The CAPM strikes back?
An equilibrium model with disasters

JFE 131 2019(2) 269–298

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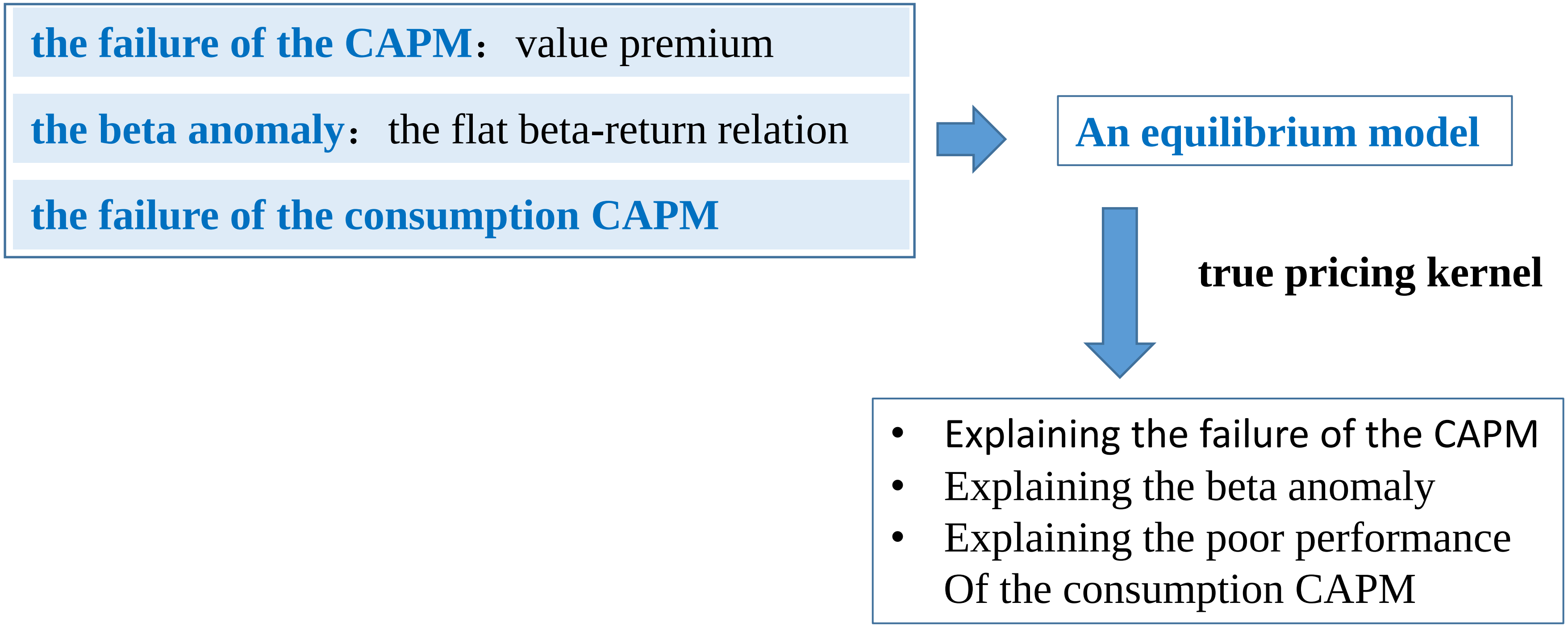

the failure of the CAPM： value premium

the beta anomaly： the flat beta-return relation

the failure of the consumption CAPM

An equilibrium model

true pricing kernel

- Explaining the failure of the CAPM
- Explaining the beta anomaly
- Explaining the poor performance Of the consumption CAPM

The CAPM strikes back? An equilibrium model with disasters

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Academic Background

- **PhD** in Finance from The Wharton School, University of Pennsylvania, 2002
- **MA** in Economics from Washington University (St. Louis), 1997
- **BA** in Economics from Jiangxi University of Finance and Economics, 1993

Current Appointments

Professor of Finance, at Fisher College of Business, The Ohio State University

Research Associate at National Bureau of Economic Research

Associate Editor for Journal of Financial Economics and Journal of Financial and Quantitative Analysis

Aggregation, capital heterogeneity, and the investment CAPM (with Goncalves and Xue), forthcoming, Review of Financial Studies

Replicating anomalies (with Hou and Xue), forthcoming, Review of Financial Studies

Endogenous disasters (with Petrosky-Nadeau and Kuehn), American Economic Review 108 (8), 2212-2245.

The investment CAPM, European Financial Management 23 (4), 545-603.

Digesting anomalies: An investment approach (with Hou and Xue), 2015, Review of Financial Studies, 28 (3), 650-705. Editor’s Choice, lead article.

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教育背景:

博士学位：芝加哥大学金融学， 2002
学士学位：中国科学技术大学电子工程学， 1995

当前任职:

金融学教授-俄亥俄州立大学 菲舍尔商学院
金融学特聘教授-上海高级金融学院

主要研究领域:资产定价、市场效率、行为金融学、实证公司金融、资本市场会计研究

在国际顶级期刊 **Journal of Finance, Journal of Financial Economics, Review of Financial Studies, Journal of Accounting and Economics和 Management Science**等发表多篇论文

1. Jack Bao, Kewei Hou, 2017, **De Facto Seniority, Credit Risk, and Corporate Bond Prices**, *Review of Financial Studies*.
2. Hou, Kewei, and Roger K. Loh, 2016**, Have We Solved the Idiosyncratic Volatility Puzzle?,** *Journal of Financial Economics*.
3. Hou, Kewei, Chen Xue, and Lu Zhang, 2015, **Digesting Anomalies: An Investment Approach, *Review of Financial Studies.***

Howard Kung 霍华德·孔

Academic Background

- **BA，** in Mathematics from University of Virginia, 2001-2005
- **PhD，** in Finance from Duke University, 2006-2012

Current Appointments

Assistant Professor of Finance, London Business School

Research interest

the intersection of asset pricing and macroeconomics

1. "**Growth Slowdowns and Recoveries**" (with F. Bianchi and G. Morales). Journal of Monetary Economics, Accepted
2. "**How Uncertainty Affects Corporate Investment: The Asset Redeployability Channel**" (with H. Kim). Review of Financial Studies, January 2017
3. "**Macroeconomic Linkages between Monetary Policy and the Term Structure of Interest Rates**". Journal of Financial Economics, January 2015
4. "**Innovation, Growth and Asset Prices**" (with L. Schmid). Journal of Finance, June 2015
5. "**Fiscal Policies and Asset Prices**" (with M. Croce, T. Nguyen and L. Schmid) Review of Financial Studies, September 2012 (lead article).

Hang Bai

Academic Background

- **PhD**, Finance, The Ohio State University ,2016
- **MA**, Business Administration, Duke University,2012
- **MS**, Engineering, University of California,2008
- **BE,** Engineering, Tsinghua University,2007

Current Appointments

Assistant Professor of Finance, School of Business, University of Connecticut

Research interest

asset pricing, credit risk, macro finance, and international finance

Working paper
“**Unemployment and Credit Risk**“, Revise and Resubmit
“**Predictable Returns over the Credit Cycle**”

李学楠 Erica X.N. Li

Academic Background

- **BS** in Physics and BA in Economics from Peking University 1998
- **PhD** in Physics from the University of Massachusetts, Amherst；also a **PhD** in Finance from university of Rochester

Current Appointments

Assistant Professor in Finance,University of Michigan, Stephen M.Ross School of Business,2007-2011

Visiting Assistant Professor in Finance,Cheung Kong Graduate School of Business,2011-present

Research interest

Capital structure, asset pricing, macroeconomics

1. **Do Underwriters Compete in IPO Pricing?** 2018, Management Science 64 (2), 925-954
2. **Anomalies**,2009, with Dmitry Livdan and Lu Zhang,Review of Financial Studies, lead article,22(11),4301-4334.
3. **Nominal Rigidities, Asset Returns and Monetary Policy**, 2014, with Francisco Palomino, Journal of Monetary Economics, 66, 210–225.

preliminary

1.value premium puzzle

despite similar market betas, firms with high BM (value firms) earn **higher** average stock returns than firms with low BM (growth firms)

2.disaster

3.beta anomaly

the empirical relation between the market beta and **the average return** is too **flat** to be consistent with the CAPM

4.true beta& true market beta

5. full sample & post-Compustat sample & （pre-）

6. true beta $(E_t[R_{it+1}] - r_{ft})/\phi_{Mt}$ & true market beta

$(E_t[R_{it+1}] - r_{ft})/(E_t[R_{Mt+1}] - r_{ft})$.

preliminary

7. Nonlinearity in the pricing kernel

If the CAPM holds exactly, the pricing kernel can be expressed as a linear function of the market excess return $M_{t+1} = l_0 + l_1 R_{Mt+1}$

The disaster risk induces strong nonlinearity in the pricing kernel, making the CAPM a poor proxy of the pricing kernel.

Abstract

①

embed disasters into a general equilibrium model with heterogeneous firms

induces

strong nonlinearity in the pricing kernel

help explain

the empirical failure of the (consumption) CAPM

②

in finite samples without disasters → our single-factor model reproduces the failure of the CAPM in explaining the value premium

in the samples with disasters → our single-factor model is relative success

Abstract

③

even though the true beta-return relation is strongly positive

the estimated beta-return relation is flat due to beta measurement errors → consistent with the beta “anomaly”

④

Finally, the consumption CAPM fails in simulations, even though a nonlinear model with the true pricing kernel holds exactly by construction

Contents

（1）background

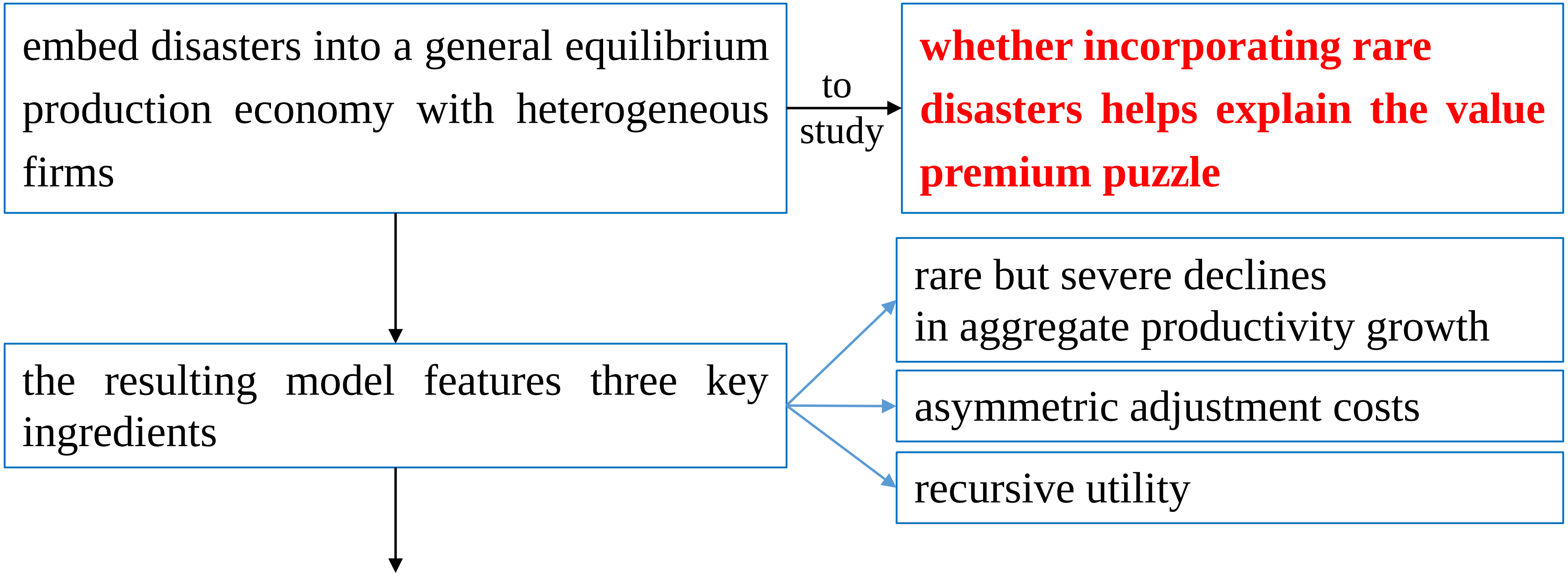
the failure of CAPM in empirical

Fama and French (1992)： In the US sample from **July 1963 to June 2017**, the high-minus-low book-to-market decile return is, on average, 0.47% per month (t=2.53). However, its **market beta is only 0.07 (t=0.86)**, giving rise to an economically **large alpha of 0.43% (t=1.89)** in the capital asset pricing model (CAPM)

the success of CAPM in empirical

Ang and Chen (2007)： The CAPM performs better in explaining the value premium in the long sample **from July 1926 onward** that contains the Great Depression. The high-minus-low return is, on average, 0.48% (t=2.5), but its **CAPM alpha is only 0.19% (t=0.99), with a large market beta of 0.45 (t=3.87)**.

（2）what to study



embed disasters into a general equilibrium production economy with heterogeneous firms

to study

whether incorporating rare disasters helps explain the value premium puzzle

the resulting model features three key ingredients

- rare but severe declines in aggregate productivity growth
- asymmetric adjustment costs
- recursive utility

- We calibrate the model to disaster moments estimated from **a historical cross-country panel dataset**
- We quantify the model’s properties **on simulated samples** in which disasters are not realized as well as on samples in which disasters are realized

（2）three key quantitative results

First, our equilibrium model succeeds in replicating the results：

In samples without disasters → failure of the CAPM in explaining the value premium

In samples with disasters → better performance

原因：

Intuitively, with asymmetric adjustment costs（价值溢价怎么产生）

More important, the disaster risk induces strong nonlinearity in the pricing kernel, making the linear CAPM a poor empirical proxy for the pricing kernel.（有灾难样本CAPM 表现好，灾难可以帮助解释价值溢价 ）

（2）three key quantitative results

Second, our equilibrium model is also consistent with the beta “anomaly”：

In simulated samples，with and without disasters

①sorting on the preranking market beta yields an **average return** spread that is economically small and statistically **insignificant**
②postranking beta spread that is economically large and significantly positive, and a CAPM **alpha** spread that is often **significantly negative**.

①the true beta often mean reverts, giving rise to a negative correlation with the rolling beta**,**
② However, while the realization of disasters makes the rolling beta more aligned with the true beta, the measurement errors remain large, and the **beta anomaly** persists even in the disaster samples.

The crux is that the estimated market beta is a poor proxy for the true beta.

（3）three key quantitative results

Third, our equilibrium model, in which a nonlinear consumption CAPM holds by construction, also largely succeeds in replicating the empirical failure of the standard, linearized consumption CAPM：

In simulated samples，with and without disasters

- the **consumption betas** from regressing excess returns on the aggregate consumption growth in the first-stage regressions are mostly **insignificant**
- In the second-stage cross-sectional regressions, the **intercepts** are **significantly** positive

Intuitively, the aggregate consumption growth is a poor proxy for the pricing kernel based on recursive utility.

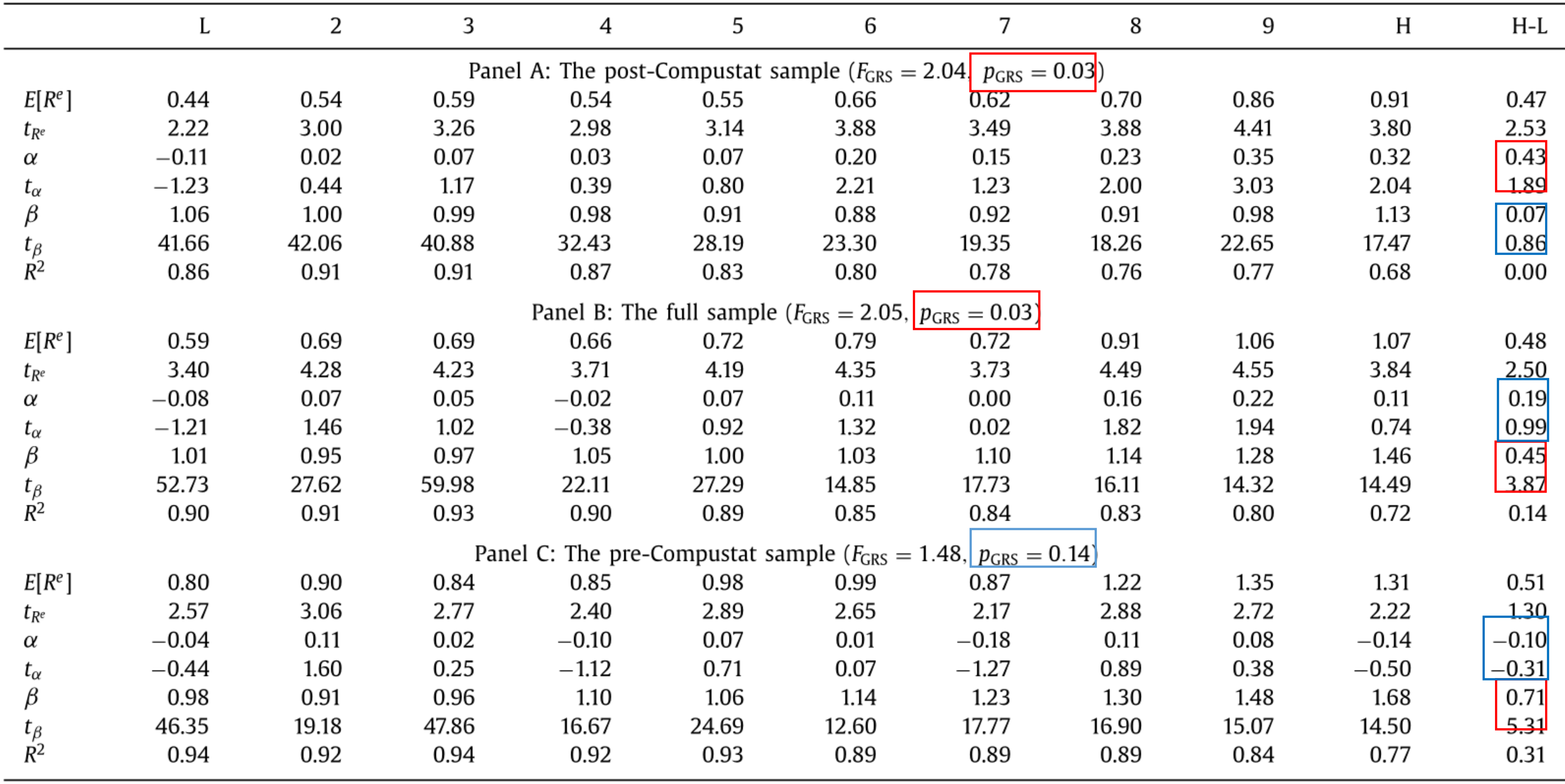
The true pricing kernel performs substantially better in the linearized consumption CAPM tests, especially in the disaster samples. However, without the extreme observations from disasters, **even the true price kernel encounters difficulty in the linear tests.**

