estimate the λ-s (third column of Table 1). In Panel B, 54 index option portfolios are used to estimate the λ-s (first column of Table 2 and third column of Table 4). In Panel C, 6 size/momentum, 6 option, and 6 currency portfolios are used to estimate the λ-s (last column of Tables 2 and 4). The test portfolios in the first five columns are the same as in Table 1, while the test portfolios in the last five columns are the same as in Table 2. Sample periods and data sources are described in Appendix A. Within each column the lowest RMSPE value is boldfaced and underlined, while the second lowest RMSPE value is boldfaced.

Stocks	S×BM	S×Mom	S,B,M	S×OP	S×INV		S×BM	S×Mom	S×BM	S×Mom
Options						✓	✓	✓	✓	✓
Currencies									✓	✓
Panel A: Stocks only										
CAPM	30.6	39.4	24.9	23.9	29.0	43.8	40.5	42.8	49.7	50.5
GDA3	27.2	25.1	19.8	19.4	25.8	**12.9**	**22.7**	**22.4**	**20.8**	**21.1**
GDA5	**25.2**	**21.8**	**18.7**	**18.4**	24.3	**12.9**	**20.8**	**20.8**	**23.0**	**22.7**
Ang et al.	25.8	28.7	21.9	19.1	23.4	22.3	25.3	27.1	27.8	29.9
Lettau et al.	26.0	31.7	23.0	19.4	23.4	26.7	28.1	29.7	35.0	36.8
VOL	26.1	35.5	23.4	19.1	**23.0**	20.9	25.8	28.3	33.1	34.0
Carhart	**14.0**	**17.7**	**9.7**	**13.9**	**15.0**	39.7	34.3	34.1	46.1	45.8
Panel B: Options only										
CAPM	28.7	38.6	24.1	21.8	26.9	43.8	39.8	42.4	48.9	49.9
GDA3	25.7	**24.9**	**18.6**	**17.9**	23.9	**11.9**	**21.7**	**21.3**	**21.3**	**21.6**
GDA5	**24.6**	**21.7**	**18.8**	**18.1**	24.7	**10.0**	**20.2**	**21.8**	**19.0**	**19.0**
Ang et al.	**25.2**	29.3	21.6	18.9	**22.7**	19.5	24.4	26.0	27.5	29.5
Lettau et al.	25.3	31.7	23.0	19.4	**22.4**	24.0	26.7	28.8	31.9	34.1
VOL	26.6	36.2	24.0	20.6	23.0	14.2	24.4	26.5	26.8	27.8
Carhart	233.4	155.5	157.7	154.6	169.0	19.4	180.9	126.0	149.2	91.8
Panel C: Stocks, options, and currencies										
CAPM	29.9	39.1	24.6	23.1	28.2	43.7	40.2	42.6	49.4	50.3
GDA3	28.1	25.5	**19.4**	19.9	26.9	**14.9**	**24.3**	**23.0**	**20.9**	**19.8**
GDA5	30.3	**25.0**	**20.3**	22.1	29.5	**13.1**	**22.3**	**22.0**	**19.2**	**17.3**
Ang et al.	**26.0**	28.6	21.9	**19.8**	23.6	24.2	26.8	29.1	25.9	28.4
Lettau et al.	**26.1**	32.1	23.9	20.7	**23.3**	28.8	29.3	32.1	30.5	33.3
VOL	27.6	36.5	24.6	21.9	23.9	16.6	26.4	27.8	26.2	26.8
Carhart	32.1	**17.0**	24.7	**17.6**	**16.2**	36.4	41.4	33.2	49.4	44.4

