Asset pricing and ambiguity：Empirical evidence

Menachem Brenner，Yehuda Izhakian

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汇报人 郑睿

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山西财经大学

Menachem Brenner

Professor of Finance, Stern School of Business, NYU
Asset Pricing, Derivatives Markets, Microstructure, Volatility

Informed Options Trading Prior to Takeover Announcements: Insider Trading?（MS, 2019）
Asset pricing and ambiguity: Empirical evidence（JFE, 2018）

Yehuda Izhakian
Zicklin School of Business, Baruch College
Assistant Professor of Economics and Finance

Ambiguity, Volatility, and Credit Risk（RFS, 2019, Forthcoming ）
An Experimental Test of the Lucas Asset Pricing Model
（The Review of Economic Studies, 2019 ）
Asset pricing and ambiguity: Empirical evidence（JFE，2018）
Risk, ambiguity, and the exercise of employee stock options （JFE，2017）
Expected Utility with Uncertain Probabilities Theory
（Journal of Mathematical Economics, 2017）

ABSTRACT

We introduce ambiguity in conjunction with risk to study the relation between risk, ambiguity, and expected returns. Distinguishing between ambiguity and attitudes toward ambiguity, we develop an empirical methodology for measuring the degree of ambiguity and for assessing attitudes toward ambiguity from market data.

The main findings indicate that ambiguity in the equity market is priced. Introducing ambiguity alongside risk provides stronger evidence on the role of risk in explaining expected returns in the equity markets.

The findings also indicate that investors’ level of aversion to or love for ambiguity is contingent on the expected probability of favorable returns.

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1. Introduction

Introduction

Risk in equity markets means that future returns are realized **with known probabilities**

Ambiguity refers to situations where the probabilities associated with these realizations are not known or not uniquely assigned.

We investigate the relation between risk, ambiguity, and expected return in the equity markets over time.

Introduction

numerous studies have investigated the fundamental relation between the risk and return of the market portfolio. The findings are conflicting:

positive relation

(e.g., French et al., 1987; Campbell and Hentschel, 1992; Guo and Whitelaw, 2006; Pástor et al., 2008)

negative relation

(e.g.,Black, 1976; Campbell, 1987; Nelson, 1991; Harvey, 2001).

Reconcile the risk-return relation

conditional variance (e.g., Glosten et al., 1993; Harvey, 2001),

risk measures (e.g., Ghysels et al., 2005).

time-varying risk aversion (e.g.,Campbell and Cochrane, 1999),

investor sentiment (e.g., Yu and Yuan, 2011).

Ambiguity ——one of the determinants of expected return

(e.g., Epstein and Schneider, 2008 , Anderson et al., 2009; Antoniou et al.,2015).

Theory but not empirical

Experiments data but not Market data

aversion to ambiguity but not the impact of ambiguity on financial decision-making.

Introduction

Based on this idea, we propose an empirical measure of ambiguity, which is independent of risk, attitudes toward risk, as well as attitudes toward ambiguity.

$$\mho^2[r] = \int \mathrm{E}[\varphi(r)]\mathrm{Var}[\varphi(r)]dr$$

where $\varphi(\cdot)$ is a probability density function, $\mathrm{E}[\varphi(r)]$ is the expected probability of a given rate of return r, and $\mathrm{Var}[\varphi(r)]$ is the variance of the probability of r.

e.g 1：
-d=−10% and u=20% .
P(d)=P(u)=0.5
The expected return is thus 5%,
The standard deviation of the return (measuring the degree of risk) is 15%.
In this case, since the probabilities are known, ambiguity is not present.
e.g 2:
P(d)=0.4 and P(u)=0.6 or P(d) = 0.6 and P(u)=0.4
Investors now face not only risk but also ambiguity.

$$\mho = \sqrt{\sum_i \mathrm{E}[\mathrm{P}(i)]\mathrm{Var}[\mathrm{P}(i)]} = \sqrt{2 \times 0.5 \times \left(0.5 \times (0.4 - 0.5)^2 + 0.5 \times (0.6 - 0.5)^2\right)} = 0.1$$

Introduction

Our measure ambiguity depends only on the probabilities of outcomes. Thereby, ambiguity is measured independently of risk.

$$\mathbb{E}_t[r_{t+1}] - r_f = \underbrace{\gamma \frac{1}{2} \mathbb{V}\mathrm{ar}_t[r_{t+1}]}_{\text{Risk premium}} + \underbrace{\eta\big(1 - \mathrm{E}_t[\mathrm{P}_{t+1}]\big)\mathbb{E}_t\big[|r_{t+1} - \mathbb{E}_t[r_{t+1}]|\big]\mho_t^2[r_{t+1}]}_{\text{Ambiguity premium}},$$

Under EUUP, there are two phases of the decision-making process:
In the first phase, the investor forms her perceived probabilities for all events that are relevant to her decision.
In the second phase, she assesses the expected value of each alternative using her perceived probabilities and chooses accordingly.

Ambiguity—the uncertainty about probabilities—dominates the first phase, while risk—the uncertainty about outcomes—dominates the second phase.