（4）contribution

- contributes to investment-based asset pricing theories
  - We turbocharge the asymmetry mechanism via disasters
  - We retain **the single-factor structure** but fail the CAPM via **disaster-induced nonlinearity in the pricing kernel**.
- Methodologically, most prior models are partial equilibrium in nature, with exogenous pricing kernels
  - We first construct **a general equilibrium** model with heterogenous firms in which **consumption and the pricing kernel are endogenously determined**.
- contribute to the disaster literature, which uses disasters to explain the equity premium puzzle
  - Integratingthe disaster literature with investment-based asset pricing,we show how disasters help resolve a long-standing puzzle in the latter literature in explaining the failure of the(consumption) CAPM

（1）The failure of the CAPM

根据BM分组



	L	2	3	4	5	6	7	8	9	H	H-L
Panel A: The post-Compustat sample ($F_{GRS} = 2.04$, $p_{GRS} = 0.03$)											
$E[R^e]$	0.44	0.54	0.59	0.54	0.55	0.66	0.62	0.70	0.86	0.91	0.47
t_{R^e}	2.22	3.00	3.26	2.98	3.14	3.88	3.49	3.88	4.41	3.80	2.53
α	−0.11	0.02	0.07	0.03	0.07	0.20	0.15	0.23	0.35	0.32	0.43
t_α	−1.23	0.44	1.17	0.39	0.80	2.21	1.23	2.00	3.03	2.04	1.89
β	1.06	1.00	0.99	0.98	0.91	0.88	0.92	0.91	0.98	1.13	0.07
t_β	41.66	42.06	40.88	32.43	28.19	23.30	19.35	18.26	22.65	17.47	0.86
R^2	0.86	0.91	0.91	0.87	0.83	0.80	0.78	0.76	0.77	0.68	0.00
Panel B: The full sample ($F_{GRS} = 2.05$, $p_{GRS} = 0.03$)											
$E[R^e]$	0.59	0.69	0.69	0.66	0.72	0.79	0.72	0.91	1.06	1.07	0.48
t_{R^e}	3.40	4.28	4.23	3.71	4.19	4.35	3.73	4.49	4.55	3.84	2.50
α	−0.08	0.07	0.05	−0.02	0.07	0.11	0.00	0.16	0.22	0.11	0.19
t_α	−1.21	1.46	1.02	−0.38	0.92	1.32	0.02	1.82	1.94	0.74	0.99
β	1.01	0.95	0.97	1.05	1.00	1.03	1.10	1.14	1.28	1.46	0.45
t_β	52.73	27.62	59.98	22.11	27.29	14.85	17.73	16.11	14.32	14.49	3.87
R^2	0.90	0.91	0.93	0.90	0.89	0.85	0.84	0.83	0.80	0.72	0.14
Panel C: The pre-Compustat sample ($F_{GRS} = 1.48$, $p_{GRS} = 0.14$)											
$E[R^e]$	0.80	0.90	0.84	0.85	0.98	0.99	0.87	1.22	1.35	1.31	0.51
t_{R^e}	2.57	3.06	2.77	2.40	2.89	2.65	2.17	2.88	2.72	2.22	1.30
α	−0.04	0.11	0.02	−0.10	0.07	0.01	−0.18	0.11	0.08	−0.14	−0.10
t_α	−0.44	1.60	0.25	−1.12	0.71	0.07	−1.27	0.89	0.38	−0.50	−0.31
β	0.98	0.91	0.96	1.10	1.06	1.14	1.23	1.30	1.48	1.68	0.71
t_β	46.35	19.18	47.86	16.67	24.69	12.60	17.77	16.90	15.07	14.50	5.31
R^2	0.94	0.92	0.94	0.92	0.93	0.89	0.89	0.89	0.84	0.77	0.31

What is the differences across the pre- and post-1963 samples?

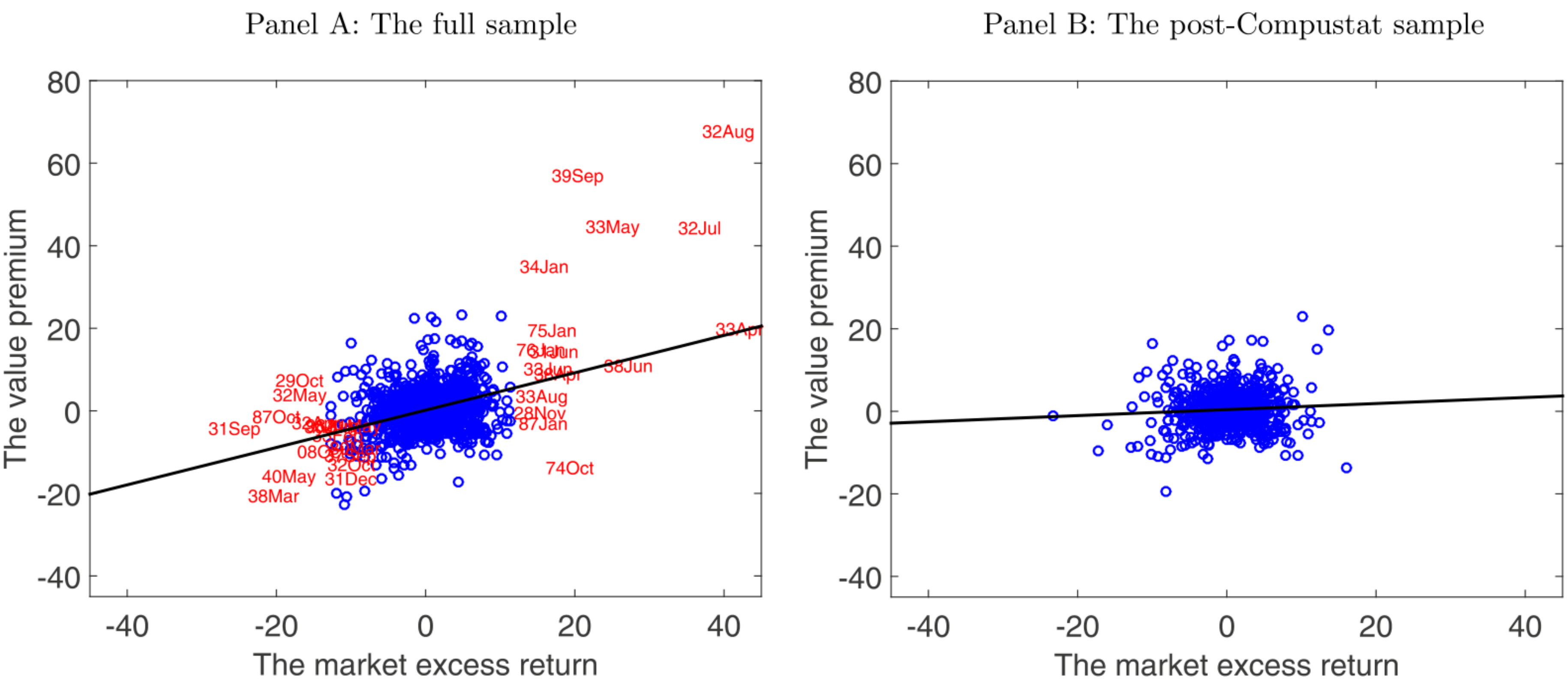
（1）The failure of the CAPM

Returns are in monthly percent

	MKT	H-L		MKT	H-L
November 1928	11.81	−0.29	August 1933	12.05	3.76
October 1929	−20.12	7.60	January 1934	12.60	35.20
June 1930	−16.27	−3.60	September 1937	−13.61	−10.56
May 1931	−13.24	−3.37	March 1938	−23.82	−20.35
June 1931	13.90	14.57	April 1938	14.51	9.16
September 1931	−29.13	−4.03	June 1938	23.87	11.15
December 1931	−13.53	−16.22	September 1939	16.88	57.22
April 1932	−17.96	−2.65	May 1940	−21.95	−15.59
May 1932	−20.51	4.09	October 1974	16.10	−13.57
July 1932	33.84	44.54	January 1975	13.66	19.72
August 1932	37.06	67.95	January 1976	12.16	15.03
October 1932	−13.17	−12.80	March 1980	−12.90	−8.78
February 1933	−15.24	−5.70	January 1987	12.47	−2.83
April 1933	38.85	20.04	October 1987	−23.24	−1.20
May 1933	21.43	44.85	August 1998	−16.08	−3.27
June 1933	13.11	10.40	October 2008	−17.23	−9.64

① 23 of 32 are from the Great Depression
② Their correlation is 0.72

（1）The failure of the CAPM

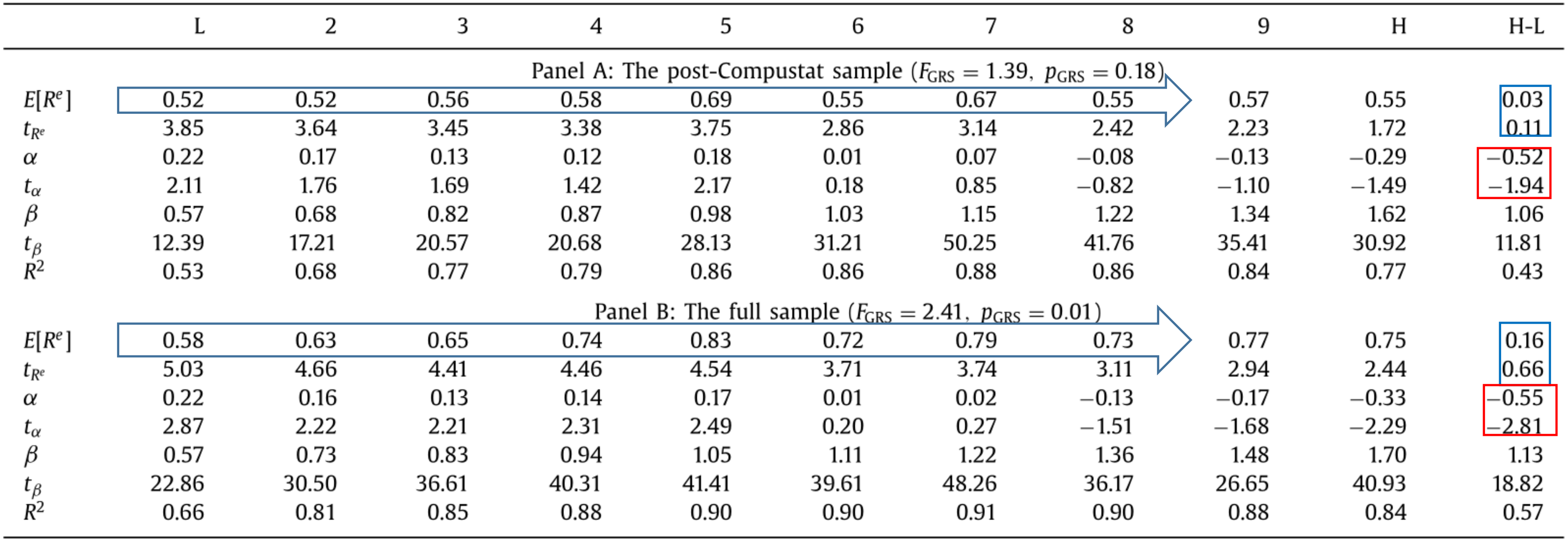


These observations clearly contribute to the market beta of 0.45 (t = 3 .87) for the value-minus-growth decile in the long sample

a largely flat regression line, **CAPM fails** in the short post-1963 sample

（2）The beta anomaly

根据preranking market beta分组

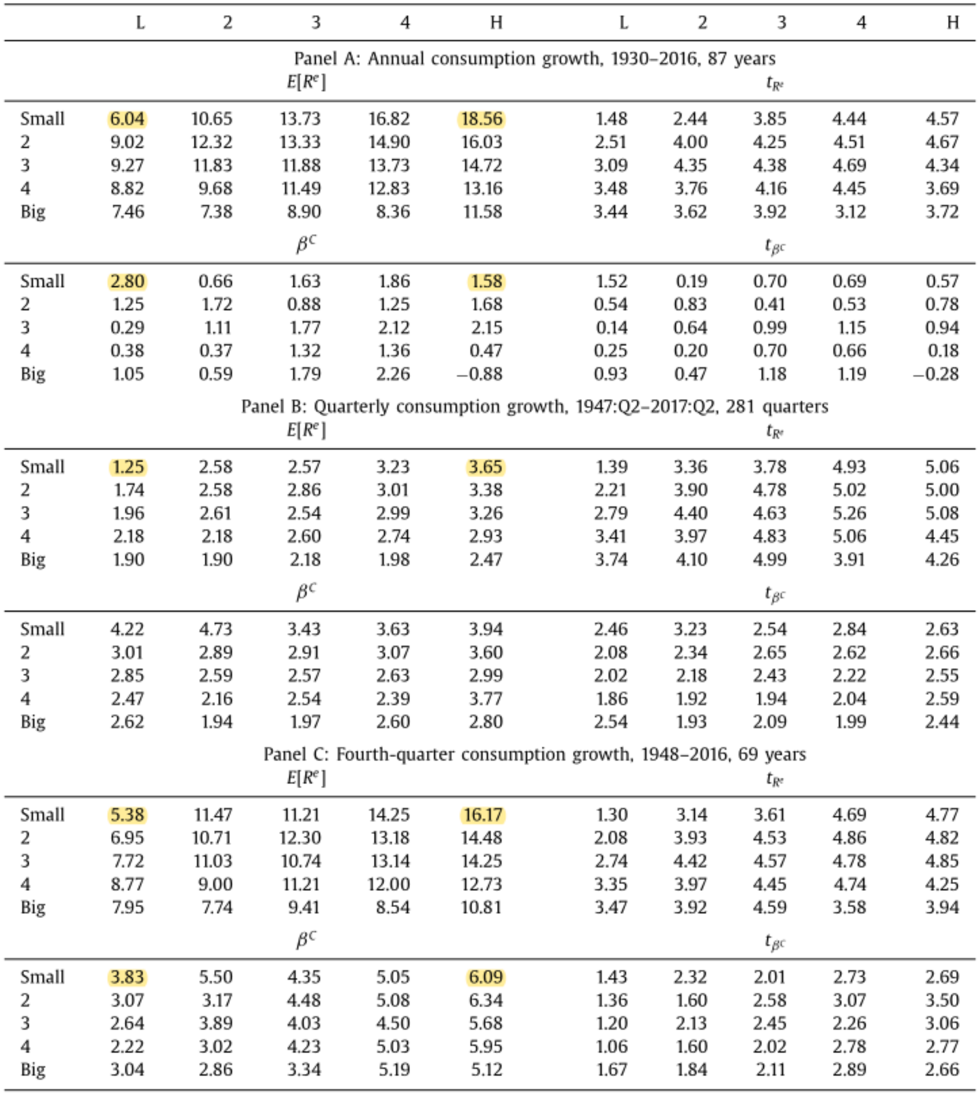


	L	2	3	4	5	6	7	8	9	H	H-L
Panel A: The post-Compustat sample ($F_{GRS} = 1.39$, $p_{GRS} = 0.18$)											
$E[R^e]$	0.52	0.52	0.56	0.58	0.69	0.55	0.67	0.55	0.57	0.55	0.03
t_{R^e}	3.85	3.64	3.45	3.38	3.75	2.86	3.14	2.42	2.23	1.72	0.11
α	0.22	0.17	0.13	0.12	0.18	0.01	0.07	−0.08	−0.13	−0.29	−0.52
t_α	2.11	1.76	1.69	1.42	2.17	0.18	0.85	−0.82	−1.10	−1.49	−1.94
β	0.57	0.68	0.82	0.87	0.98	1.03	1.15	1.22	1.34	1.62	1.06
t_β	12.39	17.21	20.57	20.68	28.13	31.21	50.25	41.76	35.41	30.92	11.81
R^2	0.53	0.68	0.77	0.79	0.86	0.86	0.88	0.86	0.84	0.77	0.43
Panel B: The full sample ($F_{GRS} = 2.41$, $p_{GRS} = 0.01$)											
$E[R^e]$	0.58	0.63	0.65	0.74	0.83	0.72	0.79	0.73	0.77	0.75	0.16
t_{R^e}	5.03	4.66	4.41	4.46	4.54	3.71	3.74	3.11	2.94	2.44	0.66
α	0.22	0.16	0.13	0.14	0.17	0.01	0.02	−0.13	−0.17	−0.33	−0.55
t_α	2.87	2.22	2.21	2.31	2.49	0.20	0.27	−1.51	−1.68	−2.29	−2.81
β	0.57	0.73	0.83	0.94	1.05	1.11	1.22	1.36	1.48	1.70	1.13
t_β	22.86	30.50	36.61	40.31	41.41	39.61	48.26	36.17	26.65	40.93	18.82
R^2	0.66	0.81	0.85	0.88	0.90	0.90	0.91	0.90	0.88	0.84	0.57

① contradicting the CAPM, the relation between the market beta and the average return in the data is largely flat

② CAPM alpha for the high-minus-low market beta decile is economically large, −0 . 52% , albeit marginally significant(t = −1 . 94)