for the Ang et al., 2006a model in Table 4) and in the case when all three asset classes are included (20 bps versus 28 bps). The model of Lettau et al. (2014) imposes a different restriction on $\lambda_{\mathcal{D}}$, and as it can be seen in Panel C of Table 4, all the implied $\lambda_{\mathcal{D}}$ values are positive. Since the downstate premium values are typically negative for the GDA3, this restriction is also not in line with the data. The Lettau et al. (2014) model provides a poorer fit than the other two models. The only exception is the size/book-to-market portfolios, where the RMSPE of the Lettau et al. (2014) model is marginally lower than the RMSPE of the Ang et al. (2006a) model (but not lower than that of the GDA3). In general, the models proposed by Ang et al. (2006a) and Lettau et al. (2014) impose restrictions, compared to the GDA3, that are not supported by the data. Panels E and F in Fig. 1 show scatter plots of actual versus predicted returns for the Ang et al. (2006a) and Lettau et al. (2014) models. These plots provide a visual evidence that the GDA3 has a better fit than the two nested models.

Panel D of Table 4 corresponds to the four-factor model of Carhart (1997), which is an important benchmark in the literature. The Carhart (1997) model does a good job in pricing the size/book-to-market and size/momentum portfolios. This is not surprising, as the four-factor model is tailor-made to price these stock portfolios correctly. When we consider other asset classes, the Carhart (1997) model is much less successful. When estimating the model using option portfolios, the pricing error is twice as much as

Table 6
Risk premiums for the GDA models using additional asset classes. The table shows risk premium estimates for the GDA models when we add corporate bond, sovereign bond, and commodity futures portfolios to our benchmark set of test assets. The benchmark set of test assets consists of 6 stock portfolios (size/book-to-market), 6 option portfolios, and 6 currency portfolios. We use monthly data and the sample periods and data sources are described in Appendix A. The premiums are estimated using GMM. Standard errors are in parentheses, and *, **, and *** denote significance at the 10%, 5%, and 1% levels. Values with the superscript *i* are imposed by the restriction that the market portfolio should be correctly priced (and by cross-price restrictions for the GDA5). RMSPE is the root-mean-squared pricing error of the model in basis points per month and the RMSPE to root-mean-squared returns ratio is reported in brackets.

Stocks	6 S × BM	6 S × BM	6 S × BM	6 S × BM
Options	6	6	6	6
Currencies	6	6	6	6
Corp. bonds	5			5
Sov. bonds		6		6
Commodities			6	6
Panel A: GDA3				
λ_W	0.0069***	0.0060***	0.0068***	0.0066***
	(0.0004)	(0.0004)	(0.0007)	(0.0006)
$\lambda_{\mathcal{D}}$	-0.1546^i	-0.3238^i	-0.1615^i	-0.1472^i
$\lambda_{W\mathcal{D}}$	0.0201***	0.0193***	0.0198***	0.0178***
	(0.0055)	(0.0051)	(0.0053)	(0.0052)
RMSPE	21.5 [0.34]	26.2 [0.42]	23.2 [0.35]	28.1 [0.50]
Panel B: GDA5				
λ_W	0.0067***	0.0063***	0.0066***	0.0066***
	(0.0007)	(0.0006)	(0.0006)	(0.0005)
$\lambda_{\mathcal{D}}$	-0.2592^i	-0.3980^i	-0.2009^i	-0.1997^i
$\lambda_{W\mathcal{D}}$	0.0213***	0.0231***	0.0193***	0.0191***
	(0.0051)	(0.0052)	(0.0040)	(0.0040)
λ_X	-0.0010^i	-0.0002^i	-0.0014^i	-0.0015^i
$\lambda_{X\mathcal{D}}$	-0.0015^i	-0.0005^i	-0.0017^i	-0.0018^i
a	0.3170	0.3115	0.2545	0.3171
	(0.6596)	(0.3642)	(0.6878)	(0.5433)
RMSPE	20.0 [0.31]	23.5 [0.37]	23.1 [0.35]	26.9 [0.47]

that of the GDA5 model, but even more importantly, the estimated risk premiums change considerably compared to the stock portfolios. In other words, the estimated risk premiums are very different, when different asset classes are used. Consequently, the Carhart (1997) model performs badly when estimated using the three asset classes jointly: only the CAPM provides higher RMSPE values. In general, the four-factor model works well for pricing stock portfolios, but it is less successful in pricing portfolios from other asset classes. This is also illustrated in Panel G of Fig. 1. The stock portfolios line up along the 45-degree line, but the portfolios from other asset classes do not.