2. The theoretical model

The theoretical model

To formally define the uncertain return r, let (S, E , P) be a probability space, where S is a state space, E is a subsets of the state space (i.e., a set of events),P is a cumulative probability P measure, and the set of probability measures P is convex.

$$\mathrm{E}[\varphi(r)] \equiv \int_{\mathcal{P}} \varphi(r)d\xi \quad \text{and} \quad \mathrm{E}[\mathrm{P}(r)] \equiv \int_{\mathcal{P}} \mathrm{P}(r)d\xi,$$

$$\mathrm{Var}[\varphi(r)] \equiv \int_{\mathcal{P}} \left(\varphi(r) - \mathrm{E}[\varphi(r)]\right)^2 d\xi$$

$$\mathbb{E}[r] \equiv \int \mathrm{E}[\varphi(r)]rdr \quad \mathbb{V}\mathrm{ar}[r] \equiv \int \mathrm{E}[\varphi(r)]\left(r - \mathbb{E}[r]\right)^2 dr.$$

ambiguity attitude ϒ(·)
ϒ [0 , 1] → R .
ambiguity aversion takes the form of a concave ϒ(·)
ambiguity loving takes the form of a convex ϒ(·),
and ambiguity neutrality the form of a linear ϒ(·).

The theoretical model

Consider a decision to invest one unit of wealth, where future consumption is determined by the one-period (uncertain) return r , which is the only source of wealth. In EUUP, when the investor does not distort perceived probabilities, the expected utility of this investment opportunity can be approximated by

$$
\begin{aligned}
&\mathrm{W}(1+r) \\
&\approx \int_{r\le r_f} \mathrm{U}(1+r)\underbrace{\mathrm{E}[\varphi(r)]\left(1-\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(r)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(r)])}\mathrm{Var}[\varphi(r)]\right)}_{\text{Perceived probability of unfavorable return}} dr \\
&\quad + \int_{r\ge r_f} \mathrm{U}(1+r)\underbrace{\mathrm{E}[\varphi(r)]\left(1+\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(r)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(r)])}\mathrm{Var}[\varphi(r)]\right)}_{\text{Perceived probability of favorable return}} dr,
\end{aligned}
$$

where P(r) is the cumulative probability of return being lower than r .

The theoretical model

The uncertainty premium of a risky and ambiguous consumption 1 + r

$$\mathcal{K} \approx \underbrace{-\frac{1}{2}\frac{U''(1+E[r])}{U'(1+E[r])}\mathbb{V}ar[r]}_{\text{Risk premium}} \underbrace{-\mathbb{E}\left[\frac{\Upsilon''(1-E[P(r)])}{\Upsilon'(1-E[P(r)])}\right]\mathbb{E}[|r-\mathbb{E}[r]|]\mho^2[r]}_{\text{Ambiguity premium}} \qquad \eta(\cdot) = -\frac{\Upsilon''(\cdot)}{\Upsilon'(\cdot)}$$

First, it distinguishes between the risk premium and the ambiguity premium. Second, within each premium, it distinguishes between the two sources of the premium: attitudes and beliefs.

$-\frac{\Upsilon''}{\Upsilon'} > 0$ Aversion to ambiguity，implies a positive ambiguity premium

$-\frac{\Upsilon''}{\Upsilon'} < 0$ Love for ambiguity，implies a negative premium.

A higher degree of ambiguity or a higher aversion to ambiguity result in a greater ambiguity premium.

$$\mathcal{K} \approx \underbrace{\gamma\frac{1}{2}\mathbb{V}ar[r]}_{\text{Risk premium}} + \underbrace{\mathbb{E}\left[\eta(1-E[P(r)])\right]\mathbb{E}\left[|r-\mathbb{E}[r]|\right]\mho^2[r]}_{\text{Ambiguity premium}}, \quad \mho^2[r] = \int E[\phi(r;\mu,\sigma)]Var[\phi(r;\mu,\sigma)]dr,$$

The theoretical model

Hypothesis 1 . When ambiguity is accounted for, the risk premium is positive, as investors typically exhibit risk aversion.

Hypothesis 2 . Investors typically exhibit aversion to ambiguity when expecting favorable returns. Therefore, for a relatively high expected probability of favorable returns, the ambiguity premium is positive.

Hypothesis 3 . Investors typically exhibit love for ambiguity when expecting unfavorable returns. Therefore, for a relatively high expected probability of unfavorable returns, the ambiguity premium is negative.

Hypothesis 4 . Aversion to ambiguity increases with the expected probability of favorable returns and love for ambiguity increases with the expected probability of unfavorable returns. Therefore, the higher the expected probability of favorable returns, the higher is the positive ambiguity premium. On the other hand, the higher the expected probability of unfavorable returns, the higher is the negative ambiguity premium.

3. Data and parameter estimation

Data and parameter estimation

Data： intraday data of the ETF SPDR（The SPDR is designed to track the S&P 500 Index）

From： TAQ database

Period: February 1993 to December 2016, 287 months

Data and parameter estimation

Estimating risk and ambiguity:

（1）prices of the SPDR every five minutes from 9:30 a.m. to 4:00 p.m. each day, which provides 79 prices for each day.