（3）The failure of the consumption CAPM



	L	2	3	4	H	L	2	3	4	H
Panel A: Annual consumption growth, 1930–2016, 87 years										
			$E[R^e]$					t_{R^e}		
Small	6.04	10.65	13.73	16.82	18.56	1.48	2.44	3.85	4.44	4.57
2	9.02	12.32	13.33	14.90	16.03	2.51	4.00	4.25	4.51	4.67
3	9.27	11.83	11.88	13.73	14.72	3.09	4.35	4.38	4.69	4.34
4	8.82	9.68	11.49	12.83	13.16	3.48	3.76	4.16	4.45	3.69
Big	7.46	7.38	8.90	8.36	11.58	3.44	3.62	3.92	3.12	3.72
			β^C					t_{β^C}		
Small	2.80	0.66	1.63	1.86	1.58	1.52	0.19	0.70	0.69	0.57
2	1.25	1.72	0.88	1.25	1.68	0.54	0.83	0.41	0.53	0.78
3	0.29	1.11	1.77	2.12	2.15	0.14	0.64	0.99	1.15	0.94
4	0.38	0.37	1.32	1.36	0.47	0.25	0.20	0.70	0.66	0.18
Big	1.05	0.59	1.79	2.26	−0.88	0.93	0.47	1.18	1.19	−0.28
Panel B: Quarterly consumption growth, 1947:Q2–2017:Q2, 281 quarters										
			$E[R^e]$					t_{R^e}		
Small	1.25	2.58	2.57	3.23	3.65	1.39	3.36	3.78	4.93	5.06
2	1.74	2.58	2.86	3.01	3.38	2.21	3.90	4.78	5.02	5.00
3	1.96	2.61	2.54	2.99	3.26	2.79	4.40	4.63	5.26	5.08
4	2.18	2.18	2.60	2.74	2.93	3.41	3.97	4.83	5.06	4.45
Big	1.90	1.90	2.18	1.98	2.47	3.74	4.10	4.99	3.91	4.26
			β^C					t_{β^C}		
Small	4.22	4.73	3.43	3.63	3.94	2.46	3.23	2.54	2.84	2.63
2	3.01	2.89	2.91	3.07	3.60	2.08	2.34	2.65	2.62	2.66
3	2.85	2.59	2.57	2.63	2.99	2.02	2.18	2.43	2.22	2.55
4	2.47	2.16	2.54	2.39	3.77	1.86	1.92	1.94	2.04	2.59
Big	2.62	1.94	1.97	2.60	2.80	2.54	1.93	2.09	1.99	2.44
Panel C: Fourth-quarter consumption growth, 1948–2016, 69 years										
			$E[R^e]$					t_{R^e}		
Small	5.38	11.47	11.21	14.25	16.17	1.30	3.14	3.61	4.69	4.77
2	6.95	10.71	12.30	13.18	14.48	2.08	3.93	4.53	4.86	4.82
3	7.72	11.03	10.74	13.14	14.25	2.74	4.42	4.57	4.78	4.85
4	8.77	9.00	11.21	12.00	12.73	3.35	3.97	4.45	4.74	4.25
Big	7.95	7.74	9.41	8.54	10.81	3.47	3.92	4.59	3.58	3.94
			β^C					t_{β^C}		
Small	3.83	5.50	4.35	5.05	6.09	1.43	2.32	2.01	2.73	2.69
2	3.07	3.17	4.48	5.08	6.34	1.36	1.60	2.58	3.07	3.50
3	2.64	3.89	4.03	4.50	5.68	1.20	2.13	2.45	2.26	3.06
4	2.22	3.02	4.23	5.03	5.95	1.06	1.60	2.02	2.78	2.77
Big	3.04	2.86	3.34	5.19	5.12	1.67	1.84	2.11	2.89	2.66

① two-stage Fama and MacBeth (1973)

$$R^e_{it} = a_i + \beta^C_i g_{Ct} + e_{it},$$

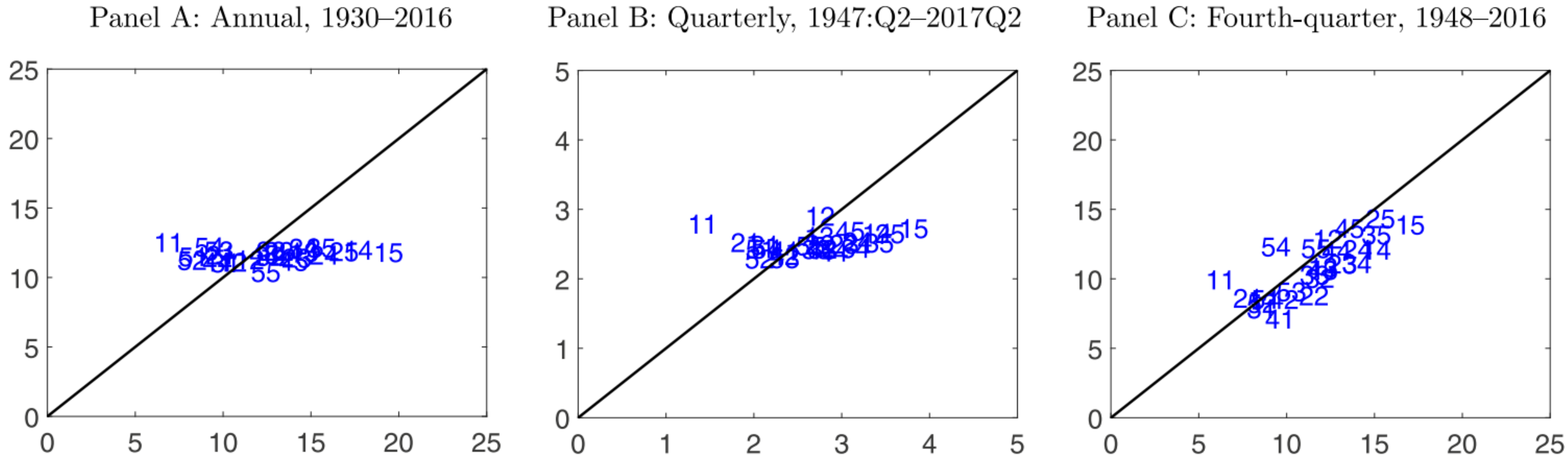
$$R^e_{it} = \phi_0 + \phi_1 \beta^C_i + \alpha_i$$

②size： small 12.52% big 4.12%

③ βc 1.58 vs 2.8

（3）The failure of the consumption CAPM

	Panel A: Annual, 1930–2016		Panel B: Quarterly, 1947:Q2–2017:Q2		Panel C: Fourth-quarter, 1948–2016	
	ϕ_0	ϕ_1	ϕ_0	ϕ_1	ϕ_0	ϕ_1
Estimates	10.97	0.58	1.88	0.22	3.30	1.75
t_{FM}	4.14	1.16	3.73	1.12	1.23	3.44
t_S	3.99	1.13	3.42	1.03	0.77	2.23
χ^2		152.19		100.00		55.85
p_{χ^2}		0.00		0.00		0.00
R^2		0.02		0.07		0.60



Average predicted excess returns versus average realized excess returns

（1）Preferences

intertemporal elasticity of substitution

$$U_t = \left[(1-\varrho) C_t^{1-\frac{1}{\psi}} + \varrho \left(E_t \left[U_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\boxed{\psi}}}, \tag{3}$$

$$M_{t+1} = \varrho \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}^{1-\gamma}}{E_t \left[U_{t+1}^{1-\gamma} \right]} \right)^{\frac{1/\psi-\gamma}{1-\gamma}}. \tag{4}$$

（2）Technology

$$Y_{it} = (X_t Z_{it})^{1-\xi} K_{it}^{\xi}, \tag{5}$$

fixed costs of production

$$\Pi_{it} = Y_{it} - \boxed{fK_{it}}, \tag{6}$$

$$\boxed{g_{xt}} = \bar{g} + g_t, \tag{7}$$

log aggregate productivity growth

$$g_{xt} \equiv \log(X_t/X_{t-1}).$$

$$g_{t+1} = \rho_g g_t + \sigma_g \epsilon^{g}_{t+1}, \tag{8}$$

$$z_{it+1} = (1-\rho_z)\bar{z} + \rho_z z_{it} + \sigma_z \epsilon^{z}_{it+1}, \tag{9}$$

（3）Disasters

$$\tilde{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{15} \\ p_{21} & p_{22} & \cdots & p_{25} \\ \vdots & \vdots & \ddots & \vdots \\ p_{51} & p_{52} & \cdots & p_{55} \end{bmatrix},$$

g_t into a fifive-point grid

$$\{g_1, g_2, g_3, g_4, g_5\}$$

$$p(g_{t+1} = g_5 \mid g_t = g_1) = p_{15} \quad (10)$$

To incorporate disasters into the model

$$P = \begin{bmatrix} \theta & 0 & 0 & \cdots & 0 & 1-\boxed{\theta} \\ \eta & p_{11}-\eta & p_{12} & \cdots & p_{15} & 0 \\ \eta & p_{21} & p_{22}-\boxed{\eta} & \cdots & p_{25} & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \eta & p_{51} & p_{52} & \cdots & p_{55}-\eta & 0 \\ 0 & (1-\nu)/5 & (1-\nu)/5 & \cdots & (1-\nu)/5 & \boxed{\nu} \end{bmatrix}. \quad (13)$$

probability of entering the disaster state from any of the normal states

probability of remaining in the disaster state next period

is the persistence of the recovery state

（4）Adjustment costs

$$K_{it+1} = I_{it} + (1-\delta)K_{it}, \qquad (14)$$

asymmetric adjustment costs

$$\Phi_{it} \equiv \Phi(I_{it}, K_{it}) = \begin{cases} a^{+}K_{it} + \frac{c^{+}}{2}\left(\frac{I_{it}}{K_{it}}\right)^{2}K_{it} & \text{for} \quad I_{it} > 0 \\ 0 & \text{for} \quad I_{it} = 0 \\ a^{-}K_{it} + \frac{c^{-}}{2}\left(\frac{I_{it}}{K_{it}}\right)^{2}K_{it} & \text{for} \quad I_{it} < 0 \end{cases}, \qquad (15)$$

（5）Firms’ problem

$$\mu_{t+1} = \Upsilon(\mu_t, X_t, X_{t+1}). \tag{16}$$

$$D_{it} \equiv \Pi_{it} - I_{it} - \Phi(I_{it}, K_{it})$$

$$\begin{aligned} V_{it} &\equiv V(K_{it}, Z_{it}; X_t, \mu_t) \\ &= \max_{\{\chi_{it}\}} \left(\max_{\{I_{it}\}} \boxed{D_{it}} + E_t[M_{t+1}V(K_{it+1}, Z_{it+1}; \right. \\ &\qquad \left. X_{t+1}, \mu_{t+1})], \boxed{s}K_{it} \right), \end{aligned} \tag{17}$$

$s > 0$ is the liquidation value parameter

when $V_{it} \geq sK_{it}$, firm *i* stays in the economy

$$V_{it} = D_{it} + E_t[M_{t+1}V_{it+1}] \qquad R_{it+1} \equiv V_{it+1}/(V_{it} - D_{it})$$

（5）Firms’ problem

$$\begin{aligned} E_t[R_{it+1}] &= r_{ft} + \overbrace{\boxed{\left(-\frac{\mathrm{Cov}_t[R_{it+1}, M_{t+1}]}{\mathrm{Var}_t[M_{t+1}]}\right)}}^{\text{true beta}} \overbrace{\boxed{\frac{\mathrm{Var}_t[M_{t+1}]}{E_t[M_{t+1}]}}}^{\text{price of consumption risk}} \\ &= r_{ft} + \beta_{it}^{M}\phi_{Mt} \qquad (18) \end{aligned}$$

when $V_{it} < sK_{it}$, firm *i* exits from the economy

The current shareholders receive sK_{it}

New shareholders take over the remainder of the firm’s capital $(1-s-\kappa)K_{it}$

Prior theoretical models, all of which have no disasters, have largely ignored the exit decision.

（6）Competitive equilibrium

The aggregate behavior of the economy isconsistent with the optimal behavior of all firms in the economy

$$Y_t = \int Y_{it}\, \mu_t(dK_{it}, dZ_{it}), \tag{19}$$

$$I_t = \int I_{it}\, \mu_t(dK_{it}, dZ_{it}), \tag{20}$$

$$K_t = \int K_{it}\, \mu_t(dK_{it}, dZ_{it}), \tag{21}$$

$$\Phi_t = \int \Phi_{it}\, \mu_t(dK_{it}, dZ_{it}). \tag{22}$$

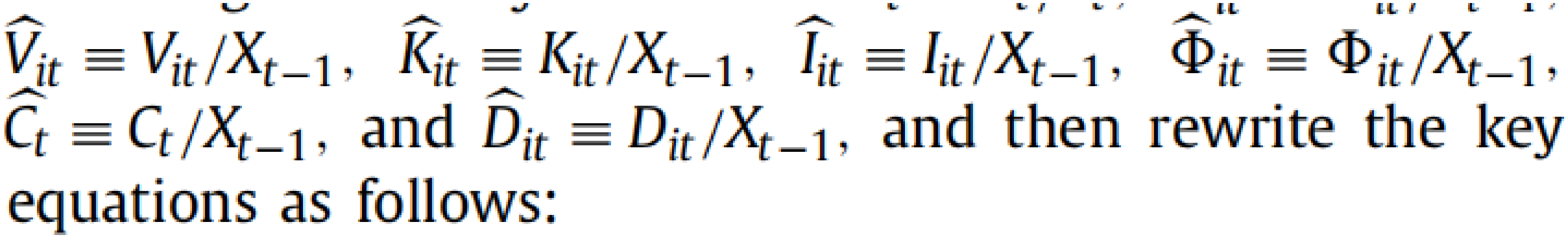
$$\mu_{t+1}(\Theta, X_{t+1}) = T(\Theta, (K_{it}, Z_{it}), X_t)\, \mu_t(K_{it}, Z_{it}, X_t), \tag{23}$$

in which

$$T(\Theta, (K_{it}, Z_{it}), X_t) \equiv \iint \mathbf{1}_{\{(I_{it}+(1-\delta)K_{it}, Z_{it+1}) \in \Theta\}}\, Q_Z(dZ_{it+1}|Z_{it}) Q_X(dX_{t+1}|X_t), \tag{24}$$

$$C_t = Y_t - I_t \Rightarrow C_t = D_t + fK_t + \Phi_t. \tag{25}$$

（7）Solving for the competitive equilibrium



$\widehat{V}_{it} \equiv V_{it}/X_{t-1}$, $\widehat{K}_{it} \equiv K_{it}/X_{t-1}$, $\widehat{I}_{it} \equiv I_{it}/X_{t-1}$, $\widehat{\Phi}_{it} \equiv \Phi_{it}/X_{t-1}$, $\widehat{C}_t \equiv C_t/X_{t-1}$, and $\widehat{D}_{it} \equiv D_{it}/X_{t-1}$, and then rewrite the key equations as follows:

（7）Solving for the competitive equilibrium

- The log utility-to-consumption ratio, $\widehat{u}_t \equiv \log(\widehat{U}_t)$:

$$\exp(\widehat{u}_t) = \left[(1-\varrho) + \varrho\left(E_t\left[\exp\left[(1-\gamma)\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\times(\widehat{u}_{t+1} + \widehat{g}_{ct+1} + g_{xt})\right]\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \qquad (26)$$

in which $\widehat{g}_{ct+1} \equiv \log(\widehat{C}_{t+1}/\widehat{C}_t)$ is the log growth rate of detrended consumption.

- The pricing kernel:

$$M_{t+1} = \varrho \exp\left[-\frac{1}{\psi}(\widehat{g}_{ct+1} + g_{xt})\right]$$
$$\times\left[\frac{\exp\left[(1-\gamma)(\widehat{u}_{t+1} + \widehat{g}_{ct+1})\right]}{E_t\left[\exp\left[(1-\gamma)(\widehat{u}_{t+1} + \widehat{g}_{ct+1})\right]\right]}\right]^{\frac{1/\psi-\gamma}{1-\gamma}}. \qquad (27)$$

- Profits: $\widehat{\Pi}_{it} \equiv \exp[(1-\xi)g_{xt}]Z_{it}^{1-\xi}\widehat{K}_{it}^{\xi} - f\widehat{K}_{it}$.

