

**Wednesday Seminar in Shanxi University**

# **Safety Transformation and the Structure of the Financial System**

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## Academic Employment

- Assistant Professor of Finance, Wharton School, University of Pennsylvania 2018-
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## Education

- Ph.D., Business Economics, Harvard University, 2018
- B.A., Economics, Mathematics, Yale University, 2011

## Publications

- “Latent Indices in Assortative Matching Models” (with Nikhil Agarwal) *Quantitative Economics*, November 2017, Winner of “Best Paper Prize” in *Quantitative Economics for 2017*
- “Safety Transformation and the Structure of the Financial System” *Journal of Finance*, December 2020

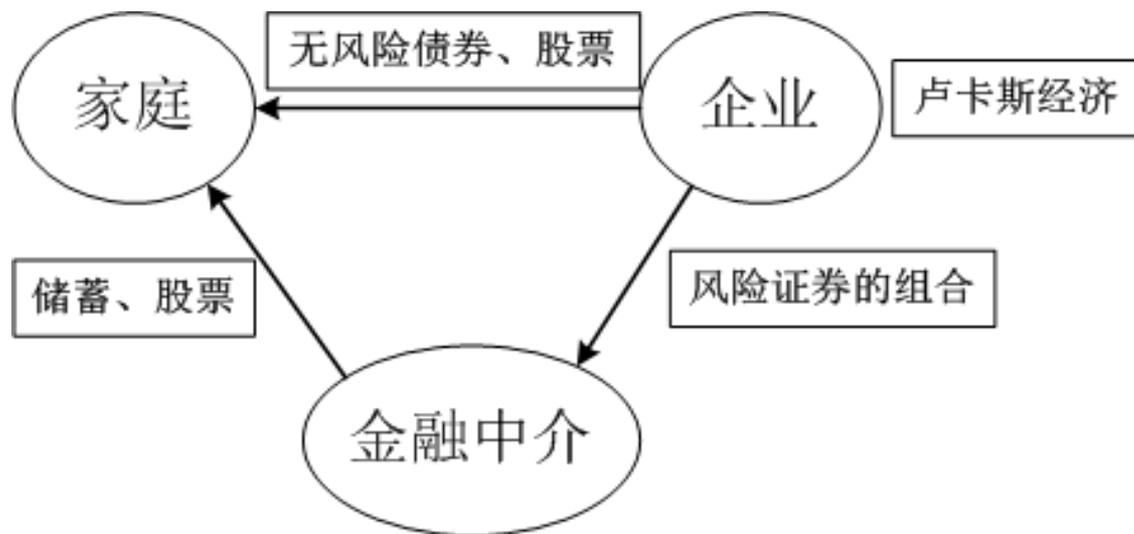
## Working Papers

- “Risk-Free Interest Rates” (with Jules van Binsbergen and Marco Grotteria) (accepted, *Journal of Financial Economics*)
- “Credit Cycles with Market-Based Household Leverage” (with Tim Landvoigt) (revise and resubmit, *Journal of Financial Economics*)



## ➤ Preliminary

- 金融系统提供金融产品，如股票、债券、储蓄等，实现保值和增值。



- 当金融系统受到宏观冲击时，家庭对安全资产的需求增加，需要金融中介发行更多的储蓄，企业发行更多的无风险债券，金融中介持有更多的风险债券组合，因此增加了企业杠杆。量化宽松能降低金融中介的规模和风险，以及企业的杠杆。



## ➤ A1. Agents

- 两期经济（现在和未来），未来有“好”和“坏”两个状态。假定卢卡斯树产出的一部分被企业管理者所攫取。

There are two classes of agents: households and firm managers. Households have expected utility

$$u(c_1) + E[u(c_2) - T] + v(d), \quad (1)$$

which depends on consumption  $(c_1, c_2)$  at times 1 and 2, on a transfer  $T$  of utility paid to firm managers at time 2, and directly on the holding  $d$  of riskless assets that pay out at time 2. The functions  $u$  and  $v$  are strictly increasing, strictly concave, twice continuously differentiable, and satisfy Inada conditions. Managers have expected utility equal to the expected transfer  $E(T)$  they receive at time 2.