Using these prices, we compute the five-minute returns, which provides a maximum of 78 returns for each day. （For each day in our sample period there are between 33 and 78 observations.）

（2）daily mean and variance of the return, denoted μ and σ^2 , respectively. assume that the intraday returns are normally distributed. For each month, there are 20 to 22 different gain probabilities.

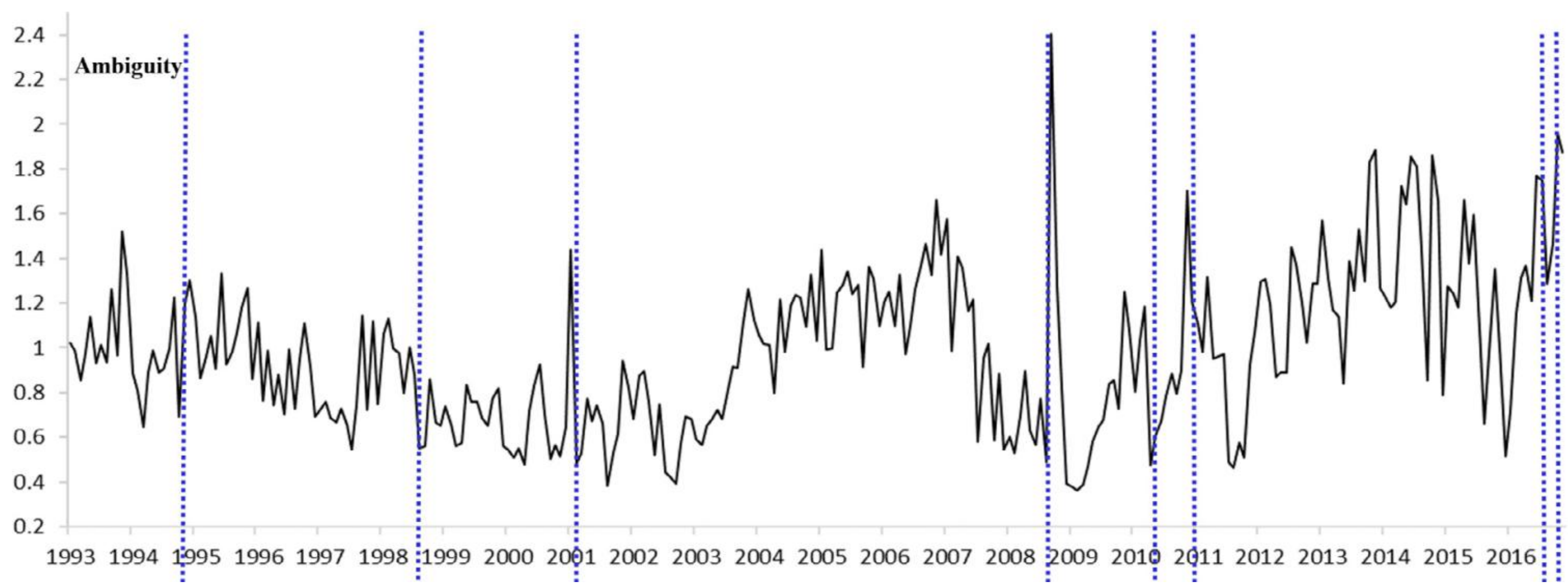
（3）To compute the monthly degree of ambiguity， we represent each daily return distribution by a histogram. we divide the range of daily returns, from −6% to +6% , into 60 intervals (bins), each of

width 0.2%. We compute the probability of the return being lower than −6% and higher than +6% . We then compute the mean and the variance of the probabilities for each of the 62 bins separately.

Data and parameter estimation

$$\mho^2[r] = \frac{1}{w(1-w)} \times \begin{pmatrix} \mathrm{E}\left[\Phi(r_0;\mu,\sigma)\right]\mathrm{Var}\left[\Phi(r_0;\mu,\sigma)\right] \\ +\sum_{i=1}^{60}\mathrm{E}\left[\Phi(r_i;\mu,\sigma)-\Phi(r_{i-1};\mu,\sigma)\right] \\ \times\mathrm{Var}[\Phi(r_i;\mu,\sigma)-\Phi(r_{i-1};\mu,\sigma)] \\ +\mathrm{E}[1-\Phi(r_{60};\mu,\sigma)]\mathrm{Var}[1-\Phi(r_{60};\mu,\sigma)] \end{pmatrix}$$

$r_0 = -0.06$, $w = r_i - r_{i-1} = 0.002$



Data and parameter estimation

Summary statistics.