3.2.4. *Variation in the risk premium estimates across test portfolios*

Risk premium estimates for all models vary across different sets of test assets, and it is hard to tell whether the variation we observe is substantial or not. To address this concern, we carry out the following exercise: for all the models, we take risk premium estimates from a given set of test portfolios, and calculate out-of-sample RMSPEs on the other sets of portfolios using these risk premiums. In other words, we assess the model fit when the same risk premium estimates are used across different sets of test portfolios. The out-of-sample RMSPE values are reported in Table 5. The shaded column in all three panels indicates which set is used for estimating the risk premiums. In Panel A, the λ-s correspond to the case when 30 stock portfolios consisting of ten size, ten book-to-market, and ten momentum portfolios are used for the estimation. Panel B corresponds to the case when 54 index option portfolios are used for the estimation of the λ-s. In Panel C, λ-s are estimated using six size/momentum, six option, and six currency portfolios.

The picture is very clear when considering sets that do not solely include stock portfolios: the lowest pricing errors are delivered by the GDA models, regardless of which set of portfolios is used for estimation. In the last five columns of Table 5, the lowest RMSPE values in all panels are produced by the GDA models, and the lowest pricing error typically corresponds to the GDA5. When considering stock-only sets in the first five columns of Table 5, the results are more mixed. In Panel A, where the λ-s are estimated using stock portfolios only, the lowest out-of-sample pricing errors are delivered by the Carhart (1997) model. Nevertheless, the second lowest RMSPE typically corresponds to the GDA5. In Panel B, where the λ-s are estimated using option portfolios only, the lowest out-of-sample pricing errors are typically delivered by the GDA models. Note also that the out-of-sample RMSPE values from the Carhart (1997) model are extremely high in this case. Finally, there are no clear tendencies for the stock-only sets in Panel C. Altogether, the results in Table 5 show that the GDA models perform well even if the same risk premiums are used across the different sets of test portfolios.

3.3. *Robustness checks*

3.3.1. *Additional portfolios as test assets*

In this section we add corporate bond, sovereign bond, and commodity futures portfolios as test assets. Five corporate bond portfolios, sorted annually on their credit spread, are from Nozawa (2012). Sovereign bond and commodity futures portfolios are from Asness et al. (2013), who create three value and three momentum portfolios in both asset classes. A more detailed description of these portfolios and the data sources can be found in Appendix A. Risk premium estimates for the GDA models are reported in Table 6.[13] The results are robust to the addition of these asset classes. All the risk premiums have the expected signs, and all the estimated risk premiums are statistically significant. In terms of pricing error, the GDA5 delivers lower RMSPE values than any of the alternative models considered in the paper.

We also consider the robustness of our results when the same asset classes are used as in Table 2, but different

[13] The corresponding results for alternative models are in the Online Appendix. Also note that stocks are represented by the six size/book-to-market portfolios in Table 6. Results with the six size/momentum portfolios are in the Online Appendix.

test portfolios are chosen to represent a given asset class. When stocks and options are jointly considered, we also use the size/operating-profit and size/investment portfolios to represent stocks. When stocks, options, and currencies are jointly considered, we use ten industry portfolios to represent stocks and also use six portfolios provided by Lustig et al. (2011) to represent currencies. The results, presented in the Online Appendix, show that the risk premium estimates are robust to the choice of test portfolios.

3.3.2. Changing the disappointment threshold

The disappointment threshold is set to $b = -0.03$ throughout the paper. As a robustness check, we consider the thresholds $b \in \{0, -0.015, -0.04\}$. The results and a more detailed assessment can be found in the Online Appendix, but we provide a brief summary here for the GDA5. The results remain very similar for the lower threshold ($b = -0.04$). When the threshold is higher ($b = -0.015$ or $b = 0$), the risk premiums, with one exception, also remain similar to those in our benchmark specification. The exception is the downstate premium, $\lambda_{\mathcal{D}}$, which comes closer to zero and can eventually turn into positive as the threshold increases. That is, disappointing events should be sufficiently out in the left tail so that the downstate factor commands a negative premium. In terms of model fit, the lowest RMSPE is typically provided by the models with low disappointment threshold (either $b = -0.03$ or $b = -0.04$).