## ➤ A2. Agency problem

- 金融中介和企业的管理者都存在代理问题，管理者能够攫取企业产出的 $C(x)$ ，其中 $x$ 是企业的产出，最终只有 $P(x)=x-C(x)$ 会付给投资者。 $C(x)$ 是一个单增凸函数。

Firm managers face an agency problem. For a firm that owns assets with a payoff of  $x$  at time 2, an increasing and (weakly) convex portion  $C(x)$  of the output is nonpledgeable and can be seized by firm managers, where  $C(0) = 0$ ,  $1 > C' > 0$ , and  $C'' \geq 0$ . As a result, only the payoff  $P(x) = x - C(x)$  can be paid to outside investors. For a nonfinancial firm, the payoff seized by management is determined only by the payoff  $\delta_i$  of the Lucas tree it owns, and the firm's pledgeable cash flows are  $\delta_i^* = P(\delta_i)$ . Because all trees must be owned by nonfinancial firms, and each tree  $i$  owned by a different firm, the payoffs seized by managers of nonfinancial firms are effectively fixed. For the intermediary, the cash flows seized by management depend on the payoff  $\delta_I$  of its (endogenously chosen) asset portfolio, yielding pledgeable cash flows of  $\delta_I^* = P(\delta_I)$ . If the payoff  $\delta_I$  of the intermediary's portfolio increases, more output  $C(\delta_I)$  can be seized by its manager.



## ➤ A3. Utility transfers

- 企业管理者会把攫取到的产出以市场价格卖给家庭。管理者不会直接从攫取的消费品得到效用，这样能够保证家庭消费了经济体的所有产出。

After firm managers seize output, they sell it to households at a market price in exchange for direct transfers of utility. If households consume  $c_2$  at time 2 and managers have  $c_{\text{seized}}$  worth of seized consumption goods, households are willing to make a transfer  $T = u'(c_2)c_{\text{seized}}$  to recover the seized consumption goods. Because managers get no utility from consuming seized goods (but do value utility transfers from households), all seized consumption goods are sold back to households. This market for utility transfers (instead of having managers consume seized goods) keeps the tractability of an endowment economy with a representative household because households consume all output. However, households are harmed by managers seizing output because they must transfer utility to buy the output back.



## ➤ A4. Securities Issuance and Portfolios

our notation. Let  $f_i$  and  $F_i$ , respectively, be the face value of senior and junior debt issued by the firm owning tree  $i$ , which I call firm  $i$ . The payoffs of the senior and junior tranches of firm  $i$ 's debt are  $d_i = \min(\delta_i^*, f_i)$  and  $D_i = \min(\delta_i^* - \min(\delta_i^*, f_i), F_i)$ , respectively. The payoff of firm  $i$ 's equity is  $E_i = \max(\delta_i^* - f_i - F_i, 0)$ . Let  $f_I$  be the face value of the intermediary's debt, so  $D_I = \min(\delta_I^*, f_I)$  and  $E_I = \max(\delta_I^* - f_I, 0)$  are the payoffs of its debt and equity, respectively.

Let  $q_I(d_i)$ ,  $q_I(D_i)$ , and  $q_I(E_i)$  be the fraction of firm  $i$ 's senior debt, junior debt, and equity held by the intermediary, with the remainder  $q_H(\cdot) = 1 - q_I(\cdot)$  held by the household.<sup>8</sup> The payoff  $\delta_I$  of the intermediary's portfolio is

$$\begin{aligned} \delta_I = & \int_0^1 \min(\delta_i^*, f_i) q_I(d_i) di + \int_0^1 \min(\delta_i^* - \min(\delta_i^*, f_i), F_i) q_I(D_i) di \\ & + \int_0^1 \max(\delta_i^* - f_i - F_i, 0) q_I(E_i) di. \end{aligned} \quad (2)$$



## ➤ A5. Regularity Conditions

I impose two regularity conditions on this environment. The first condition implies that a firm's debt is less exposed to systematic risk than its equity, which is the key reason why debt is held by financial intermediaries and equity is held by households in the model.