（7）Solving for the competitive equilibrium

$$M_{t+1} = \boxed{\varrho}\, exp\left[-\frac{1}{\boxed{\Psi}}(\hat{g}_{ct+1} + \boxed{g_{xt}})\right] \times \left[\frac{\exp[(1-\boxed{\gamma})(\boxed{\hat{u}_t} + \boxed{\hat{g}_{ct+1}})]}{E_t[\exp[(1+ \qquad \hat{g}_{ct+1})]]}\right]^{\frac{1/\Psi-\gamma}{1-\gamma}}$$

the time d

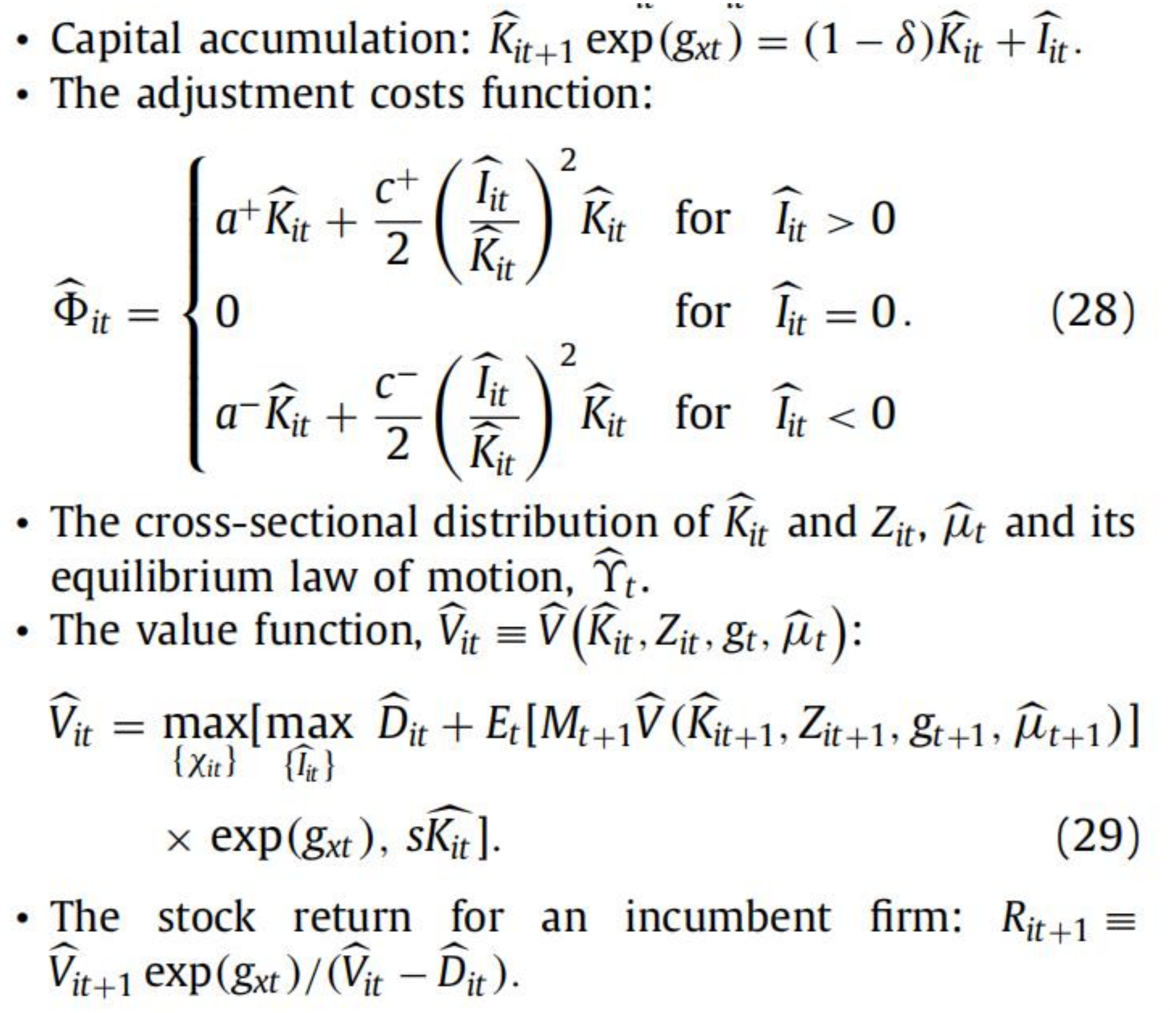
the intertemporal elastici of substitution

the log utility-to

$\hat{g}_{ct+1} \equiv \log(\hat{C}_{t+1}/\hat{C}_t)$ is the log growth rate of detrended consumption

$g_{xt} \equiv \log(X_t/X_{t-1})$ is the log aggregate productivity growth

（7）Solving for the competitive equilibrium



- Capital accumulation: $\widehat{K}_{it+1}\exp(g_{xt}) = (1-\delta)\widehat{K}_{it} + \widehat{I}_{it}$.
- The adjustment costs function:

$$\widehat{\Phi}_{it} = \begin{cases} a^{+}\widehat{K}_{it} + \dfrac{c^{+}}{2}\left(\dfrac{\widehat{I}_{it}}{\widehat{K}_{it}}\right)^{2}\widehat{K}_{it} & \text{for} \quad \widehat{I}_{it} > 0 \\ 0 & \text{for} \quad \widehat{I}_{it} = 0. \\ a^{-}\widehat{K}_{it} + \dfrac{c^{-}}{2}\left(\dfrac{\widehat{I}_{it}}{\widehat{K}_{it}}\right)^{2}\widehat{K}_{it} & \text{for} \quad \widehat{I}_{it} < 0 \end{cases} \qquad (28)$$

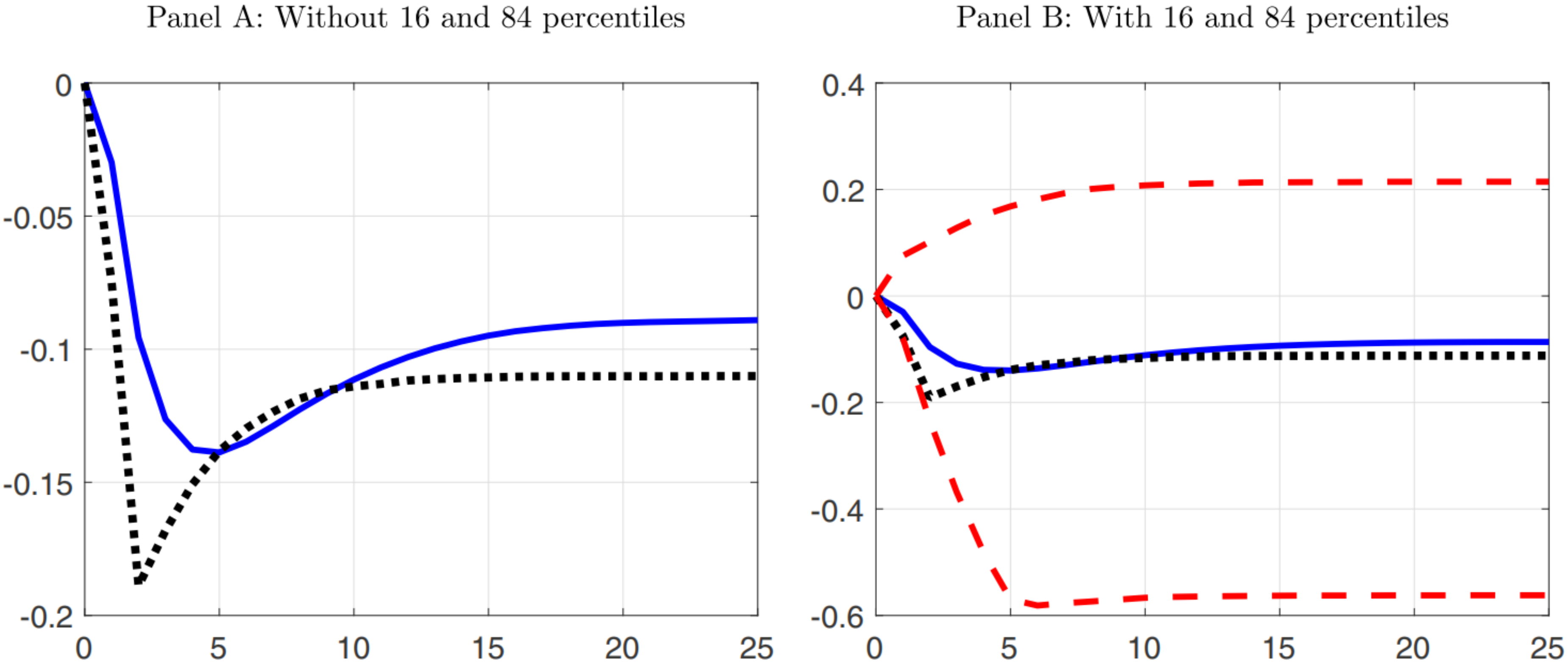
- The cross-sectional distribution of $\widehat{K}_{it}$ and Z_{it}, $\widehat{\mu}_t$ and its equilibrium law of motion, $\widehat{\Upsilon}_t$.
- The value function, $\widehat{V}_{it} \equiv \widehat{V}(\widehat{K}_{it}, Z_{it}, g_t, \widehat{\mu}_t)$:

$$\widehat{V}_{it} = \max_{\{\chi_{it}\}}[\max_{\{\widehat{I}_{it}\}} \widehat{D}_{it} + E_t[M_{t+1}\widehat{V}(\widehat{K}_{it+1}, Z_{it+1}, g_{t+1}, \widehat{\mu}_{t+1})] \times \exp(g_{xt}), s\widehat{K}_{it}]. \qquad (29)$$

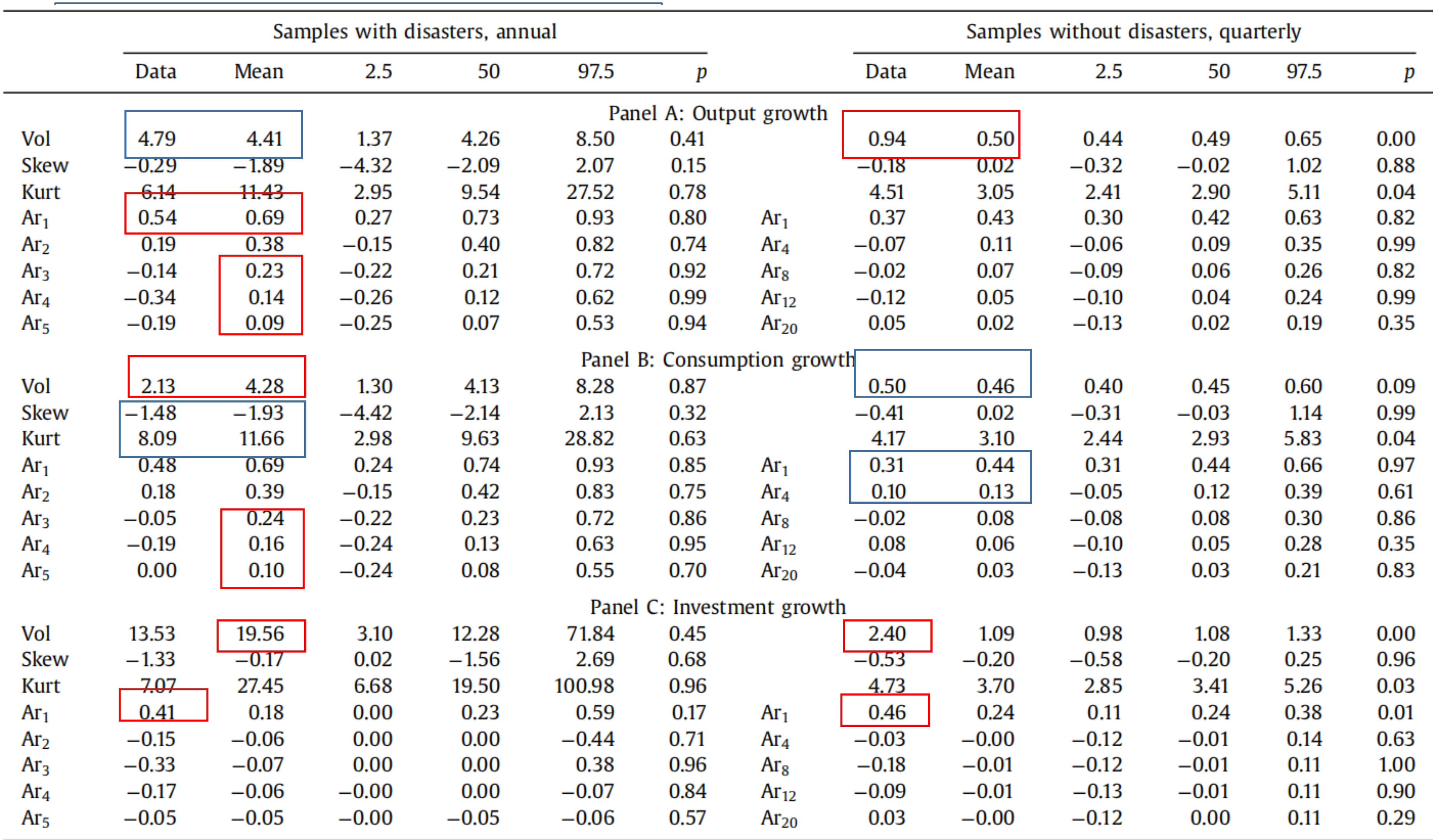
- The stock return for an incumbent firm: $R_{it+1} \equiv \widehat{V}_{it+1}\exp(g_{xt})/(\widehat{V}_{it} - \widehat{D}_{it})$.

approximate aggregation

（1）Calibration and basic moments



The impulse response of consumption to a disaster shock in the model， which are based on more than 28,000 disaster episodes.

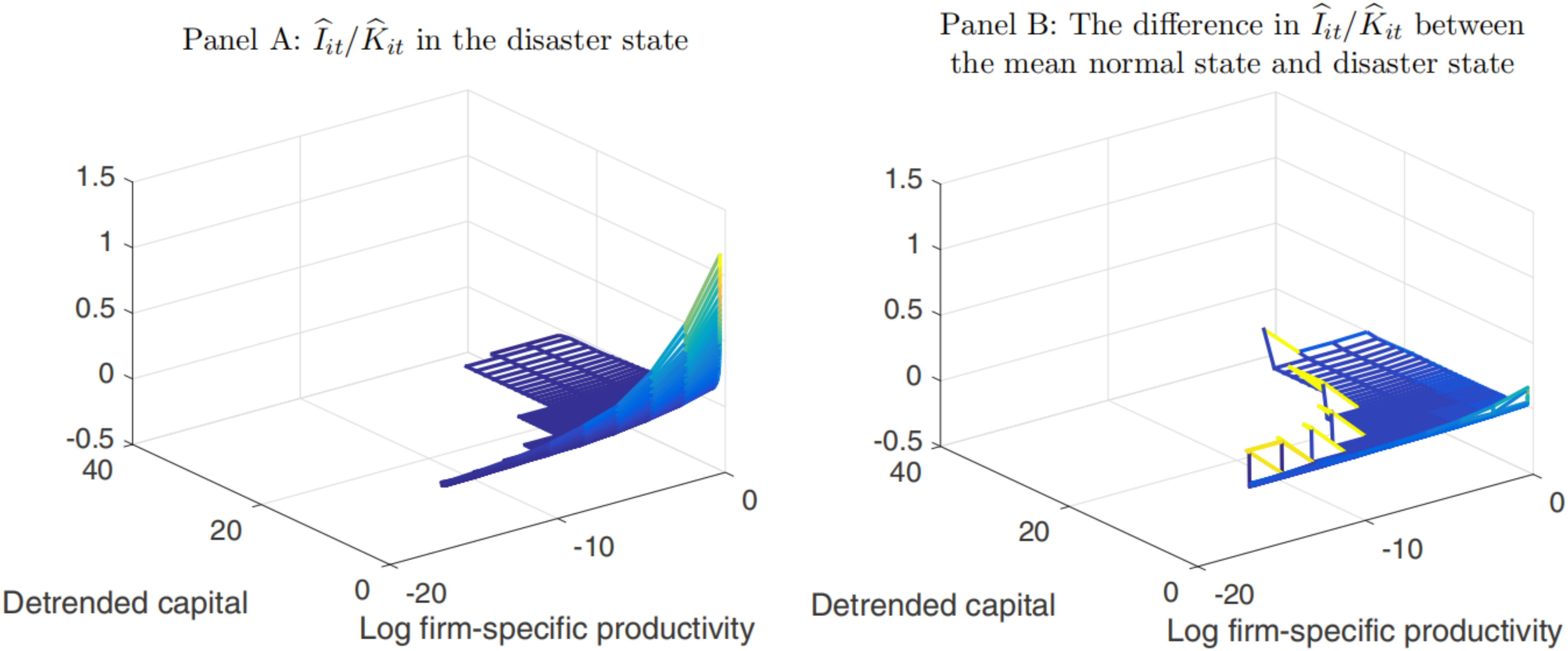


	Samples with disasters, annual							Samples without disasters, quarterly					
	Data	Mean	2.5	50	97.5	*p*		Data	Mean	2.5	50	97.5	*p*
						Panel A: Output growth							
Vol	4.79	4.41	1.37	4.26	8.50	0.41		0.94	0.50	0.44	0.49	0.65	0.00
Skew	−0.29	−1.89	−4.32	−2.09	2.07	0.15		−0.18	0.02	−0.32	−0.02	1.02	0.88
Kurt	6.14	11.43	2.95	9.54	27.52	0.78		4.51	3.05	2.41	2.90	5.11	0.04
Ar$_1$	0.54	0.69	0.27	0.73	0.93	0.80	Ar$_1$	0.37	0.43	0.30	0.42	0.63	0.82
Ar$_2$	0.19	0.38	−0.15	0.40	0.82	0.74	Ar$_4$	−0.07	0.11	−0.06	0.09	0.35	0.99
Ar$_3$	−0.14	0.23	−0.22	0.21	0.72	0.92	Ar$_8$	−0.02	0.07	−0.09	0.06	0.26	0.82
Ar$_4$	−0.34	0.14	−0.26	0.12	0.62	0.99	Ar$_{12}$	−0.12	0.05	−0.10	0.04	0.24	0.99
Ar$_5$	−0.19	0.09	−0.25	0.07	0.53	0.94	Ar$_{20}$	0.05	0.02	−0.13	0.02	0.19	0.35
						Panel B: Consumption growth							
Vol	2.13	4.28	1.30	4.13	8.28	0.87		0.50	0.46	0.40	0.45	0.60	0.09
Skew	−1.48	−1.93	−4.42	−2.14	2.13	0.32		−0.41	0.02	−0.31	−0.03	1.14	0.99
Kurt	8.09	11.66	2.98	9.63	28.82	0.63		4.17	3.10	2.44	2.93	5.83	0.04
Ar$_1$	0.48	0.69	0.24	0.74	0.93	0.85	Ar$_1$	0.31	0.44	0.31	0.44	0.66	0.97
Ar$_2$	0.18	0.39	−0.15	0.42	0.83	0.75	Ar$_4$	0.10	0.13	−0.05	0.12	0.39	0.61
Ar$_3$	−0.05	0.24	−0.22	0.23	0.72	0.86	Ar$_8$	−0.02	0.08	−0.08	0.08	0.30	0.86
Ar$_4$	−0.19	0.16	−0.24	0.13	0.63	0.95	Ar$_{12}$	0.08	0.06	−0.10	0.05	0.28	0.35
Ar$_5$	0.00	0.10	−0.24	0.08	0.55	0.70	Ar$_{20}$	−0.04	0.03	−0.13	0.03	0.21	0.83
						Panel C: Investment growth							
Vol	13.53	19.56	3.10	12.28	71.84	0.45		2.40	1.09	0.98	1.08	1.33	0.00
Skew	−1.33	−0.17	0.02	−1.56	2.69	0.68		−0.53	−0.20	−0.58	−0.20	0.25	0.96
Kurt	7.07	27.45	6.68	19.50	100.98	0.96		4.73	3.70	2.85	3.41	5.26	0.03
Ar$_1$	0.41	0.18	0.00	0.23	0.59	0.17	Ar$_1$	0.46	0.24	0.11	0.24	0.38	0.01
Ar$_2$	−0.15	−0.06	0.00	0.00	−0.44	0.71	Ar$_4$	−0.03	−0.00	−0.12	−0.01	0.14	0.63
Ar$_3$	−0.33	−0.07	0.00	0.00	0.38	0.96	Ar$_8$	−0.18	−0.01	−0.12	−0.01	0.11	1.00
Ar$_4$	−0.17	−0.06	−0.00	0.00	−0.07	0.84	Ar$_{12}$	−0.09	−0.01	−0.13	−0.01	0.11	0.90
Ar$_5$	−0.05	−0.05	−0.00	−0.05	−0.06	0.57	Ar$_{20}$	0.03	−0.00	−0.12	0.00	0.11	0.29

More important, introducing an extra aggregate state will most likely strengthen the model’s ability to explain the failure of the CAPM, which is our main focus.