The results on the model fit have implications on the preference specification in our theoretical model. Recall that in the generalized disappointment aversion framework, parameter κ determines the level of the disappointment threshold relative to the certainty equivalent. Our results that the model fit is better when disappointing events are sufficiently out in the left tail suggest that we should consider $\kappa < 1$ in a representative agent setup.

3.3.3. Alternative measures of market volatility

We also consider how the risk premiums for the GDA5 change if different measures of market volatility are used. Our alternative measures are the option-implied volatility index (VIX), realized volatility calculated from intra-daily market returns, and a model-implied volatility calculated using an Exponential GARCH specification. Details on how these alternative measures are calculated and the estimated risk premiums are presented in the Online Appendix. The conclusions are similar to our benchmark case, where monthly volatility is measured as realized volatility of the daily market returns during the month. The signs on the risk premiums are as expected, their magnitudes are similar, and the estimated premiums are statistically significant. The model fit is also similar across the different volatility measures.

4. Conclusion

This paper provides an analysis of downside risks in asset prices. Our empirical tests are motivated by the cross-sectional implications of a dynamic consumption-based general equilibrium model where the representative investor has generalized disappointment aversion preferences and macroeconomic uncertainty is time-varying. We explicitly characterize the factors that are valued by an investor in such setting. Besides the market return and market volatility, three disappointment-related factors are also priced: a downstate factor, a market downside factor, and a volatility downside factor. We also show that in addition to a fall in the market return, downside risk may also be associated with a rise in market volatility. The empirical tests confirm that these factors are priced in the cross-section of various asset classes, including stocks, options, currencies, Treasury bonds, corporate bonds, and commodity futures.

The related literature has mainly focused on the time series implications of this general equilibrium setting, discussing the preference parameter values necessary to match empirical regularities in equity returns, risk-free rate, variance premium, and options. Estimating these preference parameter values to jointly target both the time series and the cross-section of asset returns constitutes an interesting avenue for future research.

Appendix A. Data

Return data on US stock portfolios are from Kenneth French's data library.[14] We use various sets of stock portfolios in our tests. The sample period for the stock portfolios is from July 1964 to December 2016.

Index option returns are from Constantinides et al. (2013).[15] They construct a panel of S&P 500 index option portfolios. The data set contains leverage-adjusted (that is, with a targeted market beta of one) monthly returns of 54 ($2 \times 3 \times 9$) option portfolios split across two types (call and put), three targeted time to maturities (30, 60, or 90 days), and nine targeted moneyness levels (10% ITM, 7.5% ITM, 5% ITM, 2.5% ITM, ATM, 2.5% OTM, 5% OTM, 7.5% OTM, and 10% OTM). The option data are available from April 1986 to January 2012. In estimations when we use only 24 ($2 \times 3 \times 4$) option portfolios, we use only a subset of the portfolios corresponding to two types (call and put), three maturities (30, 60, or 90 days), and four moneyness levels (5% ITM, ATM, 5% OTM, and 10% OTM). In cases when we use only six (2×3) option portfolios, these contain short maturity (30 days) options split across two types (call and put) and three moneyness levels (ATM, 5% OTM, and 10% OTM).

Currency returns are from Lettau et al. (2014), who use monthly data on 53 currencies to create six portfolios by sorting them in ascending order of their respective interest rates.[16] The sixth (highest interest rate) portfolio is split into two baskets, 6A and 6B, and portfolio 6B has currencies with annualized inflation at least 10% higher than US inflation in the same month. We follow Lettau et al. (2014) and use the 6A portfolio to obtain our results.