		N	Mean	Median	Min	Max	Std. dev.	Skewness	Kurtosis
Panel A: Daily descriptive statistics									
n	Num. of obs.	6025	70.783	78.000	3.000	78.000	16.974	−2.246	3.518
μ	Mean return	6025	0.000	0.000	−0.094	0.101	0.011	0.032	7.871
σ	Std. dev.	6025	0.008	0.007	0.001	0.067	0.005	2.940	16.094
μ/σ	Mean/Std	6025	0.073	0.057	−19.003	13.915	1.356	−0.234	16.062
P	Probability	6025	0.512	0.517	0.000	1.000	0.316	−0.037	−1.309
Panel B: Monthly descriptive statistics									
r_f	Risk-free rate	287	0.002	0.002	0.000	0.006	0.002	0.187	−1.564
r	Return	287	0.008	0.013	−0.165	0.109	0.041	−0.648	1.246
$r-r_f$	Excess return	287	0.006	0.010	−0.166	0.109	0.041	−0.641	1.250
$\sqrt{\nu}$	Volatility	287	0.049	0.043	0.007	0.273	0.030	2.847	15.091
VIX	VIX	257	0.059	0.054	0.030	0.175	0.024	1.692	4.405
ϑ	Avg. absolute dev.	287	0.043	0.037	0.012	0.222	0.026	2.791	13.154
$\overline{\mathrm{P}}$	Mean Prob.	287	0.512	0.507	0.358	0.691	0.064	0.204	−0.332
$\mho$	Ambiguity	287	0.986	0.958	0.361	2.405	0.353	0.622	0.367

Panel C: Cross-correlations

	$r-r_f$	ν	ϑ	$\overline{\mathrm{P}}$	$\mho^2$
$r-r_f$	1				
ν	−0.347	1			
	(< 0.0001)				
ϑ	−0.382	0.831	1		
	(< 0.0001)	(< 0.0001)			
$\overline{\mathrm{P}}$	0.648	−0.153	−0.275	1	
	(< 0.0001)	(0.0093)	(< 0.0001)		
$\mho$	0.056	0.027	−0.327	0.244	1
	(0.3423)	(0.6506)	(< 0.0001)	(< 0.0001)	

4. Empirical methodology and results

Empirical methodology and results

The fundamental hypothesis is that expected ambiguity, in addition to expected volatility (risk), is a determinant of the expected return.

$$\ln\sqrt{\nu_t} = \psi_0 + \epsilon_t + \sum_{i=1}^{p} \psi_i \cdot \ln\sqrt{\nu_{t-i}} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{t-i},$$

$$\nu_{t+1}^{E} = \mathbb{E}_t[\nu_{t+1}] = \exp\left(2\widehat{\ln\sqrt{\nu_t}} + 2\mathbb{V}\mathrm{ar}[u_t]\right)$$

Akaike information criterion (AICC)， each p =1 , ... , 10 and q = 1 , . . . , 10

$$\ln \mho_t = \psi_0 + \epsilon_t + \sum_{i=1}^{p} \psi_i \cdot \ln \mho_{t-i} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{t-i}$$

$$\left(\mho_{t+1}^2\right)^E = \mathbb{E}_t\left[\mho_{t+1}^2\right] = \exp\left(2\widehat{\ln \mho_t} + 2\mathbb{V}\mathrm{ar}[u_t]\right)$$

To estimate the expected probability of unfavorable returns

$$\ln Q_t = \psi_0 + \epsilon_t + \sum_{i=1}^{p} \psi_i \cdot \ln Q_{t-i} + \sum_{i=1}^{q} \theta_i \cdot \epsilon_{t-i} \qquad Q_t = \frac{\overline{\mathrm{P}}_t}{1-\overline{\mathrm{P}}_t}$$

$$\mathrm{P}_{t+1}^{E} = \mathbb{E}_t[\mathrm{P}_{t+1}] = \frac{\exp\left(\widehat{\ln Q_t} + \frac{1}{2}\mathbb{V}\mathrm{ar}[u_t]\right)}{1 + \exp\left(\widehat{\ln Q_t} + \frac{1}{2}\mathbb{V}\mathrm{ar}[u_t]\right)}$$

Empirical methodology and results

However, the functional form of $\Upsilon(\cdot)$ is unknown, so we are constrained in extracting it from the data.

$$\mathbb{E}\left[\eta(1-\mathrm{E}[\mathrm{P}(r)])\right] = \mathbb{E}\left[\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(r)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(r)])}\right] = \int \mathrm{E}[\varphi(r)]\frac{\Upsilon''(1-\mathrm{E}[\mathrm{P}(r)])}{\Upsilon'(1-\mathrm{E}[\mathrm{P}(r)])}dr$$

$$\mathbb{E}\left[\eta(1-\mathrm{E}[\mathrm{P}(r)])\right] = \eta\left(\mathrm{P}^{E}\right)$$

P^{E} is the expected probability of favorable returns

Empirical methodology and results

Expected values.

Panel A: Descriptive statistics of forecasted variables

	N	Mean	Median	Min	Max	Std. dev.	Skewness	Kurtosis
$\sqrt{\nu^E}$	257	0.047	0.044	0.018	0.221	0.020	3.399	23.130
ϑ^E	257	0.042	0.036	0.020	0.201	0.020	3.418	19.693
VIX	257	0.062	0.059	0.030	0.175	0.024	1.665	4.366
$\overline{\mathrm{P}}^E$	257	0.514	0.513	0.453	0.572	0.026	−0.078	−0.865
$\mho^E$	257	0.948	0.936	0.466	1.623	0.262	0.309	−0.795

Panel B: Cross-correlations of expected values of the variables

	ν^E	ϑ^E	VIX	$\overline{\mathrm{P}}^E$	$(\mho^2)^E$
ν^E	1				
ϑ^E	0.850	1			
	(< 0.0001)				
VIX	0.783	0.908	1		
	(< 0.0001)	(< 0.0001)			
$\overline{\mathrm{P}}^E$	−0.305	−0.458	−0.322	1	
	(< 0.0001)	(< 0.0001)	(< 0.0001)		
$(\mho^2)^E$	−0.314	−0.537	−0.451	0.570	1
	(< 0.0001)	(< 0.0001)	(< 0.0001)	(< 0.0001)	

Empirical methodology and results

The expected probabilities of favorable returns range from 0.453 to 0.572 Winsorizing the very few outlier values provides the range [0.46,0.56] of expected probabilities. This range is divided into ten equal intervals (bins) of 0.01 each, indexed by i .