（2）Key properties of the competitive equilibrium

Optimal policy functions

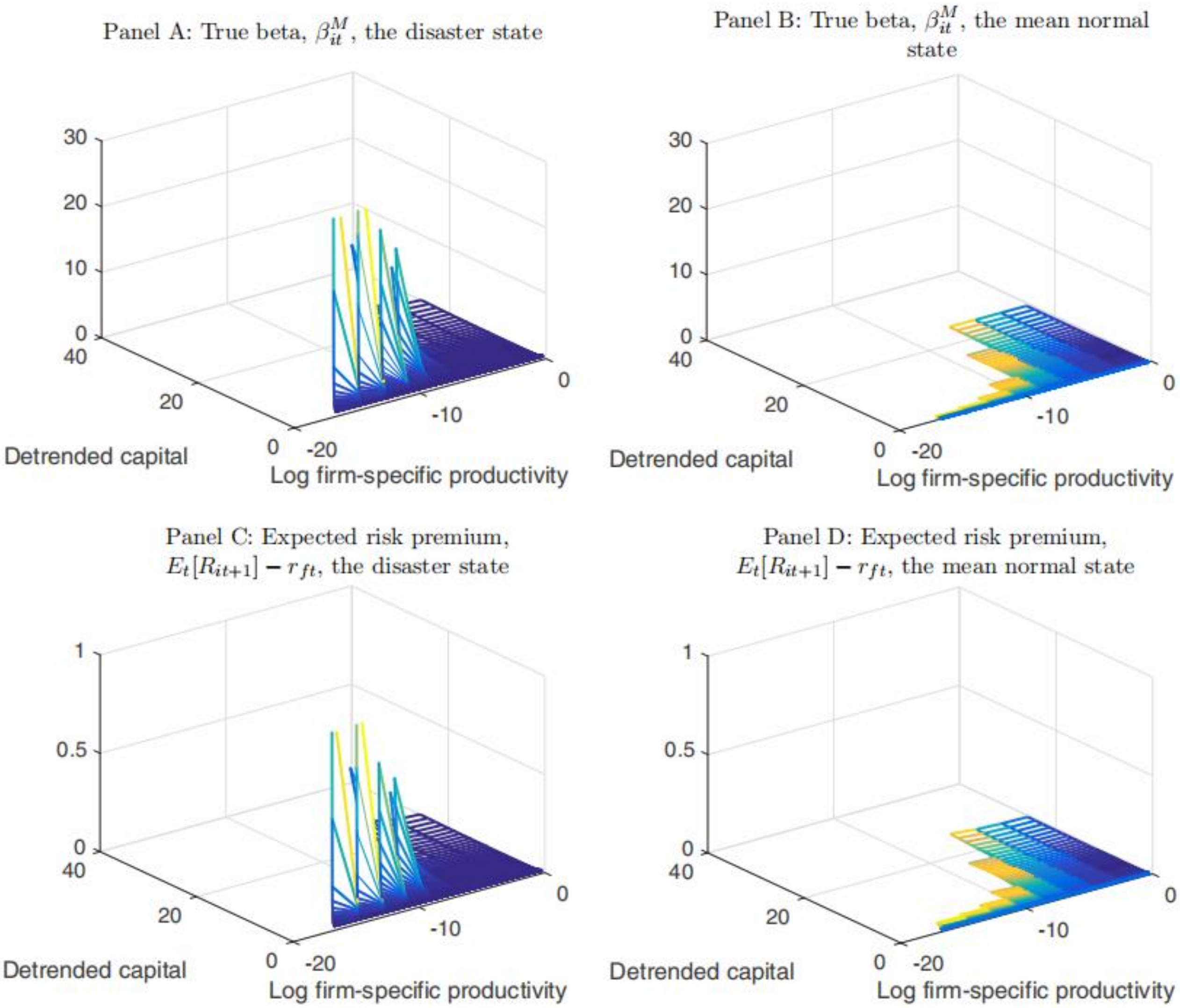


Panel A: $\widehat{I}_{it}/\widehat{K}_{it}$ in the disaster state

Panel B: The difference in $\widehat{I}_{it}/\widehat{K}_{it}$ between the mean normal state and disaster state

（2）Key properties of the competitive equilibrium

Risk and risk premiums



Panel A: True beta, β_{it}^{M}, the disaster state

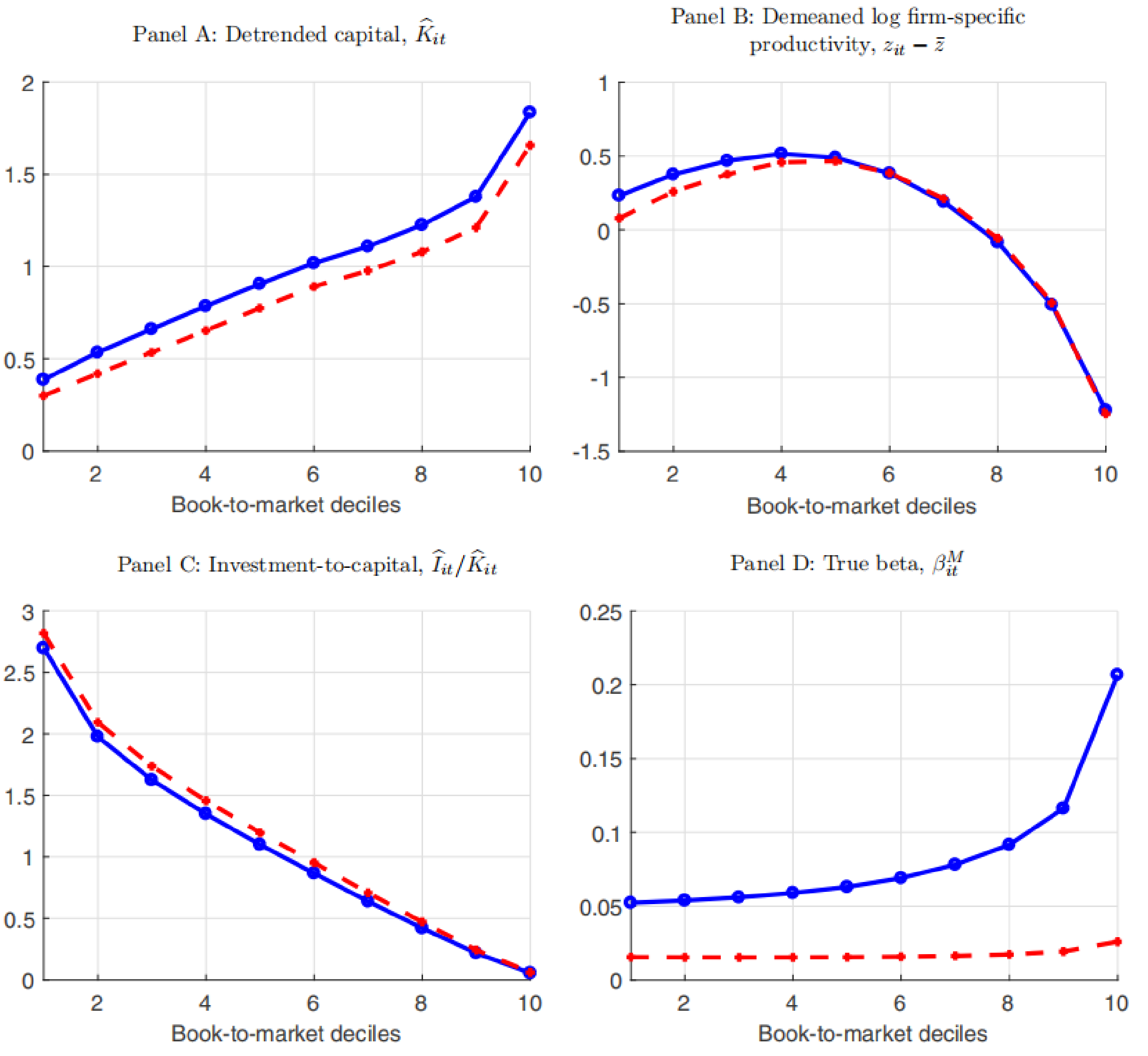
Panel B: True beta, β_{it}^{M}, the mean normal state

Panel C: Expected risk premium, $E_t[R_{it+1}] - r_{ft}$, the disaster state

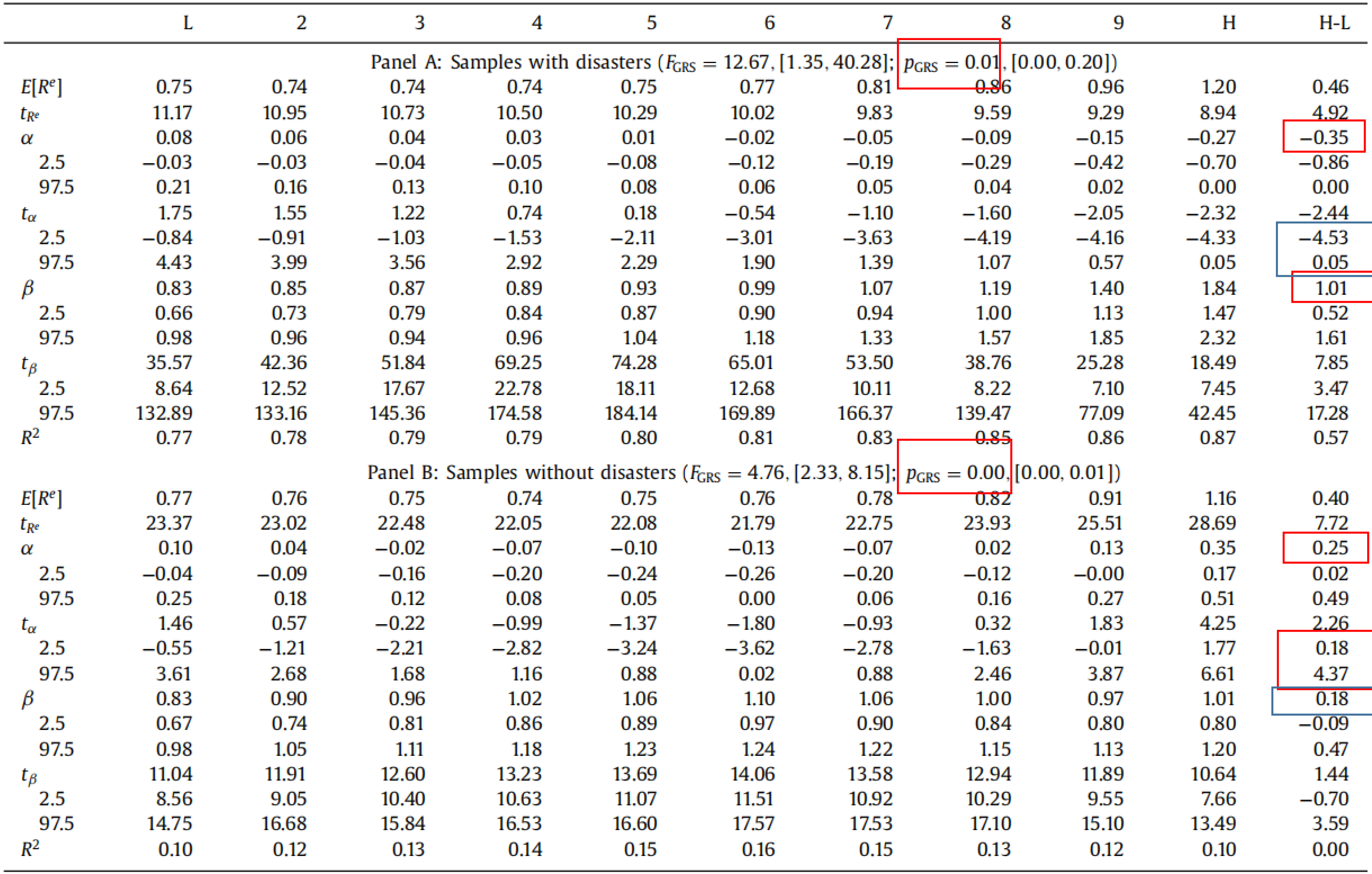
Panel D: Expected risk premium, $E_t[R_{it+1}] - r_{ft}$, the mean normal state