Condition 1: For each  $i \in [0, 1]$ , there is a constant  $\bar{\delta}_i \geq 0$  such that (i)  $Pr(\delta_i > \bar{\delta}_i | bad) = Pr(\delta_i > \bar{\delta}_i | good) = 1$  and (ii)  $\frac{Pr(\delta_i > u | good)}{Pr(\delta_i > u | bad)}$  is continuously differentiable with respect to  $u$  on  $[\bar{\delta}_i, \infty)$ , with the derivative strictly positive on  $(\bar{\delta}_i, \infty)$  and  $\lim_{u \rightarrow \infty} \frac{Pr(\delta_i > u | good)}{Pr(\delta_i > u | bad)} = \infty$ .



## ➤ A5. Regularity Conditions (cont.)

The second condition ensures that the benefit of the first unit of riskless assets issued by intermediaries exceeds its cost, so that financial intermediaries actually exist.

Condition 2: If the total supply of riskless assets is only the quantity  $\mu$  created by nonfinancial firms, the marginal cost  $E[u'(\int_0^1 \delta_i di)C'(0)]$  of the intermediary increasing the supply of riskless assets is strictly less than the marginal benefit  $v'(\mu)$ .



## ➤ B1. Statement of Social Planner's Problem

Given these portfolio and issuance decisions, the payoff  $\delta_I(q_I(\cdot), f_i, F_i)$  of the intermediary's portfolio is given by equation (2). The amount of riskless debt the intermediary can issue is  $\bar{\delta}_I^*(q_I(\cdot), f_i, F_i) = P[\min(\delta_I^{\text{bad}}(q_I(\cdot), f_i, F_i), \delta_I^{\text{good}}(q_I(\cdot), f_i, F_i))]$ , where  $(\delta_I^{\text{bad}}, \delta_I^{\text{good}})$  are the realizations of  $\delta_I$  in the bad and good aggregate states. This yields a total quantity of riskless debt held by the household of

$$d(q_I(\cdot), f_i, F_i, f_I) = f_I \mathbb{1}\{f_I \leq \bar{\delta}_I^*(q_I(\cdot), f_i, F_i)\} + \int_0^1 (1 - q_I(d_i)) f_i \mathbb{1}\{f_i \leq \bar{\delta}_i^*\} di. \quad (3)$$

To see this, note that the indicator function  $\mathbb{1}\{f_i \leq \bar{\delta}_i^*\}$  is defined to equal 1 if  $f_i \leq \bar{\delta}_i^*$  and 0 otherwise, which means that the senior tranche of firm  $i$ 's debt has a face value no greater than the (exogenous) worst realization  $\bar{\delta}_i^*$  of its pledgeable cash flows. This is equivalent to the debt being riskless. Similarly,  $\mathbb{1}\{f_I \leq \bar{\delta}_I^*\}$  is equivalent to the intermediary's debt being riskless.<sup>11</sup>



## ➤ B1. Statement of Social Planner's Problem (cont.)

Given the payoff  $\delta_I(q_I(\cdot), f_i, F_i)$  of the intermediary's portfolio,  $C(\delta_I(q_I(\cdot), f_i, F_i))$  is seized by the intermediary's manager. In addition,  $C(\delta_i)$  is seized by the manager of nonfinancial firm  $i$ . To repurchase all of these seized resources requires a transfer of utility from the household equal to

$$T(q_I(\cdot), f_i, F_i, c_2) = u'(c_2) \left[ \int_0^1 C(\delta_i) di + C(\delta_I(q_I(\cdot), f_i, F_i)) \right]. \quad (4)$$

The planner's problem can be now be written as

$$\max_{q_I(\cdot), f_i, F_i, c_1, c_2} u(c_1) + E[u(c_2) - T(q_I(\cdot), f_i, F_i, c_2)] + v(d(q_I(\cdot), f_i, F_i, f_I)). \quad (5)$$

subject to  $c_1 \leq Y_1$ ,  $c_2 \leq \int_0^1 \delta_i di$ , and  $0 \leq q_I(\cdot) \leq 1$ .