[14] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[15] Data are from Alexi Savov's website at http://pages.stern.nyu.edu/asavov/alexisavov/Alexi_Savov.html.

[16] Data are from Michael Weber's website at http://faculty.chicagobooth.edu/michael.weber.

Currency returns are available from January 1974 to March 2010.

Corporate bond portfolios, sorted annually on their credit spread, are from Nozawa (2012). Five portfolios are obtained by equally weighting the ten portfolios in the benchmark analysis of Nozawa (2012) into five baskets.[17] The corporate bond returns are available from October 1975 to March 2010.

Sovereign bond portfolios are from Asness et al. (2013) who sort government bond indexes into three portfolios based on value and three portfolios based on momentum, separately. We use all six portfolios in our estimation.[18] The portfolio returns are available from January 1983 to December 2016.

Commodity futures portfolios are also from Asness et al. (2013) who sort commodity futures into three portfolios based on value and three portfolios based on momentum, separately. We use all six portfolios in our estimation. The portfolio returns are available from January 1972 to December 2016.

In cases when multiple asset classes are used at the same time, the sample period is always the longest possible period for which all asset classes have data available.

Appendix B. The GDA3 and nested models

To calculate betas in the GDA3 model, the following regression is estimated:

$$R_{it}^e = \alpha_i + \beta_{iW} r_{Wt} + \beta_{i\mathcal{D}} I(\mathcal{D}_t) + \beta_{iW\mathcal{D}} r_{Wt} I(\mathcal{D}_t) + \varepsilon_{it} \ . \quad \text{(B.1)}$$

The mechanics of the ordinary least squares (OLS) implies $E[\varepsilon_{it}] = E[\varepsilon_{it} r_{Wt}] = E[\varepsilon_{it} I(\mathcal{D}_t)] = E[\varepsilon_{it} r_{Wt} I(\mathcal{D}_t)] = 0$, where ε_{it} denotes residuals from the estimation. Then, with the estimated α_i and β_i-s,

$$E\left[R_{it}^e\right] = \alpha_i + \beta_{iW} E[r_{Wt}] + \beta_{i\mathcal{D}} \pi + \beta_{iW\mathcal{D}} E[r_{Wt} \mid \mathcal{D}_t] \pi \quad \text{(B.2)}$$

$$\begin{aligned} E\left[R_{it}^e r_{Wt}\right] =& \alpha_i E[r_{Wt}] + \beta_{iW} E\left[r_{Wt}^2\right] + \beta_{i\mathcal{D}} E[r_{Wt} \mid \mathcal{D}_t] \pi \\ &+ \beta_{iW\mathcal{D}} E\left[r_{Wt}^2 \mid \mathcal{D}_t\right] \pi \end{aligned} \quad \text{(B.3)}$$

$$E\left[R_{it}^e \mid \mathcal{D}_t\right] = (\alpha_i + \beta_{i\mathcal{D}}) + (\beta_{iW} + \beta_{iW\mathcal{D}}) E[r_{Wt} \mid \mathcal{D}_t] \quad \text{(B.4)}$$

$$\begin{aligned} E\left[R_{it}^e r_{Wt} \mid \mathcal{D}_t\right] =& (\alpha_i + \beta_{i\mathcal{D}}) E[r_{Wt} \mid \mathcal{D}_t] \\ &+ (\beta_{iW} + \beta_{iW\mathcal{D}}) E\left[r_{Wt}^2 \mid \mathcal{D}_t\right] , \end{aligned} \quad \text{(B.5)}$$

where $\pi \equiv E[I(\mathcal{D}_t)]$ is the unconditional probability of disappointment. Also note that the occurrence of the upside event, the complement of the disappointing event, can be written as $I(\mathcal{U}_t) = 1 - I(\mathcal{D}_t)$, hence (B.1) can be rewritten as

$$\begin{aligned} R_{it}^e =& \alpha_i + \beta_{iW} r_{Wt} + \beta_{iW\mathcal{D}} r_{Wt} \cdot [1 - I(\mathcal{U}_t)] \\ &+ \beta_{i\mathcal{D}}[1 - I(\mathcal{U}_t)] + \varepsilon_{it} \\ =& (\alpha_i + \beta_{i\mathcal{D}}) + (\beta_{iW} + \beta_{iW\mathcal{D}}) r_{Wt} - \beta_{iW\mathcal{D}} r_{Wt} \cdot I(\mathcal{U}_t) \\ &- \beta_{i\mathcal{D}} I(\mathcal{U}_t) + \varepsilon_{it} . \end{aligned} \quad \text{(B.6)}$$