The few values lower than 0.46 are indexed as i = 1 , while the few values higher than 0.56 are indexed as i = 10 .

The decision to use a ten-bin resolution and not a higher one is dictated by the number of observations, 257 (of expected values).

$$\mathbb{E}_t[r_{t+1}] - r_f = \underbrace{\gamma \frac{1}{2} \mathbb{V}\mathrm{ar}_t[r_{t+1}]}_{\text{Risk premium}} + \underbrace{\eta \big(1 - \mathrm{E}_t[\mathrm{P}_{t+1}]\big) \mathbb{E}_t\Big[|r_{t+1} - \mathbb{E}_t[r_{t+1}]|\Big] \mho_t^2[r_{t+1}]}_{\text{Ambiguity premium}},$$

we can write the coefficient of ambiguity attitude as

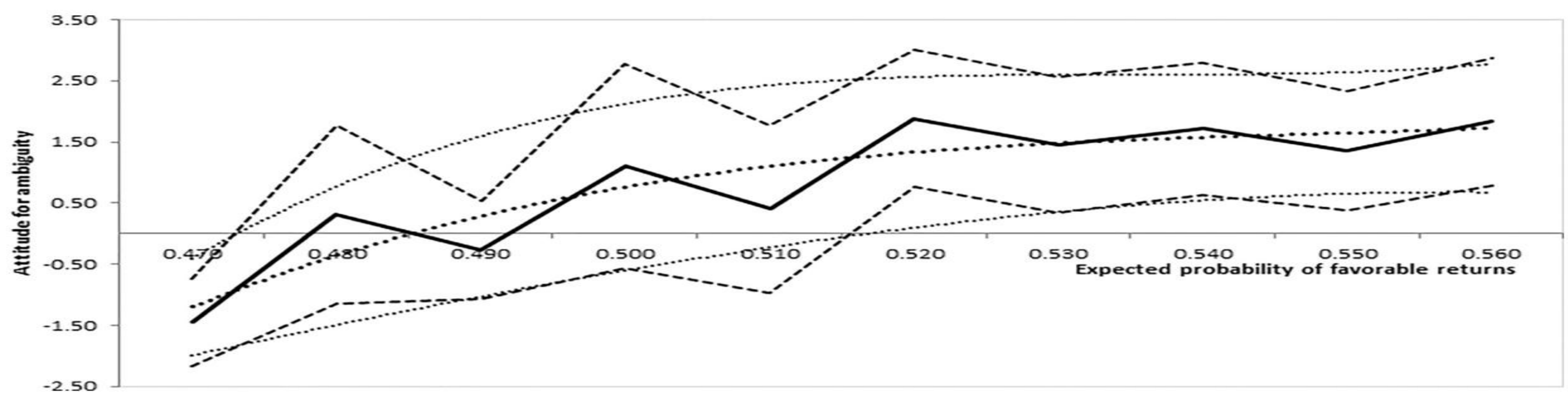
$$\eta\left(\mathrm{P}_i^E\right) = \hat{\eta} + \hat{\eta}_i$$

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot \nu_t^E + \eta \cdot \left(\left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \sum_{i=2}^{10} \eta_i \cdot \left(D_{i,t} \times \mathrm{P}_i^E \times \left(\mho_t^2\right)^E \times \vartheta_t^E \right) + \epsilon_t$$

Empirical methodology and results

Main OLS regression tests.

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\eta}_2$	$\hat{\eta}_3$	$\hat{\eta}_4$	$\hat{\eta}_5$	$\hat{\eta}_6$	$\hat{\eta}_7$	$\hat{\eta}_8$	$\hat{\eta}_9$	$\hat{\eta}_{10}$	N	R^2	Adj. R^2
Panel A: Regression results																
1	0.006	−0.098												257	0.000	−0.004
	(2.360)	(−0.160)														
2	0.005		0.001											257	0.000	−0.004
	(0.690)		(0.160)													
3	0.012			−1.255	1.676	1.788	2.310	1.451	2.902	2.373	2.697	2.256	2.725	257	0.067	0.029
	(1.130)			(−3.310)	(2.170)	(2.690)	(2.700)	(1.800)	(4.130)	(3.520)	(3.760)	(3.350)	(3.840)			
4	0.005	3.400		−1.446	1.762	1.176	2.542	1.853	3.325	2.897	3.159	2.806	3.275	257	0.095	0.055
	(0.420)	(3.580)		(−3.970)	(2.290)	(1.670)	(2.790)	(2.200)	(4.640)	(4.050)	(4.240)	(3.990)	(4.460)			
Panel B: Coefficients of ambiguity attitude																
P^E				−0.460	−0.470-	−0.480	−0.490	−0.500	−0.510	−0.520	−0.530	−0.540	−0.550			
				0.470	0.480	0.490	0.500	0.510	0.520	0.530	0.540	0.550	0.560			
3				−1.255	0.421	0.533	1.055	0.196	1.647	1.118	1.442	1.001	1.470			
4				−1.446	0.316	−0.270	1.096	0.407	1.879	1.451	1.713	1.360	1.829			



Empirical methodology and results

Main WLS regression tests.