The first two constraints are resource constraints requiring that consumption is no greater than total output, which is  $Y_1$  at time 1 and the sum  $\int_0^1 \delta_i di$  of all Lucas tree payoffs at time 2. The third constraint requires that each investor owns a nonnegative share of every financial asset.



## ➤ B2. A Simplified Planner's Problem

$$\begin{aligned}
 & E \left[ \overbrace{-u' \left( \int_0^1 \delta_i di \right) \left( C \left( \int_0^1 \min(\delta_i^* - \bar{\delta}_i^*, F_i) q_I(D_i) di + \int_0^1 \max(\delta_i^* - \bar{\delta}_i^* - F_i, 0) q_I(E_i) di \right) \right)}^{\text{utility transferred to intermediary manager}} \right] \\
 & + v \left( \overbrace{P \left( \int_0^1 E(\min(\delta_i^* - \bar{\delta}_i^*, F_i) | bad) q_I(D_i) di + \int_0^1 E(\max(\delta_i^* - \bar{\delta}_i^* - F_i, 0) | bad) q_I(E_i) di + \int_0^1 \bar{\delta}_i^* di \right)}^{\text{utility of holding safe assets}} \right) \quad (6)
 \end{aligned}$$

subject to  $0 \leq q_I(\cdot) \leq 1$ .

If we write the planner's objective function (expression (6)) in terms of the payoff  $\delta_I$  of the intermediary's portfolio rather than the planner's exogenous choice variables, we obtain

$$\begin{aligned}
 & E \left[ -u' \left( \int_0^1 \delta_i di \right) C(\delta_I) \right] + v \left( P(\delta_I^{\text{bad}}) + \int_0^1 \bar{\delta}_i^* di \right) = -\frac{1}{2} u' \left( \int_0^1 E(\delta_i | good) di \right) C(\delta_I^{\text{good}}) \\
 & - \frac{1}{2} u' \left( \int_0^1 E(\delta_i | bad) di \right) C(\delta_I^{\text{bad}}) + v \left( P(\delta_I^{\text{bad}}) + \int_0^1 \bar{\delta}_i^* di \right). \quad (7)
 \end{aligned}$$



## ➤ B2. A Simplified Planner's Problem (cont.)

If the planner wants the intermediary to create a given quantity of safe assets, it faces the lowest cost of managerial diversion by giving the nonfinancial firms' risky debt to the intermediary, all riskless assets and all equity to households, and choosing the capital structure of nonfinancial firms appropriately. The following proposition illustrates this result, which is depicted in Figure 2.

**PROPOSITION 1:** *The social planner's optimal allocation satisfies the following conditions. (1) All riskless assets are held by households ( $q_I(d_i) = 0$ ). (2) All risky debt securities are held by the financial intermediary ( $q_I(D_i) = 1$ ). (3) All equity securities are held by the household ( $q_I(E_i) = 0$ ). (4) Each nonfinancial firm issues as much riskless debt as it possibly can. It also issues an additional risky debt security as well as an equity security.*



## ➤ B3. Optimal Nonfinancial Sector Leverage

Proposition 1 characterizes the intermediary's optimal portfolio  $q_I$ , leaving only the face value  $F_i$  of risky debt issued by each nonfinancial firm for the planner to choose. The intermediary's portfolio consisting only of risky debt pays  $\delta_I = \int_0^1 \min(\delta_i^* - \bar{\delta}_i^*, F_i) di$ . Plugging this into the planner's objective function (equation (6)) yields

$$\begin{aligned} \max_{F_i} E \left[ -u' \left( \int_0^1 \delta_i di \right) C \left( \int_0^1 \min[\delta_i^* - \bar{\delta}_i^*, F_i] di \right) \right] + \\ v \left( \int_0^1 \bar{\delta}_i^* di + P \left( \int_0^1 E \{ \min[\delta_i^* - \bar{\delta}_i^*, F_i] | bad \} di \right) \right). \end{aligned} \quad (8)$$