Again, the mechanics of the OLS, namely, $E[\varepsilon_{it} I(\mathcal{U}_t)] = E[\varepsilon_{it} r_{Wt} I(\mathcal{U}_t)] = 0$, gives us

$$E\left[R_{it}^e | \mathcal{U}_t\right] = \alpha_i + \beta_{iW} E[r_{Wt} | \mathcal{U}_t] \quad \text{(B.7)}$$

$$E\left[R_{it}^e r_{Wt} | \mathcal{U}_t\right] = \alpha_i E[r_{Wt} | \mathcal{U}_t] + \beta_{iW} E\left[r_{Wt}^2 | \mathcal{U}_t\right] . \quad \text{(B.8)}$$

Using (B.4) and (B.5), it can be shown that the market downside beta is

$$\begin{aligned} \beta_i^- &\equiv \frac{Cov\left(R_{it}^e, r_{Wt} | \mathcal{D}_t\right)}{Var(r_{Wt} | \mathcal{D}_t)} \\ &= \frac{E\left[R_{it}^e r_{Wt} | \mathcal{D}_t\right] - E\left[R_{it}^e | \mathcal{D}_t\right] E[r_{Wt} | \mathcal{D}_t]}{Var(r_{Wt} | \mathcal{D}_t)} = \beta_{iW} + \beta_{iW\mathcal{D}} \ . \end{aligned} \quad \text{(B.9)}$$

Using (B.7) and (B.8), the upside beta is

$$\begin{aligned} \beta_i^+ &\equiv \frac{Cov\left(R_{it}^e, r_{Wt} | \mathcal{U}_t\right)}{Var(r_{Wt} | \mathcal{U}_t)} \\ &= \frac{E\left[R_{it}^e r_{Wt} | \mathcal{U}_t\right] - E\left[R_{it}^e | \mathcal{U}_t\right] E[r_{Wt} | \mathcal{U}_t]}{Var(r_{Wt} | \mathcal{U}_t)} = \beta_{iW} \ . \end{aligned} \quad \text{(B.10)}$$

Finally, using (B.2) and (B.3) it can be shown that

$$\begin{aligned} Cov\left(R_{it}^e, r_{Wt}\right) =& \beta_{iW} Var(r_{Wt}) + \beta_{iW\mathcal{D}} Cov(r_{Wt} I(\mathcal{D}_t), r_{Wt}) \\ &+ \beta_{i\mathcal{D}} Cov(I(\mathcal{D}_t), r_{Wt}) . \end{aligned} \quad \text{(B.11)}$$

Hence, the CAPM beta is

$$\begin{aligned} \beta_i \equiv& \frac{Cov\left(R_{it}^e, r_{Wt}\right)}{Var(r_{Wt})} = \beta_{iW} + \beta_{iW\mathcal{D}} \underbrace{\frac{Cov(r_{Wt} I(\mathcal{D}_t), r_{Wt})}{Var(r_{Wt})}}_{\equiv \gamma_1} \\ &+ \beta_{i\mathcal{D}} \underbrace{\frac{Cov(I(\mathcal{D}_t), r_{Wt})}{Var(r_{Wt})}}_{\equiv \gamma_2} . \end{aligned} \quad \text{(B.12)}$$