（2）Key properties of the competitive equilibrium

Value versus growth

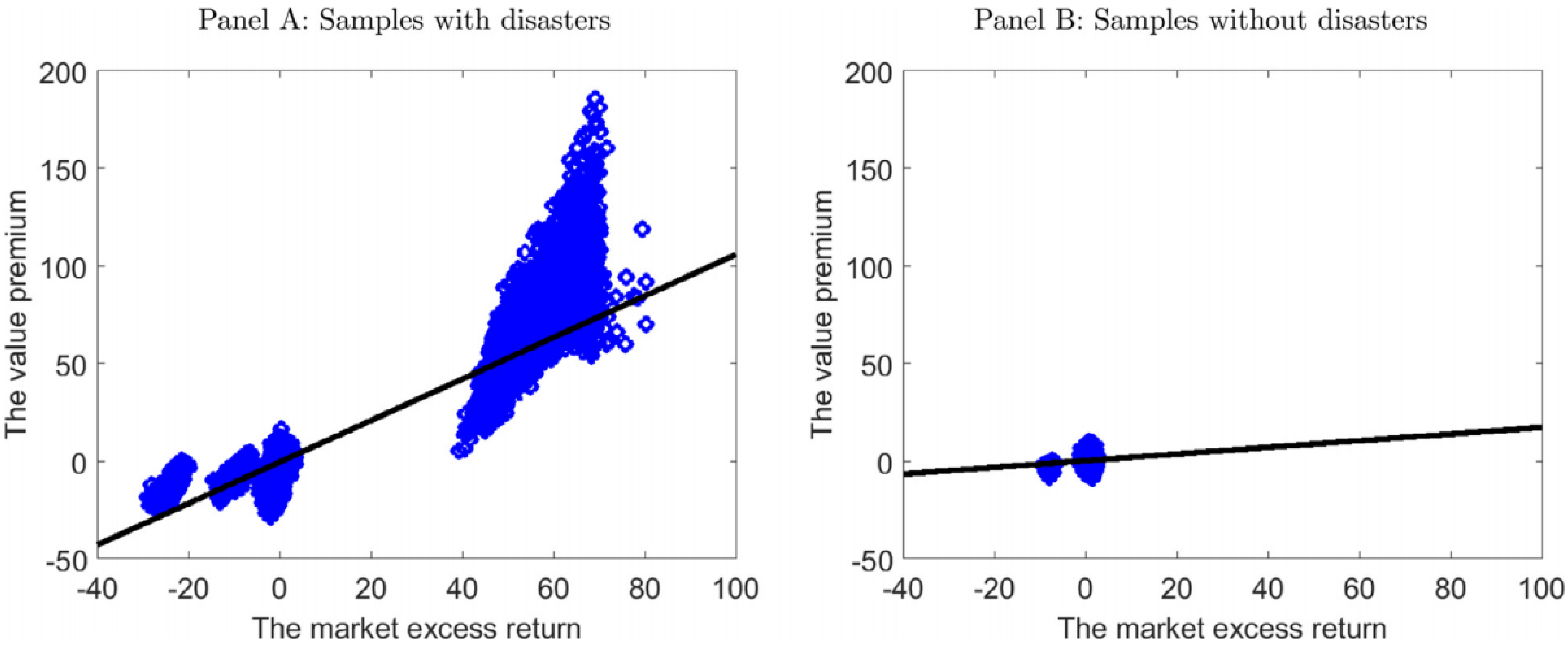


（3）Explaining the failure of the CAPM



	L	2	3	4	5	6	7	8	9	H	H-L
Panel A: Samples with disasters ($F_{GRS} = 12.67$, [1.35, 40.28]; $p_{GRS} = 0.01$, [0.00, 0.20])											
$E[R^e]$	0.75	0.74	0.74	0.74	0.75	0.77	0.81	0.86	0.96	1.20	0.46
t_{R^e}	11.17	10.95	10.73	10.50	10.29	10.02	9.83	9.59	9.29	8.94	4.92
α	0.08	0.06	0.04	0.03	0.01	−0.02	−0.05	−0.09	−0.15	−0.27	−0.35
2.5	−0.03	−0.03	−0.04	−0.05	−0.08	−0.12	−0.19	−0.29	−0.42	−0.70	−0.86
97.5	0.21	0.16	0.13	0.10	0.08	0.06	0.05	0.04	0.02	0.00	0.00
t_α	1.75	1.55	1.22	0.74	0.18	−0.54	−1.10	−1.60	−2.05	−2.32	−2.44
2.5	−0.84	−0.91	−1.03	−1.53	−2.11	−3.01	−3.63	−4.19	−4.16	−4.33	−4.53
97.5	4.43	3.99	3.56	2.92	2.29	1.90	1.39	1.07	0.57	0.05	0.05
β	0.83	0.85	0.87	0.89	0.93	0.99	1.07	1.19	1.40	1.84	1.01
2.5	0.66	0.73	0.79	0.84	0.87	0.90	0.94	1.00	1.13	1.47	0.52
97.5	0.98	0.96	0.94	0.96	1.04	1.18	1.33	1.57	1.85	2.32	1.61
t_β	35.57	42.36	51.84	69.25	74.28	65.01	53.50	38.76	25.28	18.49	7.85
2.5	8.64	12.52	17.67	22.78	18.11	12.68	10.11	8.22	7.10	7.45	3.47
97.5	132.89	133.16	145.36	174.58	184.14	169.89	166.37	139.47	77.09	42.45	17.28
R^2	0.77	0.78	0.79	0.79	0.80	0.81	0.83	0.85	0.86	0.87	0.57
Panel B: Samples without disasters ($F_{GRS} = 4.76$, [2.33, 8.15]; $p_{GRS} = 0.00$, [0.00, 0.01])											
$E[R^e]$	0.77	0.76	0.75	0.74	0.75	0.76	0.78	0.82	0.91	1.16	0.40
t_{R^e}	23.37	23.02	22.48	22.05	22.08	21.79	22.75	23.93	25.51	28.69	7.72
α	0.10	0.04	−0.02	−0.07	−0.10	−0.13	−0.07	0.02	0.13	0.35	0.25
2.5	−0.04	−0.09	−0.16	−0.20	−0.24	−0.26	−0.20	−0.12	−0.00	0.17	0.02
97.5	0.25	0.18	0.12	0.08	0.05	0.00	0.06	0.16	0.27	0.51	0.49
t_α	1.46	0.57	−0.22	−0.99	−1.37	−1.80	−0.93	0.32	1.83	4.25	2.26
2.5	−0.55	−1.21	−2.21	−2.82	−3.24	−3.62	−2.78	−1.63	−0.01	1.77	0.18
97.5	3.61	2.68	1.68	1.16	0.88	0.02	0.88	2.46	3.87	6.61	4.37
β	0.83	0.90	0.96	1.02	1.06	1.10	1.06	1.00	0.97	1.01	0.18
2.5	0.67	0.74	0.81	0.86	0.89	0.97	0.90	0.84	0.80	0.80	−0.09
97.5	0.98	1.05	1.11	1.18	1.23	1.24	1.22	1.15	1.13	1.20	0.47
t_β	11.04	11.91	12.60	13.23	13.69	14.06	13.58	12.94	11.89	10.64	1.44
2.5	8.56	9.05	10.40	10.63	11.07	11.51	10.92	10.29	9.55	7.66	−0.70
97.5	14.75	16.68	15.84	16.53	16.60	17.57	17.53	17.10	15.10	13.49	3.59
R^2	0.10	0.12	0.13	0.14	0.15	0.16	0.15	0.13	0.12	0.10	0.00

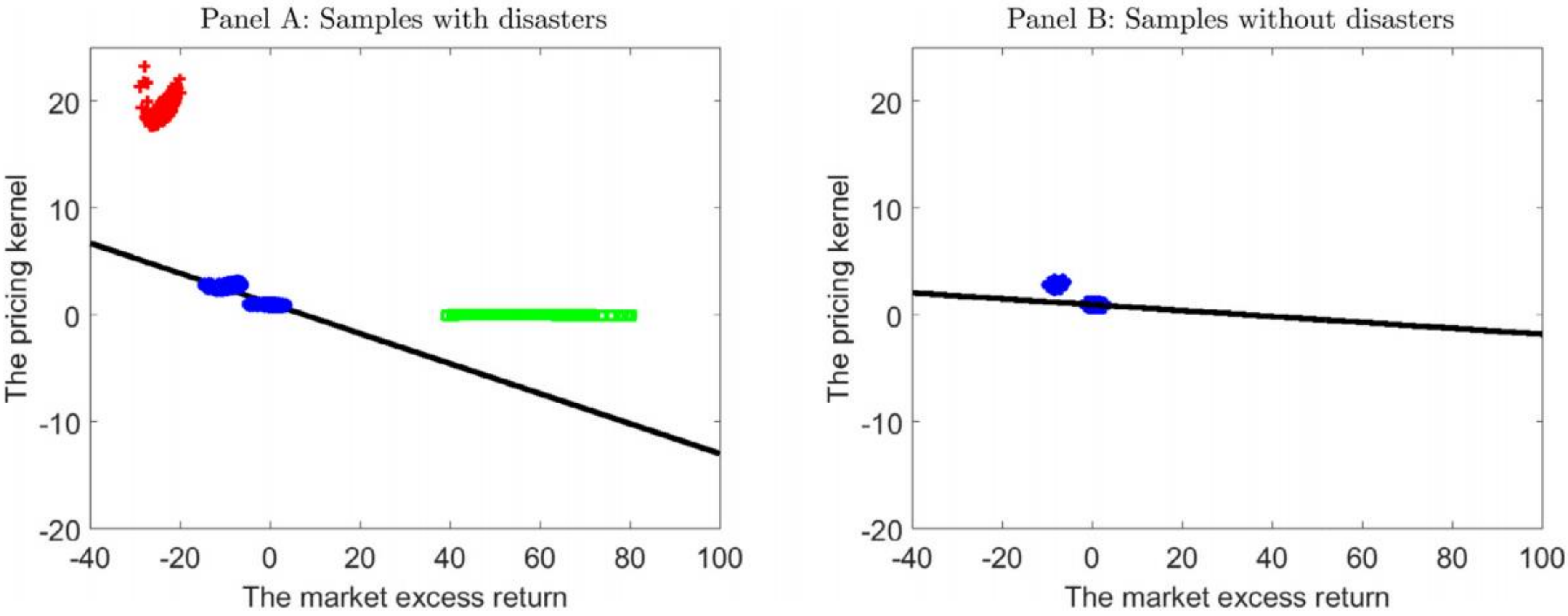
① Nonlinearity in the CAPM regressions



However, the CAPM alpha is −0.39% per month, implying that the unconditional CAPM does not hold in our dynamic single-factor model.

the CAPM regression line is largely flat

②Nonlinearity in the pricing kernel

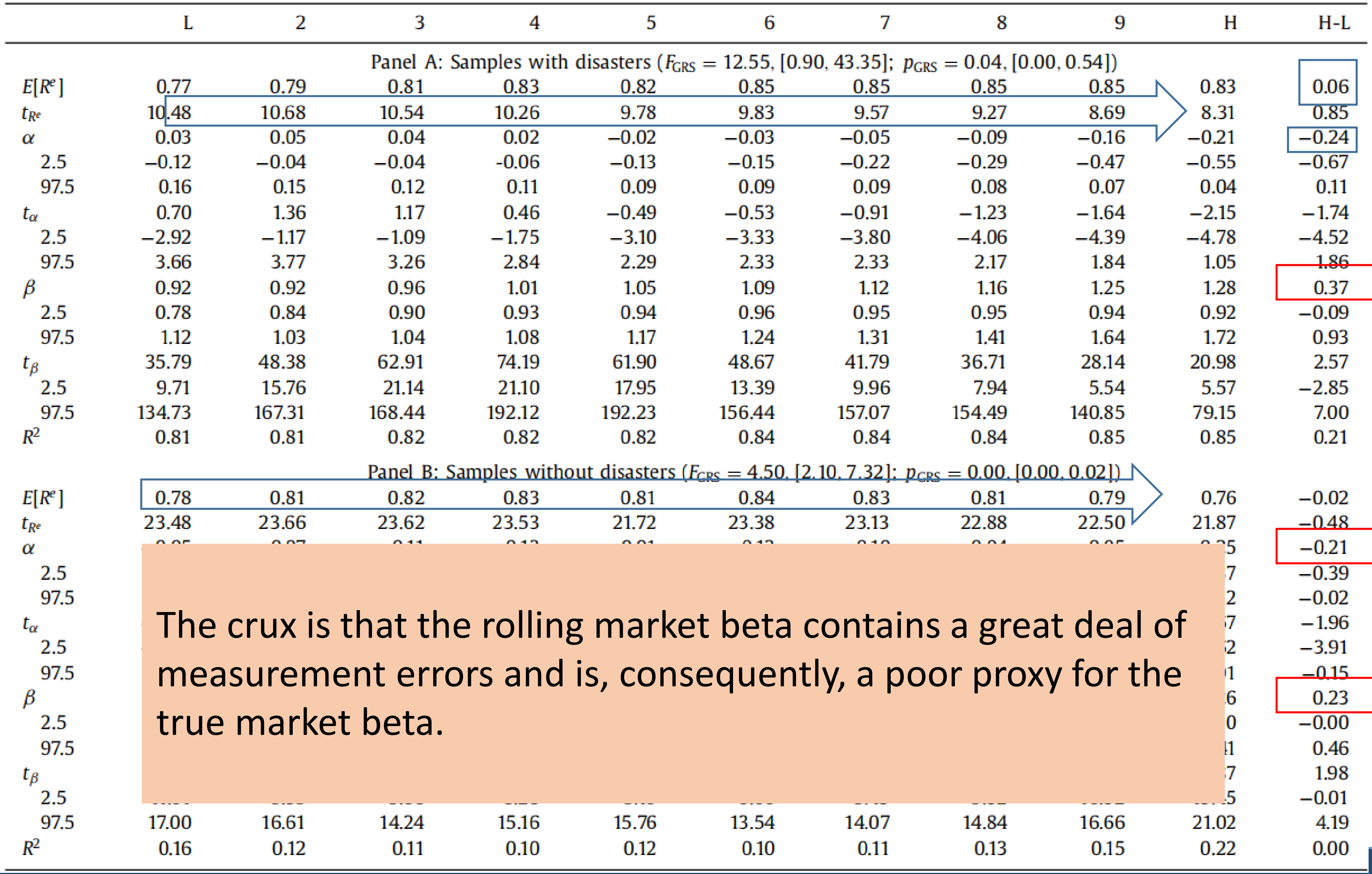


The linear CAPM fits poorly the observations from the disaster state, **with high** realizations of the pricing kernel, and the observations from the recovery state, **with low** realizations of the pricing kernel.

CAPM is an even worse proxy for the pricing kernel in the no-disaster samples. The regression slope is **only −0.03**. As such, the CAPM fails badly in the no-disaster samples.

（4）Explaining the beta anomaly

根据preranking market beta分组



	L	2	3	4	5	6	7	8	9	H	H-L
Panel A: Samples with disasters (F_{GRS} = 12.55, [0.90, 43.35]; p_{GRS} = 0.04, [0.00, 0.54])											
$E[R^e]$	0.77	0.79	0.81	0.83	0.82	0.85	0.85	0.85	0.85	0.83	0.06
t_{R^e}	10.48	10.68	10.54	10.26	9.78	9.83	9.57	9.27	8.69	8.31	0.85
α	0.03	0.05	0.04	0.02	−0.02	−0.03	−0.05	−0.09	−0.16	−0.21	−0.24
2.5	−0.12	−0.04	−0.04	-0.06	−0.13	−0.15	−0.22	−0.29	−0.47	−0.55	−0.67
97.5	0.16	0.15	0.12	0.11	0.09	0.09	0.09	0.08	0.07	0.04	0.11
t_α	0.70	1.36	1.17	0.46	−0.49	−0.53	−0.91	−1.23	−1.64	−2.15	−1.74
2.5	−2.92	−1.17	−1.09	−1.75	−3.10	−3.33	−3.80	−4.06	−4.39	−4.78	−4.52
97.5	3.66	3.77	3.26	2.84	2.29	2.33	2.33	2.17	1.84	1.05	1.86
β	0.92	0.92	0.96	1.01	1.05	1.09	1.12	1.16	1.25	1.28	0.37
2.5	0.78	0.84	0.90	0.93	0.94	0.96	0.95	0.95	0.94	0.92	−0.09
97.5	1.12	1.03	1.04	1.08	1.17	1.24	1.31	1.41	1.64	1.72	0.93
t_β	35.79	48.38	62.91	74.19	61.90	48.67	41.79	36.71	28.14	20.98	2.57
2.5	9.71	15.76	21.14	21.10	17.95	13.39	9.96	7.94	5.54	5.57	−2.85
97.5	134.73	167.31	168.44	192.12	192.23	156.44	157.07	154.49	140.85	79.15	7.00
R^2	0.81	0.81	0.82	0.82	0.82	0.84	0.84	0.84	0.85	0.85	0.21
Panel B: Samples without disasters (F_{GRS} = 4.50, [2.10, 7.32]; p_{GRS} = 0.00, [0.00, 0.02])											
$E[R^e]$	0.78	0.81	0.82	0.83	0.81	0.84	0.83	0.81	0.79	0.76	−0.02
t_{R^e}	23.48	23.66	23.62	23.53	21.72	23.38	23.13	22.88	22.50	21.87	−0.48
α	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	−0.21
2.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	−0.39
97.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	−0.02
t_α	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	−1.96
2.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	−3.91
97.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	−0.15
β	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.23
2.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	−0.00
97.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.46
t_β	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.98
2.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	−0.01
97.5	17.00	16.61	14.24	15.16	15.76	13.54	14.07	14.84	16.66	21.02	4.19
R^2	0.16	0.12	0.11	0.10	0.12	0.10	0.11	0.13	0.15	0.22	0.00

The crux is that the rolling market beta contains a great deal of measurement errors and is, consequently, a poor proxy for the true market beta.

（4）Explaining the beta anomaly

In untabulated results, sorting on the true market beta yields large average return spreads across extreme deciles in the model, with and without disasters.

In samples with disasters → The unconditional CAPM fails to price these deciles, as the postranking beta overshoots, then a negative CAPM alpha of −0.69%

In samples without disasters → The postranking beta moves in the opposite direction as the true market beta, with a spread of −0.83. alphais 1.6%, which is substantially higher than the average return spread.

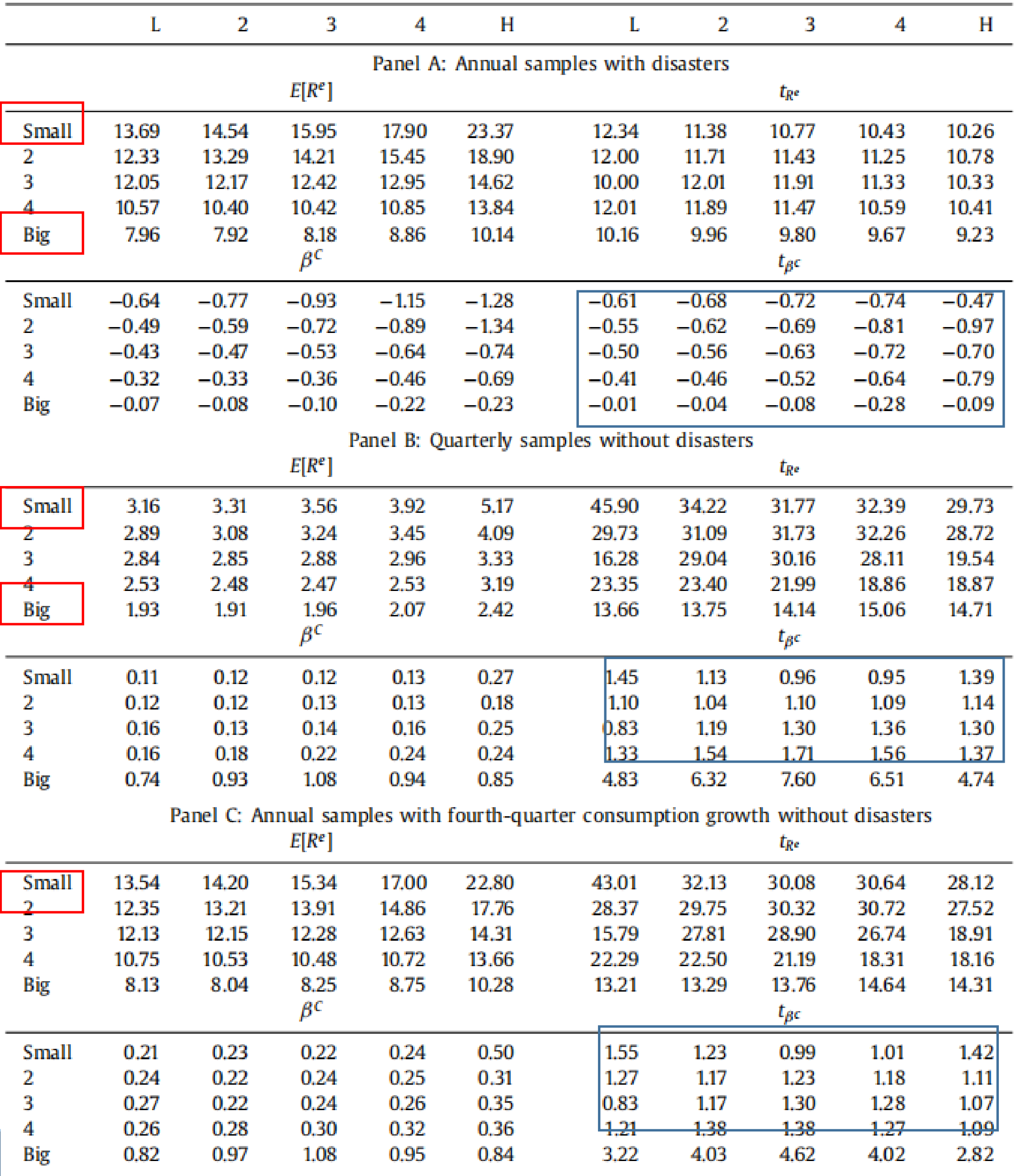
（4）Explaining the beta anomaly

To illustrate the measurement errors of rolling market betas as the proxy for the true market betas:

the correlation between the true and rolling market betas is weakly positive, 2.84%, across the preranking market beta deciles in the disaster samples, but weakly negative, −5.43%, in the no-disaster samples.

the true market beta accurately and immediately reflects changes in aggregate and firm-specific conditions. Within a given rolling window, the true market beta often even mean reverts, giving rise to opposite rankings in rolling betas.