## ➤ B3. Optimal Nonfinancial Sector Leverage (cont.)

The first-order condition for firm  $i$ 's optimal capital structure is

$$E \left[ \overbrace{u' \left( \int_0^1 \delta_i di \right) C' \left( \int_0^1 \min[\delta_i^* - \bar{\delta}_i^*, F_i] di \right) \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}}^{\text{agency cost of increasing firm } i\text{'s debt}} \right] = \quad (9)$$

$$\overbrace{v' \left( \mu + P \left( \int_0^1 E\{\min[\delta_i^* - \bar{\delta}_i^*, F_i] | bad\} di \right) \right) P' \left( \int_0^1 E\{\min[\delta_i^* - \bar{\delta}_i^*, F_i] | bad\} di \right) * \Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i\} | bad)}^{\text{utility benefit of additional safe assets backed by firm } i\text{'s debt}}.$$



## ➤ B3. Optimal Nonfinancial Sector Leverage (cont.)

After breaking the expectation in equation (9) into good-state and bad-state payoffs (each weighted by their probability  $\frac{1}{2}$  of occurring), the expression can be rearranged to yield

$$\frac{\Pr\{\delta_i^* - \bar{\delta}_i^* \geq F_i | good\}}{\Pr\{\delta_i^* - \bar{\delta}_i^* \geq F_i | bad\}} = \frac{-u'(c_2^{bad})C'(\int_0^1 E\{\min[\delta_i^* - \bar{\delta}_i^*, F_i] | bad\}di))}{u'(c_2^{good})C'(\int_0^1 E\{\min[\delta_i^* - \bar{\delta}_i^*, F_i] | good\}di))} +$$

$$\frac{2v'(\mu + P(\int_0^1 E\{\min[\delta_i^* - \bar{\delta}_i^*, F_i] | bad\}di))P'(\int_0^1 E\{\min[\delta_i^* - \bar{\delta}_i^*, F_i] | bad\}di)}{u'(c_2^{good})C'(\int_0^1 E\{\min[\delta_i^* - \bar{\delta}_i^*, F_i] | good\}di)}. \quad (10)$$



## ➤ B4. Discussion of Social Planner's Problem

- Three features of the solution to the planner's problem are particularly relevant.
- First, a financial intermediary exists to produce safe assets that nonfinancial firms cannot produce on their own. This is because nonfinancial firms face idiosyncratic risk, which the intermediary diversifies away by holding a pool of securities issued by all nonfinancial firms. If nonfinancial firms faced no idiosyncratic risk, intermediation would not be necessary.
- Second, the intermediary holds a diversified portfolio of all risky debt securities because this is the least costly way to create riskless assets. The agency cost of diversion by the intermediary's manager increases with the size of the intermediary's portfolio.



## ➤ B4. Discussion of Social Planner's Problem (cont.)

- Third, the nonfinancial sector's leverage is indirectly determined by the demand for safe assets. Risky debt securities in the model are intermediate inputs for the intermediary to create riskless deposits. Because the intermediary holds all risky debt, expanding the intermediary's balance sheet requires an increase in nonfinancial firms' leverage.



## ➤ C. Decentralized Market Equilibrium

- A competitive equilibrium in this economy, where households maximize their expected utility and firms maximize their profits, yields the same allocation of resources as the planner's problem.
- Riskless assets are held by the household, and the risk-free rate is pushed down by the household's demand for safe assets.
- Low-risk assets are held by the intermediary, which uses them to back the issuance of riskless assets, and the intermediary's agency problem makes it endogenously risk averse.
- High-risk assets are held by the household, and the price of risk for these assets is lower than for the assets held by the intermediary, resulting in segmented asset markets.