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\eta}_2$	$\hat{\eta}_3$	$\hat{\eta}_4$	$\hat{\eta}_5$	$\hat{\eta}_6$	$\hat{\eta}_7$	$\hat{\eta}_8$	$\hat{\eta}_9$	$\hat{\eta}_{10}$	N	R^2	Adj. R^2
Panel A: Regression results																
1	0.005	0.459												257	0.001	−0.003
	(1.450)	(0.490)														
2	0.006		0.000											257	0.000	−0.004
	(0.700)		(0.020)													
3	0.011			−1.244	1.785	1.772	2.422	1.301	2.993	2.446	2.746	2.260	2.745	257	0.065	0.027
	(0.990)			(−2.930)	(2.050)	(2.310)	(2.650)	(1.460)	(3.890)	(3.220)	(3.470)	(3.090)	(3.530)			
4	0.002	4.114		−1.480	1.817	1.043	2.630	1.763	3.475	3.070	3.282	2.920	3.400	257	0.102	0.062
	(0.220)	(3.770)		(−3.720)	(2.080)	(1.330)	(2.740)	(1.890)	(4.520)	(3.850)	(4.030)	(3.840)	(4.290)			
Panel B: Coefficients of ambiguity attitude																
P^E				−0.460	−0.470	−0.480	−0.490	−0.500	−0.510	−0.520	−0.530	−0.540	−0.550			
				0.470	0.480	0.490	0.500	0.510	0.520	0.530	0.540	0.550	0.560			
3				−1.244	0.541	0.528	1.178	0.057	1.749	1.202	1.502	1.016	1.501			
4				−1.480	0.337	−0.437	1.150	0.282	1.994	1.590	1.802	1.440	1.920			

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot v_t^E + \eta \cdot \left(\left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \eta_s \cdot \left(\mathrm{P}_t^E \times \left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \epsilon_t.$$

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\eta}_s$	N	R^2	Adj. R^2
Panel A: OLS								
1	0.006	−0.098				257	0.000	−0.004
	(2.370)	(−0.160)						
2	0.005		0.001			257	0.000	-0.004
	(0.700)		(0.170)					
3	0.012			−2.656	4.786	257	0.024	0.016
	(2.010)			(-2.640)	(2.300)			
4	0.013	2.448		−5.070	9.089	257	0.045	0.033
	(2.210)	(2.970)		(−3.550)	(3.360)			
Panel B: WLS								
1	0.005	0.459				257	0.001	−0.003
	(1.450)	(0.490)						
2	0.006		0.000			257	0.000	−0.004
	(0.710)		(0.020)					
3	0.012			−3.085	5.639	257	0.021	0.013
	(1.730)			(−2.600)	(2.290)			
4	0.011	3.207		−6.250	11.316	257	0.052	0.041
	(1.770)	(3.310)		(−4.070)	(3.880)			

5. Robustness tests

Robustness tests

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot VIX_{t+1} + \eta \cdot \left(\left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \eta_s \cdot \left(\mathrm{P}_t^E \times \left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \epsilon_t$$

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\theta}$	$\hat{\eta}$	$\hat{\eta}_s$	N	R^2	Adj. R^2
Panel A: OLS								
1	0.007	−0.167				257	0.000	−0.004
	(1.630)	(−0.140)						
2	0.005		0.001			257	0.000	−0.004
	(0.700)		(0.170)					
3	0.012			−2.656	4.786	257	0.024	0.016
	(2.010)			(−2.640)	(2.300)			
4	0.006	1.224		−5.040	9.516	257	0.040	0.027
	(1.060)	(0.980)		(−3.380)	(3.230)			
Panel B: WLS								
1	0.004	0.424				257	0.001	−0.003
	(0.670)	(0.300)						
2	0.005		0.001			257	0.000	−0.004
	(0.530)		(0.160)					
3	0.010			−4.024	7.645	257	0.026	0.017
	(1.350)			(−2.730)	(2.480)			
4	0.004	1.834		−5.992	11.351	257	0.044	0.032
	(0.580)	(1.310)		(−3.400)	(3.260)			

Robustness tests

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot \nu_t^E + \eta \cdot \left(\left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \eta_s \cdot \left(\mathrm{P}_t^E \times \left(\mho_t^2 \right)^E \times \vartheta_t^E \right) + \beta_1 \cdot Skew_t^E + \beta_2 \cdot Kurt_t^E + \beta_3 \cdot VolM_t^E + \beta_4 \cdot VolV_t^E + \epsilon_t$$