（5）Explaining the poor performance of the consumption CAPM



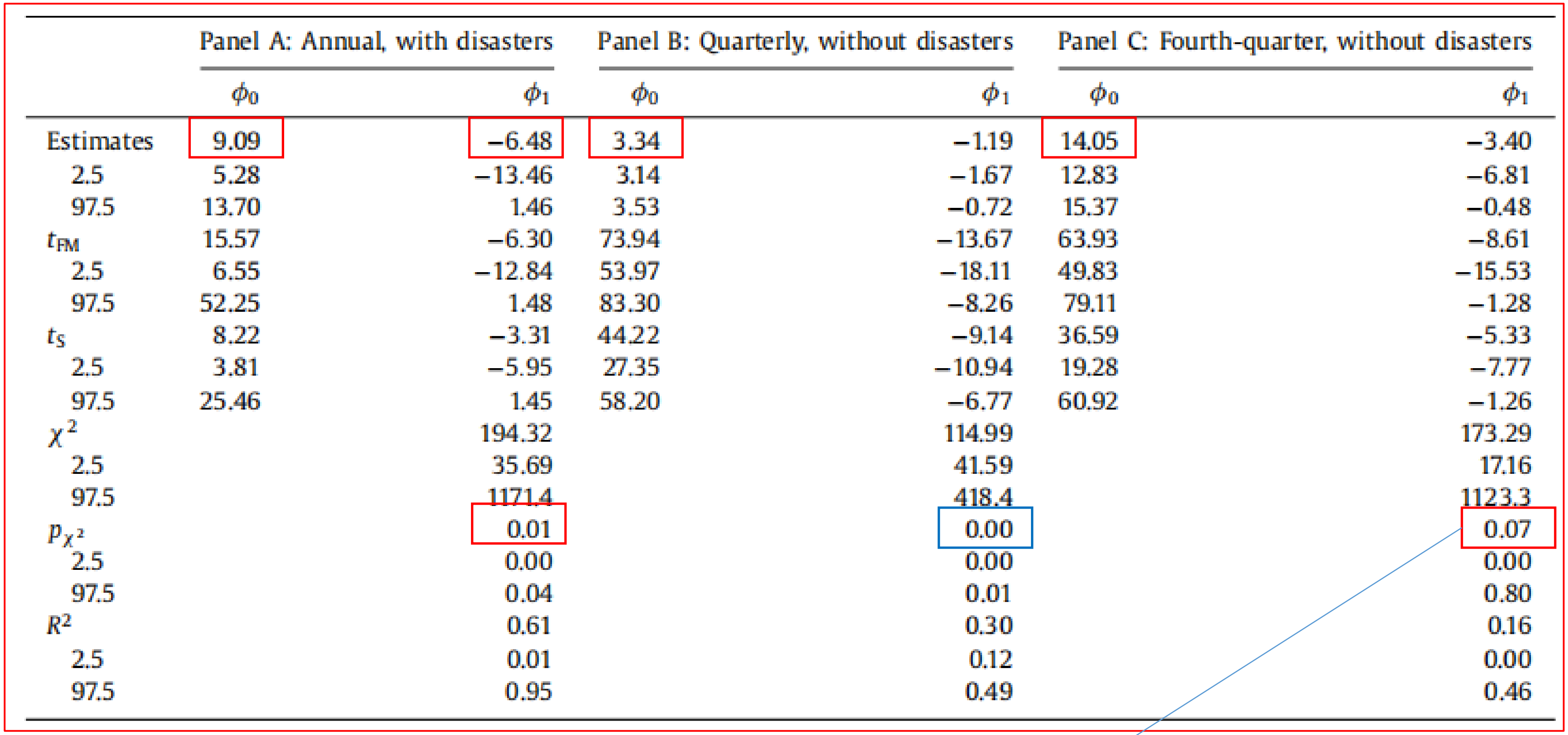
	L	2	3	4	H	L	2	3	4	H
	Panel A: Annual samples with disasters									
			$E[R^e]$					t_{R^e}		
Small	13.69	14.54	15.95	17.90	23.37	12.34	11.38	10.77	10.43	10.26
2	12.33	13.29	14.21	15.45	18.90	12.00	11.71	11.43	11.25	10.78
3	12.05	12.17	12.42	12.95	14.62	10.00	12.01	11.91	11.33	10.33
4	10.57	10.40	10.42	10.85	13.84	12.01	11.89	11.47	10.59	10.41
Big	7.96	7.92	8.18	8.86	10.14	10.16	9.96	9.80	9.67	9.23
			β^C					t_{β^C}		
Small	−0.64	−0.77	−0.93	−1.15	−1.28	−0.61	−0.68	−0.72	−0.74	−0.47
2	−0.49	−0.59	−0.72	−0.89	−1.34	−0.55	−0.62	−0.69	−0.81	−0.97
3	−0.43	−0.47	−0.53	−0.64	−0.74	−0.50	−0.56	−0.63	−0.72	−0.70
4	−0.32	−0.33	−0.36	−0.46	−0.69	−0.41	−0.46	−0.52	−0.64	−0.79
Big	−0.07	−0.08	−0.10	−0.22	−0.23	−0.01	−0.04	−0.08	−0.28	−0.09
	Panel B: Quarterly samples without disasters									
			$E[R^e]$					t_{R^e}		
Small	3.16	3.31	3.56	3.92	5.17	45.90	34.22	31.77	32.39	29.73
2	2.89	3.08	3.24	3.45	4.09	29.73	31.09	31.73	32.26	28.72
3	2.84	2.85	2.88	2.96	3.33	16.28	29.04	30.16	28.11	19.54
4	2.53	2.48	2.47	2.53	3.19	23.35	23.40	21.99	18.86	18.87
Big	1.93	1.91	1.96	2.07	2.42	13.66	13.75	14.14	15.06	14.71
			β^C					t_{β^C}		
Small	0.11	0.12	0.12	0.13	0.27	1.45	1.13	0.96	0.95	1.39
2	0.12	0.12	0.13	0.13	0.18	1.10	1.04	1.10	1.09	1.14
3	0.16	0.13	0.14	0.16	0.25	0.83	1.19	1.30	1.36	1.30
4	0.16	0.18	0.22	0.24	0.24	1.33	1.54	1.71	1.56	1.37
Big	0.74	0.93	1.08	0.94	0.85	4.83	6.32	7.60	6.51	4.74
	Panel C: Annual samples with fourth-quarter consumption growth without disasters									
			$E[R^e]$					t_{R^e}		
Small	13.54	14.20	15.34	17.00	22.80	43.01	32.13	30.08	30.64	28.12
2	12.35	13.21	13.91	14.86	17.76	28.37	29.75	30.32	30.72	27.52
3	12.13	12.15	12.28	12.63	14.31	15.79	27.81	28.90	26.74	18.91
4	10.75	10.53	10.48	10.72	13.66	22.29	22.50	21.19	18.31	18.16
Big	8.13	8.04	8.25	8.75	10.28	13.21	13.29	13.76	14.64	14.31
			β^C					t_{β^C}		
Small	0.21	0.23	0.22	0.24	0.50	1.55	1.23	0.99	1.01	1.42
2	0.24	0.22	0.24	0.25	0.31	1.27	1.17	1.23	1.18	1.11
3	0.27	0.22	0.24	0.26	0.35	0.83	1.17	1.30	1.28	1.07
4	0.26	0.28	0.30	0.32	0.36	1.21	1.38	1.38	1.27	1.09
Big	0.82	0.97	1.08	0.95	0.84	3.22	4.03	4.62	4.02	2.82

① Explaining the higher average value premium in small firms

②Explaining the failure of the consumption CAPM

（5）Explaining the poor performance of the consumption CAPM

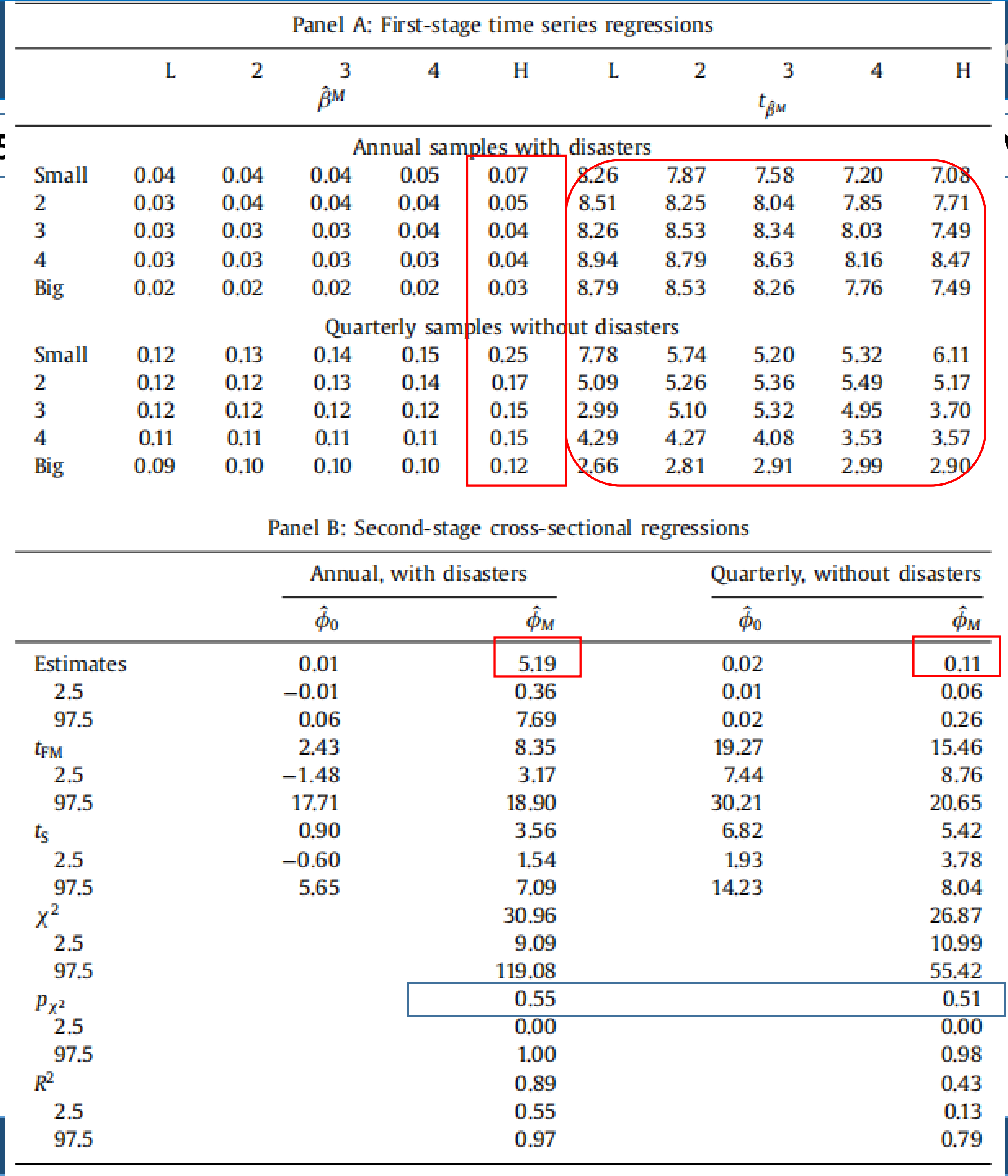
②Explaining the failure of the consumption CAPM



	Panel A: Annual, with disasters		Panel B: Quarterly, without disasters		Panel C: Fourth-quarter, without disasters	
	ϕ_0	ϕ_1	ϕ_0	ϕ_1	ϕ_0	ϕ_1
Estimates	9.09	−6.48	3.34	−1.19	14.05	−3.40
2.5	5.28	−13.46	3.14	−1.67	12.83	−6.81
97.5	13.70	1.46	3.53	−0.72	15.37	−0.48
t_{FM}	15.57	−6.30	73.94	−13.67	63.93	−8.61
2.5	6.55	−12.84	53.97	−18.11	49.83	−15.53
97.5	52.25	1.48	83.30	−8.26	79.11	−1.28
t_S	8.22	−3.31	44.22	−9.14	36.59	−5.33
2.5	3.81	−5.95	27.35	−10.94	19.28	−7.77
97.5	25.46	1.45	58.20	−6.77	60.92	−1.26
χ^2		194.32		114.99		173.29
2.5		35.69		41.59		17.16
97.5		1171.4		418.4		1123.3
p_{χ^2}		0.01		0.00		0.07
2.5		0.00		0.00		0.00
97.5		0.04		0.01		0.80
R^2		0.61		0.30		0.16
2.5		0.01		0.12		0.00
97.5		0.95		0.49		0.46

We interpret the insignifificance as probably due to the lack of power of the test, as only 25% of the observations are used

M



Panel A: First-stage time series regressions

	L	2	3	4	H	L	2	3	4	H
			$\hat{\beta}^M$					$t_{\hat{\beta}^M}$		
Annual samples with disasters										
Small	0.04	0.04	0.04	0.05	0.07	8.26	7.87	7.58	7.20	7.08
2	0.03	0.04	0.04	0.04	0.05	8.51	8.25	8.04	7.85	7.71
3	0.03	0.03	0.03	0.04	0.04	8.26	8.53	8.34	8.03	7.49
4	0.03	0.03	0.03	0.03	0.04	8.94	8.79	8.63	8.16	8.47
Big	0.02	0.02	0.02	0.02	0.03	8.79	8.53	8.26	7.76	7.49
Quarterly samples without disasters										
Small	0.12	0.13	0.14	0.15	0.25	7.78	5.74	5.20	5.32	6.11
2	0.12	0.12	0.13	0.14	0.17	5.09	5.26	5.36	5.49	5.17
3	0.12	0.12	0.12	0.12	0.15	2.99	5.10	5.32	4.95	3.70
4	0.11	0.11	0.11	0.11	0.15	4.29	4.27	4.08	3.53	3.57
Big	0.09	0.10	0.10	0.10	0.12	2.66	2.81	2.91	2.99	2.90

Panel B: Second-stage cross-sectional regressions

	Annual, with disasters		Quarterly, without disasters	
	$\hat{\phi}_0$	$\hat{\phi}_M$	$\hat{\phi}_0$	$\hat{\phi}_M$
Estimates	0.01	5.19	0.02	0.11
2.5	−0.01	0.36	0.01	0.06
97.5	0.06	7.69	0.02	0.26
t_{FM}	2.43	8.35	19.27	15.46
2.5	−1.48	3.17	7.44	8.76
97.5	17.71	18.90	30.21	20.65
t_S	0.90	3.56	6.82	5.42
2.5	−0.60	1.54	1.93	3.78
97.5	5.65	7.09	14.23	8.04
χ^2		30.96		26.87
2.5		9.09		10.99
97.5		119.08		55.42
p_{χ^2}		0.55		0.51
2.5		0.00		0.00
97.5		1.00		0.98
R^2		0.89		0.43
2.5		0.55		0.13
97.5		0.97		0.79

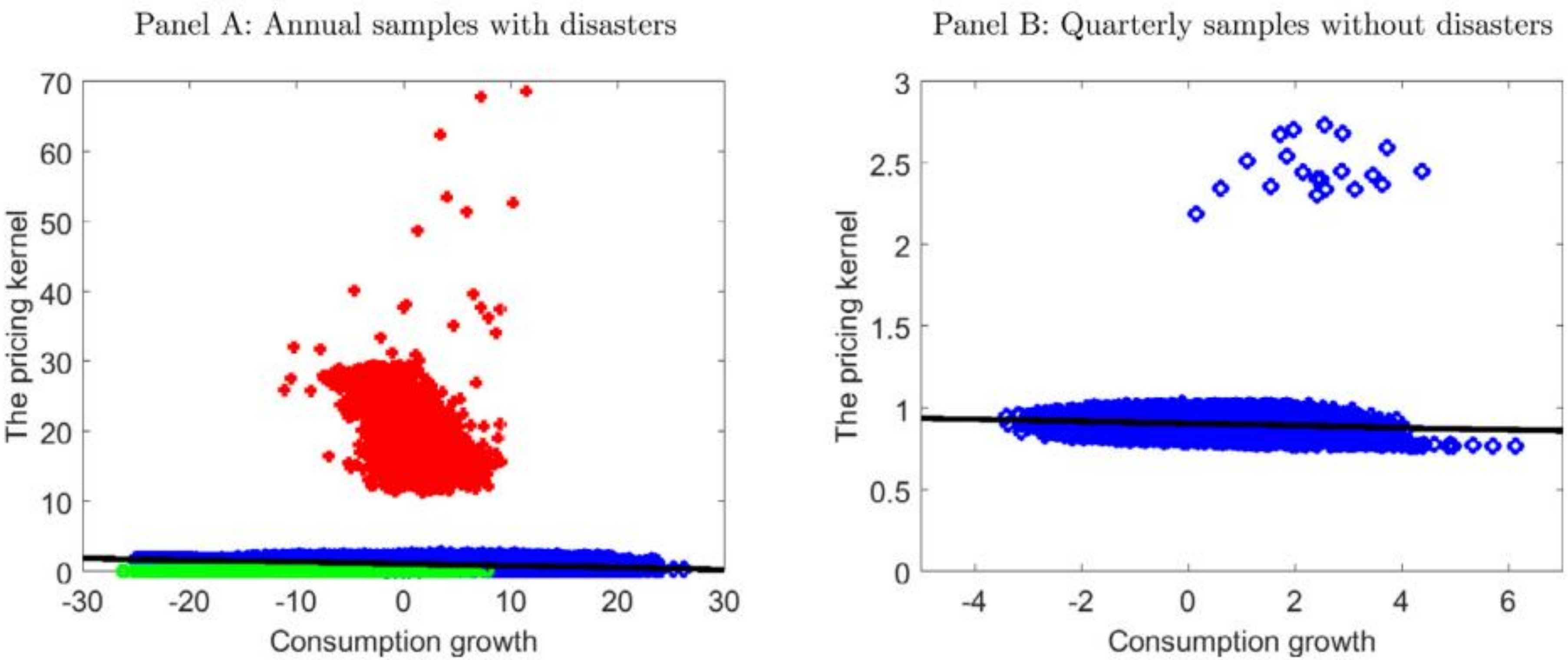
table11
the true pricing kernel in the model

① magnitude of the regression-based estimates of β is **largely in line with** that of the true beta calculated on the grid (Panel D of Fig. 6).