Using (B.9) and (B.10), the model in (19) can be written as

$$E\left[R_{it}^e\right] = \lambda^+ \beta_i^+ + \lambda^- \beta_i^- = \left(\lambda^+ + \lambda^-\right) \beta_{iW} + \lambda^- \beta_{iW\mathcal{D}} \ . \quad \text{(B.13)}$$

Using (B.9) and (B.12), the model in (21) can be written as

$$\begin{aligned} E\left[R_i^e\right] &= \lambda \beta_i + \lambda^- \left(\beta_i^- - \beta_i\right) \\ &= \lambda \beta_{iW} + \left(\gamma_1 \lambda + (1 - \gamma_1) \lambda^-\right) \beta_{iW\mathcal{D}} + \gamma_2 \left(\lambda - \lambda^-\right) \beta_{i\mathcal{D}} . \end{aligned} \quad \text{(B.14)}$$

[17] Data are from Michael Weber's website, from the replication data set connected to Lettau et al. (2014).

[18] An updated and extended version of the portfolios used by Asness et al. (2013) is available from the website of AQR Capital Management at https://www.aqr.com/library/data-sets.

References

Adrian, T., Rosenberg, J., 2008. Stock returns and volatility: pricing the short-run and long-run components of market risk. J. Finance 63 (6), 2997–3030.

Ang, A., Chen, J., Xing, Y., 2006a. Downside risk. Rev. Financial Stud. 19 (4), 1191–1239.

Ang, A., Hodrick, R.J., Xing, Y., Zhang, X., 2006b. The cross-section of volatility and expected returns. J. Finance 61 (1), 259–299.

Ang, A., Liu, J., Schwarz, K., 2017. Using Stocks or Portfolios in Tests of Factor Models Unpublished Working Paper. Columbia University, University of California - San Diego, University of Pennsylvania.

Asness, C.S., Moskowitz, T.J., Pedersen, L.H., 2013. Value and momentum everywhere. J. Finance 68 (3), 929–985.