## ➤ C. Decentralized Market Equilibrium (cont.)

- Nonfinancial firms exploit this segmentation when choosing what securities to issue, and each nonfinancial firm has a unique optimal capital structure.
- Nonfinancial firms optimally issue a riskless senior debt security (if they can) that is held by the household, a low-risk junior debt security that is held by the intermediary, and a high-risk equity security that is held by the household.



## ➤ C1. Setup

All financial securities trade at competitive market prices and can be bought by households or financial intermediaries subject to a no-short-sales constraint.<sup>13</sup> The risk-free rate is  $i_d$ , and a security paying cash flows  $x_s$  trades at a price  $p_s$ . At time 2, there is a competitive price  $p_{\text{transfer}}$  of a utility transfer, stating how much consumption can be purchased by transferring one unit of utility to managers.

- Households maximize their expected utility by investing at these competitive market prices. Firms maximize their profits, taking as given how the market prices the securities they issue and the fact that managers seize all nonpledgeable cash flows that they generate. Financial intermediaries choose both the assets that they purchase and the liabilities that they issue to maximize their profits. Nonfinancial firms own Lucas trees whose cash flows are exogenous, so they only choose which securities to issue.



## ➤ C2. Household's Problem

The household is endowed with wealth  $W_H$  and chooses a quantity  $q_H(s)$  of each risky asset  $s$ , a quantity  $d$  of riskless assets, consumption  $c_1$ , and utility transfer  $T$  to solve

$$\max_{q_H(\cdot), d, c_1, T} u(c_1) + E \left[ u \left( \int q_H(s) x_s ds + d + p_{\text{transfer}} T \right) - T \right] + v(d)$$

$$\text{subject to } c_1 + \frac{d}{1+i_d} + \int q_H(s) p_s ds = W_H \text{ (budget constraint)}$$

$$q_H(\cdot) \geq 0 \text{ (short-sale constraint).} \tag{11}$$



## ➤ C2. Household's Problem (cont.)

If the household puts  $q_H(s)$  in each risky asset  $s$ , its total risky asset portfolio pays  $\int q_H(s)x_s ds$  and sells for a price of  $\int q_H(s)p_s ds$ . In addition, the transfer  $T$  at time 2 buys  $p_{\text{transfer}}T$  of consumption goods.

The first-order conditions for the quantity of riskless assets  $d^{14}$ , for the quantity  $q_H(s)$  of risky asset  $s$ , and for the amount of utility  $T$  to transfer in exchange for consumption are

$$u'(c_1) = (1 + i_d)(E[u'(c_2)] + v'(d)), \quad (12)$$

$$p_s \geq E\left[\frac{u'(c_2)}{u'(c_1)}x_s\right], \quad (13)$$

$$u'(c_2)p_{\text{transfer}} = 1, \quad (14)$$

where inequality (13) must be an equality for any risky asset held in positive quantity by the household.



### ➤ C3. Intermediary's Problem

One risky security that must be held by the household in equilibrium is the equity of the financial intermediary. As a result, inequality (13) must be an equality for this security. If the intermediary pays a dividend of  $E_I$  to equity-holders at time 2 and raises equity  $e_I$  at time 1, then its market value at time 1 is (net of equity issuance)

$$E\left(\frac{u'(c_2)}{u'(c_1)}E_I\right) - e_I. \quad (15)$$

The intermediary maximizes this market value of its equity by choosing to buy a quantity  $q_I(s)$  of each risky asset  $s$ , to buy a quantity  $d_I$  of riskless securities, and to issue quantities  $e_I$  of risky equity and  $D_I$  of riskless deposits. The intermediary's problem can be written as

$$\begin{aligned} \max_{q_I(\cdot), d_I, e_I, D_I} & E\left[\frac{u'(c_2)}{u'(c_1)}\left\{P\left(\int q_I(s)x_s ds + d_I\right) - D_I\right\}\right] - e_I \\ \text{subject to} & \int q_I(s)p_s ds + \frac{d_I}{1+i_d} = D_I\frac{1}{1+i_d} + e_I \text{ (budget constraint),} \\ & D_I \leq P\left(\int q_I(s)E(x_s|bad) ds + d_I\right) \text{ (deposit issuance constraint),} \\ & q_I(\cdot) \geq 0 \text{ (short-sale constraint).} \end{aligned} \quad (16)$$