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\eta}$	$\hat{\eta}_s$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	N	R^2	Adj. R^2
Panel B: OLS regressions											
1	0.012	2.461	−4.987	8.901	−0.022				257	0.053	0.038
	(2.190)	(3.110)	(−3.530)	(3.320)	(−1.300)						
2	0.012	2.423	−4.954	8.851		0.001			257	0.045	0.030
	(2.210)	(2.990)	(−3.210)	(2.990)		(0.260)					
3	0.013	2.797	−4.840	8.673			−17.117		257	0.045	0.030
	(2.250)	(2.180)	(−3.260)	(3.100)			(−0.320)				
4	0.014	1.940	−5.162	9.206				128890.8	257	0.045	0.030
	(1.840)	(0.750)	(−3.750)	(3.500)				(0.250)			
Panel C: WLS regressions											
1	0.011	3.187	−5.987	10.762	−0.030				257	0.065	0.050
	(1.760)	(3.510)	(−4.070)	(3.840)	(−1.600)						
2	0.011	3.181	−6.125	11.059		0.001			257	0.052	0.037
	(1.760)	(3.420)	(−3.730)	(3.480)		(0.210)					
3	0.011	2.976	−6.397	11.583			11.155		257	0.052	0.037
	(1.740)	(2.410)	(−3.740)	(3.610)			(0.200)				
4	0.013	2.690	−6.385	11.499				143229.1	257	0.052	0.037
	(1.420)	(1.010)	(−4.160)	(3.950)				(0.240)			

Robustness tests

the width of the set of priors

$$DRE = \max_{\Phi_a, \Phi_b \in \mathcal{P}} \begin{pmatrix} \Phi(r_0; \mu_a, \sigma_a) \ln \dfrac{\Phi(r_0; \mu_a, \sigma_a)}{\Phi(r_0; \mu_b, \sigma_b)} \\ + \sum_{i=1}^{60} \big(\Phi(r_i; \mu_a, \sigma_a) - \Phi(r_{i-1}; \mu_a, \sigma_a)\big) \ln \dfrac{\Phi(r_i; \mu_a, \sigma_a) - \Phi(r_{i-1}; \mu_a, \sigma_a)}{\Phi(r_i; \mu_b, \sigma_b) - \Phi(r_{i-1}; \mu_b, \sigma_b)} \\ + \big(1 - \Phi(r_{60}; \mu_a, \sigma_a)\big) \ln \dfrac{1 - \Phi(r_{60}; \mu_a, \sigma_a)}{1 - \Phi(r_{60}; \mu_b, \sigma_b)} \end{pmatrix}$$

$$DKS = \max_{\Phi_a, \Phi_b \in \mathcal{P}} \left[\max_{i \in 0, \ldots, 60} \left| \Phi(r_i; \mu_a, \sigma_a) - \Phi(r_i; \mu_b, \sigma_b) \right| \right]$$

$$DDS = \max_{\Phi_a, \Phi_b \in \mathcal{P}} \begin{pmatrix} \left| \Phi(r_0; \mu_a, \sigma_a) - \Phi(r_0; \mu_b, \sigma_b) \right| \\ + \sum_{i=1}^{60} \left| (\Phi(r_i; \mu_a, \sigma_a) - \Phi(r_{i-1}; \mu_a, \sigma_a)) - (\Phi(r_i; \mu_b, \sigma_b) - \Phi(r_{i-1}; \mu_b, \sigma_b)) \right| \\ + \left| (1 - \Phi(r_{60}; \mu_a, \sigma_a)) - (1 - \Phi(r_{60}; \mu_b, \sigma_b)) \right| \end{pmatrix}$$

Robustness tests

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot \nu_t^E + \eta \cdot ((\mho_t^2)^E \times \vartheta_t^E) + \eta_s \cdot (\mathrm{P}_t^E \times (\mho_t^2)^E \times \vartheta_t^E) + +\beta_1 \cdot DRE_t^E + \beta_2 \cdot DKS_t^E + \beta_3 \cdot DDS_t^E + \epsilon_t .$$

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\eta}$	$\hat{\eta}_s$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	N	R^2	Adj. R^2
Panel B: OLS regressions										
1	0.003	0.651			0.000			258	−0.003	−0.011
	(0.870)	(0.740)			(1.680)					
2	0.048	0.411				−0.042		258	−0.005	−0.013
	(0.380)	(0.530)				(−0.320)				
3	0.007	0.426					0.000	258	−0.006	-0.014
	(0.060)	(0.540)					(0.000)			
4	0.008	2.530	−5.075	9.111	0.000			258	0.048	0.033
	(1.070)	(3.070)	(−3.520)	(3.360)	(1.760)					
5	−0.030	2.457	−5.187	9.316		0.044		258	0.045	0.030
	(−0.220)	(2.980)	(−3.410)	(3.220)		(0.310)				
6	−0.091	2.492	−5.350	9.646			0.054	258	0.046	0.031
	(−0.670)	(3.070)	(−3.580)	(3.390)			(0.760)			
Panel C: WLS regressions										
1	0.004	1.299			0.000			257	−0.009	−0.017
	(1.170)	(1.050)			(1.180)					
2	0.093	1.052				−0.091		257	−0.007	−0.015
	(0.590)	(0.950)				(−0.550)				
3	0.030	1.073					−0.012	257	−0.009	-0.017
	(0.200)	(0.950)					(−0.150)			
4	0.009	3.276	−6.216	11.251	0.000			257	0.053	0.038
	(1.170)	(3.380)	(−4.050)	(3.870)	(1.180)					
5	−0.027	3.212	−6.340	11.494		0.039		257	0.052	0.037
	(−0.160)	(3.340)	(−3.900)	(3.720)		(0.230)				
6	−0.098	3.238	−6.492	11.799			0.057	257	0.053	0.038
	(−0.600)	(3.430)	(−4.100)	(3.910)			(0.670)			