Bandi, F. M., Moise, C. E., Russell, J. R., 2006. Market Volatility, Market Frictions, and the Cross-section of Stock Returns Unpublished Working Paper. University of Chicago, Case Western Reserve University
Bansal, R., Khatchatrian, V., Yaron, A., 2005. Interpretable asset markets? Eur. Econ. Rev. 49 (3), 531–560.
Bansal, R., Yaron, A., 2004. Risks for the long run: A potential resolution of asset pricing puzzles. J. Finance 59 (4), 1481–1509.
Bawa, V.S., Lindenberg, E.B., 1977. Capital market equilibrium in a mean-lower partial moment framework. J. Financial Econ. 5 (2), 189–200.
Beeler, J., Campbell, J.Y., 2012. The long-run risks model and aggregate asset prices: An empirical assessment. Crit. Finance Rev. 1 (1), 141–182.
Black, F., 1976. Studies of stock price volatility changes. In: Proceedings of the 1976 Meeting of the American Statistical Association, Business and Economic Statistics Section, pp. 177–181.
Bollerslev, T., Tauchen, G., Zhou, H., 2009. Expected stock returns and variance risk premia. Rev. Financial Stud. 22 (11), 4463–4492.
Bonomo, M., Garcia, R., Meddahi, N., Tédongap, R., 2011. Generalized disappointment aversion, long-run volatility risk and aggregate asset prices. Rev. Financial Stud. 24 (1), 82–122.
Calvet, L.E., Fisher, A.J., 2007. Multifrequency news and stock returns. J. Financial Econ. 86 (1), 178–212.
Campbell, J.Y., 1993. Intertemporal asset pricing without consumption data. Am. Econ. Rev. 83 (3), 487–512.
Carhart, M.M., 1997. On persistence in mutual fund performance. J. Finance 52 (1), 57–82.
Christie, A.A., 1982. The stochastic behavior of common stock variances: Value, leverage and interest rate effects. J. Financial Econ. 10 (4), 407–432.
Clarida, R., Davis, J., Pedersen, N., 2009. Currency carry trade regimes: Beyond the Fama regression. J. Inter. Money Finance 28 (8), 1375–1389.
Cochrane, J.C., 2000. Asset Pricing. Princeton University Press, Princeton, NJ.
Constantinides, G.M., Jackwerth, J.C., Savov, A., 2013. The puzzle of index option returns. Rev. Asset Pricing Stud. 1, 1–29.
Daniel, K., Moskowitz, T.J., 2016. Momentum crashes. J. Financial Econ. 122 (2), 221–247.
Delikouras, S., 2014. Why Corporate Bonds May Disappoint: Disappointment Aversion and the Credit Spread Puzzle Unpublished Working Paper. University of Miami.
Delikouras, S., 2017. Where the kink? Disappointment events in consumption growth and equilibrium asset prices. Rev. Financial Stud. 30 (8), 2851–2889.
Epstein, L.G., Zin, S.E., 1989. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical framework. Econ. 57 (4), 937–969.
Fama, E.F., MacBeth, J.D., 1973. Risk, return, and equilibrium: empirical tests. J. Political Econ. 81 (3), 607–636.
Gagliardini, P., Ossola, E., Scaillet, O., 2016. Time-varying risk premium in large cross-sectional equity data sets. Econometrica 84 (3), 985–1046.
Gul, F., 1991. A theory of disappointment aversion. Econometrica 59 (3), 667–686.
Hansen, L.P., Heaton, J., Lee, J., Roussanov, N., 2007. Intertemporal substitution and risk aversion. In: Heckman, J., Leamer, E. (Eds.), Handbook of Econometrics, 6a (Chapter 61). Elsevier, North Holland, pp. 3967–4056.
Hansen, L.P., Singleton, K.J., 1982. Generalized instrumental variables estimation of nonlinear rational expectations models. Econometrica 50 (5), 1269–1286.
Hansen, L.P., Singleton, K.J., 1983. Stochastic consumption, risk aversion, and the temporal behavior of asset returns. J. Political Econ. 91 (2), 249–265.
Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decision under risk. Econometrica 47 (2), 263–291.
Lettau, M., Maggiori, M., Weber, M., 2014. Conditional risk premia in currency markets and other asset classes. J. Financial Econ. 114 (2), 197–225.
Lewellen, J., Nagel, S., Shanken, J., 2010. A skeptical appraisal of asset pricing tests. J. Financial Econ. 96 (2), 175–194.
Lintner, J., 1965. The valuation of risk assets and selection of risky investments in stock portfolios and capital budgets. Rev. Econ. Stat. 47, 13–37.
Lustig, H., Roussanov, N., Verdelhan, A., 2011. Common risk factors in currency markets. Rev. Financial Stud. 24 (11), 3731–3777.
Markowitz, H., 1959. Portfolio Selection. Yale University Press, New Haven, CT..
Nozawa, Y., 2012. Corporate Bond Premia Unpublished Working Paper. University of Chicago.
Parker, J., Julliard, C., 2005. Consumption risk and the cross section of expected returns. J. Political Econ. 113 (1), 185–222.
Routledge, B.R., Zin, S.E., 2010. Generalized disappointment aversion and asset prices. J. Finance 65 (4), 1303–1332.
Roy, A.D., 1952. Safety first and the holding of assets. Econometrica 20, 431–449.
Schreindorfer, D., 2014. Tails, Fears, and Equilibrium Option Prices Unpublished Working Paper. Arizona State University.
Shanken, J., 1992. On the estimation of beta-pricing models. Rev. Financial Stud. 5 (1), 1–55.
Sharpe, W., 1964. Capital asset prices: a theory of market equilibrium under conditions of risk. J. Finance 19, 425–442.
Tauchen, G., 2011. Stochastic volatility in general equilibrium. Q. J. Finance 1 (4), 707–731.
Weil, P., 1989. The equity premium puzzle and the risk-free rate puzzle. J. Monet. Econ. 24 (3), 401–421.
Wilcox, D.W., 1992. The construction of U.S. consumption data: some facts and their implications for empirical work. Am. Econ. Rev. 82 (4), 922–941.