### ➤ C3. Intermediary's Problem (cont.)

The intermediary's budget constraint states that the sum of the equity  $e_I$  issued and the proceeds  $D_I \frac{1}{1+i_d}$  from issuing riskless deposits must equal the price  $\int q_I(s)p_s ds$  of the intermediary's risky asset portfolio plus the price  $\frac{a_I}{1+i_d}$  of the riskless assets it buys. Of the payoff  $\int q_I(s)x_s ds + d_I$  of the intermediary's portfolio, only  $P(\int q_I(s)x_s ds + d_I)$  remains after managers seize the nonpledgeable output, so this is what remains to sell to outside investors. The realization of  $P(\int q_I(s)E(x_s|bad)ds + d_I)$  in the bad aggregate state is the largest riskless payoff the intermediary can promise to outside investors, so this is the amount of riskless deposits  $D_I$  the intermediary is able to issue.



## ➤ C4. Nonfinancial Firm's Problem

of face value  $F_i$ , and an equity security.<sup>15</sup> The firm chooses the face values of its debt securities to maximize the total market value of the securities it issues. The price of each security is the maximum of what the household and intermediary are willing to pay for it. If  $p_H(x)$  and  $p_I(x)$  are, respectively, the household's and the intermediary's willingness to pay for a cash flow  $x$ , and  $p_{\max}[x] = \max(p_H(x), p_I(x))$ , the firm's problem can be written as

$$\begin{aligned} \max_{f_i, F_i} \{ & p_{\max} [\min(\delta_i^*, f_i)] + p_{\max} [\min(\delta_i^* - \min(\delta_i^*, f_i), F_i)] \\ & + p_{\max} [\max(\delta_i^* - f_i - F_i, 0)] \}. \end{aligned} \quad (17)$$

The payoffs  $d_i = \min(\delta_i^*, f_i)$  of the firm's senior debt,  $D_i = \min(\delta_i^* - \min(\delta_i^*, f_i), F_i)$  of its junior debt, and  $E_i = \max(\delta_i^* - f_i - F_i, 0)$  of its equity satisfy  $d_i + D_i + E_i = \delta_i^*$ . The total cash flow paid out by the firm is independent of the securities it issues. In a frictionless asset market, where a single pricing kernel prices all assets,  $p_H(\cdot)$  and  $p_I(\cdot)$  would both be equal to some linear function  $p(\cdot)$ . This would imply that  $p(d_i) + p(D_i) + p(E_i) = p(d_i + D_i + E_i) = p(\delta_i^*)$ , so the firm's value would be the same regardless of which securities it issues.



## ➤ C5. Asset Prices and Portfolio Choices

To see how the intermediary prices assets, note that it chooses to issue as many riskless securities as it can and to not buy any riskless securities, as shown in the Appendix. This is because the household's utility  $v(d)$  from holding riskless assets means that it is willing to invest at a lower risk-free rate, which makes it attractive to borrow at the rate and unattractive to lend at it. Taking this as given, and using equation (12) to determine the risk-free rate, the intermediary's problem reduces to

$$\begin{aligned} \max_{q_I(\cdot) \geq 0} E & \left[ \frac{u'(c_2)}{u'(c_1)} \left\{ \int q_I(s) x_s ds - C \left( \int q_I(s) x_s ds \right) \right\} \right] \\ & + P \left( \int q_I(s) E(x_s | bad) ds \right) \frac{v'(d)}{u'(c_1)} - \int q_I(s) p_s ds. \end{aligned} \quad (18)$$