Robustness tests

$$r_{t+1} - r_{f,t+1} = \alpha + \gamma \cdot \nu_t^E + \sum_{j=2}^{10} \gamma_j \cdot \left(C_{j,t} \times w_j \times \nu_t^E\right) + \eta \cdot \left(\left(\mho_t^2\right)^E \times \vartheta_t^E\right) + \sum_{i=2}^{10} \eta_i \cdot \left(D_{i,t} \times P_i^E \times \left(\mho_t^2\right)^E \times \vartheta_t^E\right) + \epsilon_t,$$

#	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\gamma}_2$ $\hat{\eta}$	$\hat{\gamma}_3$ $\hat{\eta}_2$	$\hat{\gamma}_4$ $\hat{\eta}_3$	$\hat{\gamma}_5$ $\hat{\eta}_4$	$\hat{\gamma}_6$ $\hat{\eta}_5$	$\hat{\gamma}_7$ $\hat{\eta}_6$	$\hat{\gamma}_8$ $\hat{\eta}_7$	$\hat{\gamma}_9$ $\hat{\eta}_8$	$\hat{\gamma}_{10}$ $\hat{\eta}_9$	$\hat{\eta}_{10}$	N	R^2	Adj. R^2
Panel A: OLS															
1	0.006	4.299	32.318	15.657	0.152	−5.548	−4.415	-4.664	−7.998	0.130	−1.443		257	0.056	0.018
	(1.090)	(0.500)	(1.640)	(1.070)	(0.010)	(−0.550)	(−0.550)	(−0.680)	(−1.280)	(0.020)	(−0.340)				
2	0.012		−1.255	1.676	1.788	2.310	1.451	2.902	2.373	2.697	2.256	2.725	257	0.067	0.029
	(1.130)		(−3.310)	(2.170)	(2.690)	(2.700)	(1.800)	(4.130)	(3.520)	(3.760)	(3.350)	(3.840)			
3	0.008	1.526	45.766	26.075	6.078	2.315	0.186	−0.974	−3.791	0.767	1.174				
	(0.660)	(0.140)	(1.620)	(1.520)	(0.450)	(0.190)	(0.020)	(−0.110)	(−0.500)	(0.090)	(0.180)				
			−1.443	1.812	1.073	2.467	1.844	3.291	2.635	3.088	2.728	3.184	257	0.144	0.072
			(−4.020)	(2.360)	(1.530)	(2.670)	(2.310)	(4.750)	(3.700)	(4.040)	(3.770)	(4.130)			
Panel B: WLS															
1	0.005	5.311	33.054	13.954	−0.083	−6.491	−5.244	−5.319	−9.203	−1.527	−0.925		257	0.075	0.037
	(0.840)	(0.580)	(1.640)	(0.910)	(−0.010)	(−0.590)	(−0.610)	(−0.730)	(−1.390)	(−0.250)	(−0.210)				
2	0.011		−1.244	1.785	1.772	2.422	1.301	2.993	2.446	2.746	2.260	2.745	257	0.065	0.027
	(0.990)		(−2.930)	(2.050)	(2.310)	(2.650)	(1.460)	(3.890)	(3.220)	(3.470)	(3.090)	(3.530)			
3	0.006	2.912	49.417	25.159	5.490	1.078	−0.662	−1.662	−4.677	−1.577	1.226				
	(0.490)	(0.270)	(1.630)	(1.450)	(0.390)	(0.090)	(−0.070)	(−0.190)	(−0.610)	(−0.190)	(0.180)				
			−1.451	1.856	0.974	2.522	1.753	3.404	2.734	3.214	2.814	3.287	257	0.162	0.091
			(−3.720)	(2.130)	(1.240)	(2.550)	(1.970)	(4.580)	(3.430)	(3.790)	(3.580)	(3.950)			

Conclusion

In this study, we introduce ambiguity into the traditional risk-return relation. Our results show that the excess return on the market as a whole, known as the equity premium, is determined by two distinct factors: ambiguity and risk.

We find that in the case of a high expected probability of gains, the effect of ambiguity is positive and highly significant, while for a high expected probability of losses, it is negative and highly significant. Furthermore, our findings indicate that aversion to ambiguity increases with the expected probability of gains, while love for ambiguity increases with the expected probability of losses.

When we include ambiguity in the pricing model, the effect of risk is positive and significant, while its effect is insignificant when ambiguity is not accounted.

Thank You !