②pricing kernel's volatility is higher in samples with disasters than without disasters, meaning that the realized pricing of risk, ϕMt, is lower in samples without disasters.

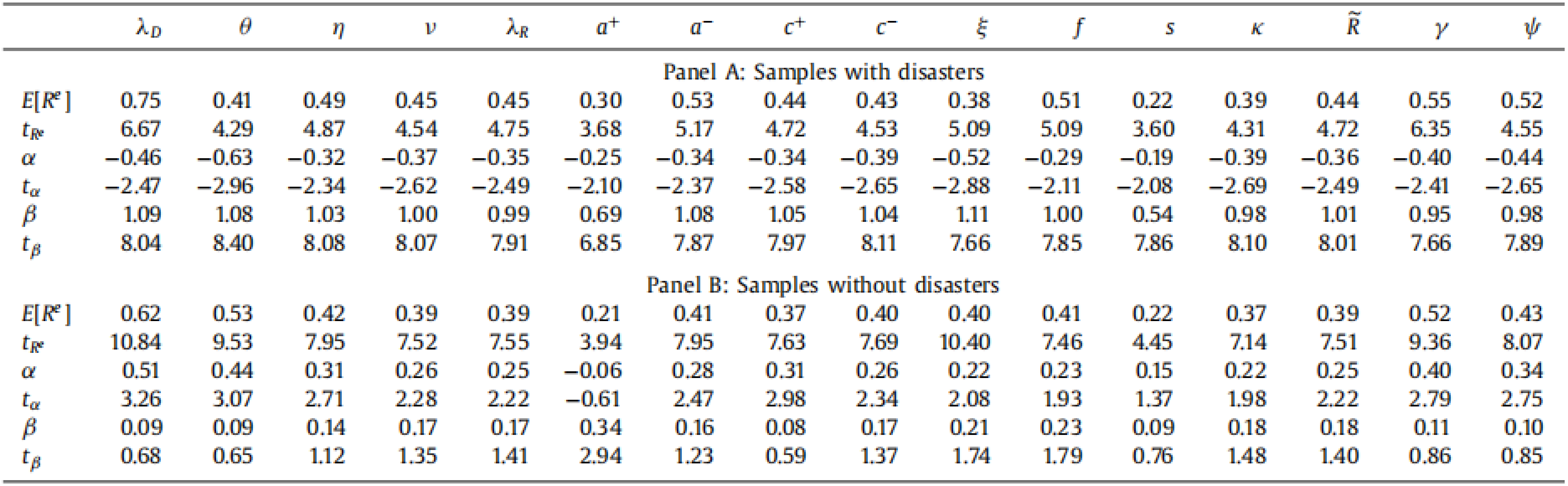
（5）Explaining the poor performance of the consumption CAPM

②Explaining the failure of the consumption CAPM



（6）Comparative statics

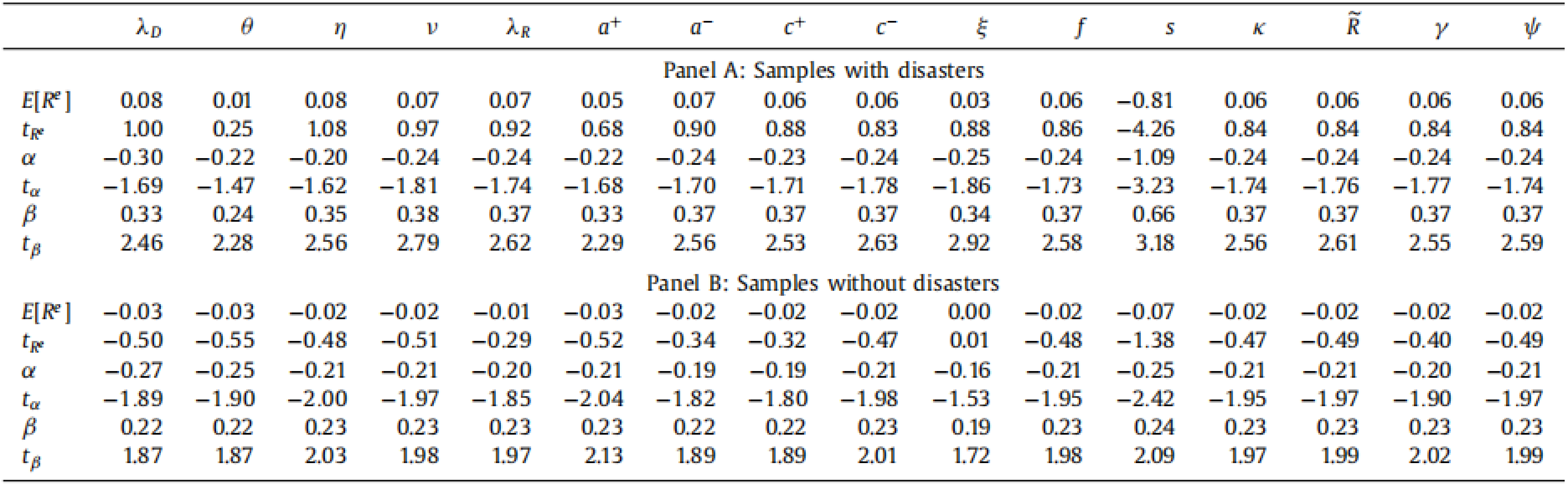
①the CAPM regressions of the book-to-market deciles.



	λ_D	θ	η	ν	λ_R	a^+	a^-	c^+	c^-	ξ	f	s	κ	$\widetilde{R}$	γ	ψ
Panel A: Samples with disasters																
$E[R^e]$	0.75	0.41	0.49	0.45	0.45	0.30	0.53	0.44	0.43	0.38	0.51	0.22	0.39	0.44	0.55	0.52
t_{R^e}	6.67	4.29	4.87	4.54	4.75	3.68	5.17	4.72	4.53	5.09	5.09	3.60	4.31	4.72	6.35	4.55
α	−0.46	−0.63	−0.32	−0.37	−0.35	−0.25	−0.34	−0.34	−0.39	−0.52	−0.29	−0.19	−0.39	−0.36	−0.40	−0.44
t_α	−2.47	−2.96	−2.34	−2.62	−2.49	−2.10	−2.37	−2.58	−2.65	−2.88	−2.11	−2.08	−2.69	−2.49	−2.41	−2.65
β	1.09	1.08	1.03	1.00	0.99	0.69	1.08	1.05	1.04	1.11	1.00	0.54	0.98	1.01	0.95	0.98
t_β	8.04	8.40	8.08	8.07	7.91	6.85	7.87	7.97	8.11	7.66	7.85	7.86	8.10	8.01	7.66	7.89
Panel B: Samples without disasters																
$E[R^e]$	0.62	0.53	0.42	0.39	0.39	0.21	0.41	0.37	0.40	0.40	0.41	0.22	0.37	0.39	0.52	0.43
t_{R^e}	10.84	9.53	7.95	7.52	7.55	3.94	7.95	7.63	7.69	10.40	7.46	4.45	7.14	7.51	9.36	8.07
α	0.51	0.44	0.31	0.26	0.25	−0.06	0.28	0.31	0.26	0.22	0.23	0.15	0.22	0.25	0.40	0.34
t_α	3.26	3.07	2.71	2.28	2.22	−0.61	2.47	2.98	2.34	2.08	1.93	1.37	1.98	2.22	2.79	2.75
β	0.09	0.09	0.14	0.17	0.17	0.34	0.16	0.08	0.17	0.21	0.23	0.09	0.18	0.18	0.11	0.10
t_β	0.68	0.65	1.12	1.35	1.41	2.94	1.23	0.59	1.37	1.74	1.79	0.76	1.48	1.40	0.86	0.85

（6）Comparative statics

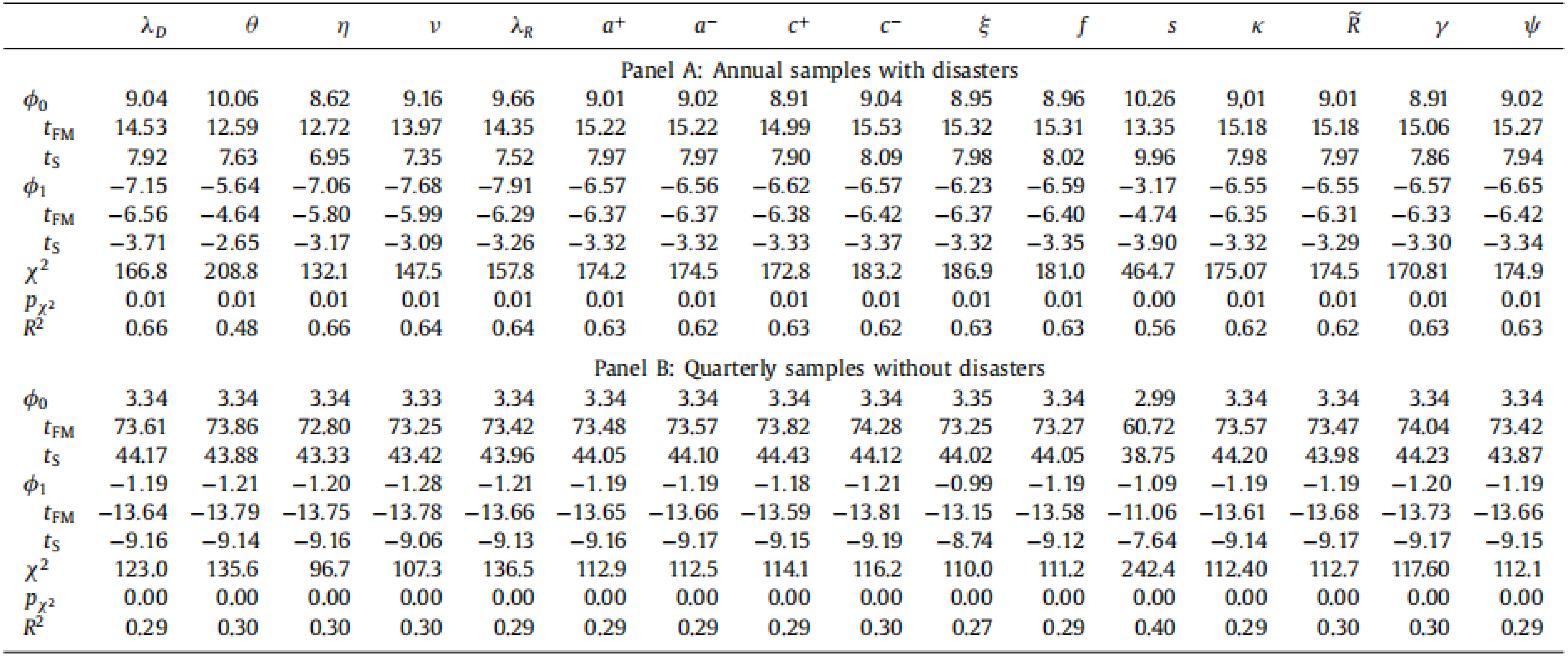
②the CAPM regressions of the preranking market beta deciles.



	λ_D	θ	η	ν	λ_R	a^+	a^-	c^+	c^-	ξ	f	s	κ	$\widetilde{R}$	γ	ψ
								Panel A: Samples with disasters								
$E[R^e]$	0.08	0.01	0.08	0.07	0.07	0.05	0.07	0.06	0.06	0.03	0.06	−0.81	0.06	0.06	0.06	0.06
t_{R^e}	1.00	0.25	1.08	0.97	0.92	0.68	0.90	0.88	0.83	0.88	0.86	−4.26	0.84	0.84	0.84	0.84
α	−0.30	−0.22	−0.20	−0.24	−0.24	−0.22	−0.24	−0.23	−0.24	−0.25	−0.24	−1.09	−0.24	−0.24	−0.24	−0.24
t_α	−1.69	−1.47	−1.62	−1.81	−1.74	−1.68	−1.70	−1.71	−1.78	−1.86	−1.73	−3.23	−1.74	−1.76	−1.77	−1.74
β	0.33	0.24	0.35	0.38	0.37	0.33	0.37	0.37	0.37	0.34	0.37	0.66	0.37	0.37	0.37	0.37
t_β	2.46	2.28	2.56	2.79	2.62	2.29	2.56	2.53	2.63	2.92	2.58	3.18	2.56	2.61	2.55	2.59
								Panel B: Samples without disasters								
$E[R^e]$	−0.03	−0.03	−0.02	−0.02	−0.01	−0.03	−0.02	−0.02	−0.02	0.00	−0.02	−0.07	−0.02	−0.02	−0.02	−0.02
t_{R^e}	−0.50	−0.55	−0.48	−0.51	−0.29	−0.52	−0.34	−0.32	−0.47	0.01	−0.48	−1.38	−0.47	−0.49	−0.40	−0.49
α	−0.27	−0.25	−0.21	−0.21	−0.20	−0.21	−0.19	−0.19	−0.21	−0.16	−0.21	−0.25	−0.21	−0.21	−0.20	−0.21
t_α	−1.89	−1.90	−2.00	−1.97	−1.85	−2.04	−1.82	−1.80	−1.98	−1.53	−1.95	−2.42	−1.95	−1.97	−1.90	−1.97
β	0.22	0.22	0.23	0.23	0.23	0.23	0.22	0.22	0.23	0.19	0.23	0.24	0.23	0.23	0.23	0.23
t_β	1.87	1.87	2.03	1.98	1.97	2.13	1.89	1.89	2.01	1.72	1.98	2.09	1.97	1.99	2.02	1.99

（6）Comparative statics

③the consumption CAPM test on the 25 size and book-to-market portfolios



	λ_D	θ	η	ν	λ_R	a^+	a^-	c^+	c^-	ξ	f	s	κ	$\tilde{R}$	γ	ψ
Panel A: Annual samples with disasters																
ϕ_0	9.04	10.06	8.62	9.16	9.66	9.01	9.02	8.91	9.04	8.95	8.96	10.26	9,01	9.01	8.91	9.02
t_{FM}	14.53	12.59	12.72	13.97	14.35	15.22	15.22	14.99	15.53	15.32	15.31	13.35	15.18	15.18	15.06	15.27
t_S	7.92	7.63	6.95	7.35	7.52	7.97	7.97	7.90	8.09	7.98	8.02	9.96	7.98	7.97	7.86	7.94
ϕ_1	−7.15	−5.64	−7.06	−7.68	−7.91	−6.57	−6.56	−6.62	−6.57	−6.23	−6.59	−3.17	−6.55	−6.55	−6.57	−6.65
t_{FM}	−6.56	−4.64	−5.80	−5.99	−6.29	−6.37	−6.37	−6.38	−6.42	−6.37	−6.40	−4.74	−6.35	−6.31	−6.33	−6.42
t_S	−3.71	−2.65	−3.17	−3.09	−3.26	−3.32	−3.32	−3.33	−3.37	−3.32	−3.35	−3.90	−3.32	−3.29	−3.30	−3.34
χ^2	166.8	208.8	132.1	147.5	157.8	174.2	174.5	172.8	183.2	186.9	181.0	464.7	175.07	174.5	170.81	174.9
p_{χ^2}	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.01
R^2	0.66	0.48	0.66	0.64	0.64	0.63	0.62	0.63	0.62	0.63	0.63	0.56	0.62	0.62	0.63	0.63
Panel B: Quarterly samples without disasters																
ϕ_0	3.34	3.34	3.34	3.33	3.34	3.34	3.34	3.34	3.34	3.35	3.34	2.99	3.34	3.34	3.34	3.34
t_{FM}	73.61	73.86	72.80	73.25	73.42	73.48	73.57	73.82	74.28	73.25	73.27	60.72	73.57	73.47	74.04	73.42
t_S	44.17	43.88	43.33	43.42	43.96	44.05	44.10	44.43	44.12	44.02	44.05	38.75	44.20	43.98	44.23	43.87
ϕ_1	−1.19	−1.21	−1.20	−1.28	−1.21	−1.19	−1.19	−1.18	−1.21	−0.99	−1.19	−1.09	−1.19	−1.19	−1.20	−1.19
t_{FM}	−13.64	−13.79	−13.75	−13.78	−13.66	−13.65	−13.66	−13.59	−13.81	−13.15	−13.58	−11.06	−13.61	−13.68	−13.73	−13.66
t_S	−9.16	−9.14	−9.16	−9.06	−9.13	−9.16	−9.17	−9.15	−9.19	−8.74	−9.12	−7.64	−9.14	−9.17	−9.17	−9.15
χ^2	123.0	135.6	96.7	107.3	136.5	112.9	112.5	114.1	116.2	110.0	111.2	242.4	112.40	112.7	117.60	112.1
p_{χ^2}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
R^2	0.29	0.30	0.30	0.30	0.29	0.29	0.29	0.29	0.30	0.27	0.29	0.40	0.29	0.30	0.30	0.29

① Rare disasters help explain the value premium puzzle --------value stocks earn higher average returns than growth stocks, despite their similar market betas.

② the model also explains the beta anomaly---------due to severe beta measurement errors, the relation between the preranking market beta and the average return is flat in the model’s simulations

③ A fundamental innovation of our work relative to prior theoretical models is general equilibrium in which consumption and the pricing kernel are endogenous.

In addition， the widely documented empirical failures of standard asset pricing models might have more to do with the deficiencies of standard empirical tests rather than deficiencies of economic theory

THANKS
Q&A