## ➤ C5. Asset Prices and Portfolio Choices (cont.)

The intermediary's first-order condition for buying risky asset  $s$  is thus

$$\begin{aligned}
 & \overbrace{E\left(\frac{u'(c_2)}{u'(c_1)}x_s\right)}^{\text{household's willingness to pay}} \quad - \quad \overbrace{E\left(\frac{u'(c_2)}{u'(c_1)}C'\left(\int q_I(s)x_s ds\right)x_s\right)}^{\text{agency cost of buying asset}} \\
 & + \overbrace{P'\left(\int q_I(s)E(x_s|bad)ds\right)E(x_s|bad)}^{\text{additional riskless payoff backed by asset}} \quad \overbrace{\frac{(v'(d))}{u'(c_1)}}^{\text{safety premium}} \leq p_s, \quad (19)
 \end{aligned}$$

with equality if the intermediary holds a positive quantity of the asset. The intermediary's willingness to pay for an asset differs from that of the household for two reasons. First, a portion  $C(\int q_I(s)x_s ds)$  of the intermediary's portfolio is seized by its manager, and this agency cost grows when the intermediary buys more assets. Second, as part of the intermediary's diversified portfolio, an asset  $x_s$  increases the amount of riskless securities that the intermediary can issue by  $P'(\int q_I(s)E(x_s|bad)ds)E(x_s|bad)$ . Because the risk-free rate is low



## ➤ C5. Asset Prices and Portfolio Choices (cont.)

(due to the household's demand for safe assets), the intermediary benefits from issuing more riskless securities. The intermediary buys an asset if the agency cost of holding the asset is less than the benefit of the extra riskless securities the asset allows the intermediary to issue. As shown in the Appendix, the intermediary buys all assets for which

$$\frac{2P'(\int q_I(s)E(x_s|bad)ds)v'(d) - u'(c_2^{bad})C'(\int q_I(s)E(x_s|bad)ds)}{u'(c_2^{good})C'(\int q_I(s)E(x_s|good)ds)} \geq \frac{E(x_s|good)}{E(x_s|bad)}. \quad (20)$$



## ➤ C5. Asset Prices and Portfolio Choices (cont.)

### PROPOSITION 2:

(1) All riskless assets are bought by the household. The risk-free rate  $i_d$  is given by

$$u'(c_1) = (1 + i_d)[Eu'(c_2) + v'(d)]. \quad (21)$$

(2) For some cutoff value  $\tau$ , all risky assets whose payoffs  $x_s$  have sufficiently low systematic risk ( $\frac{E(x_s|good)}{E(x_s|bad)} < \tau$ ) are bought by the intermediary. The price  $p_s$  of such an asset equals

$$\begin{aligned} E\left(\frac{u'(c_2)}{u'(c_1)}x_s\right) - E\left(\frac{u'(c_2)}{u'(c_1)}C'\left(\int q_I(s)x_s ds\right)x_s\right) \\ + P'\left(\int q_I(s)E(x_s|bad)ds\right)E(x_s|bad)\frac{v'(d)}{u'(c_1)}. \end{aligned} \quad (22)$$

(3) All risky assets with sufficiently high systematic risk ( $\frac{E(x_s|good)}{E(x_s|bad)} > \tau$ ) are bought by the household. For these assets, the price  $p_s$  is given by

$$E\left(\frac{u'(c_2)}{u'(c_1)}x_s\right) = p_s. \quad (23)$$

## ➤ C6. Capital Structure Choices of Nonfinancial Firms

$$\max_{F_i} E \left( \frac{u'(c_2) + v'(d)}{u'(c_1)} \delta_i^* \right) + \left( \frac{u'(c_2)}{u'(c_1)} (\delta_i^* - \bar{\delta}_i^*) \right) - E \left[ \frac{u'(c_2)}{u'(c_1)} C' \left( \int q_I(s) x_s ds \right) * \right. \quad (24)$$

$$\left. \min (\delta_i^* - \bar{\delta}_i^*, F_i) \right] + P' \left( \int q_I(s) E(x_s | bad) ds \right) E(\min (\delta_i^* - \bar{\delta}_i^*, F_i) | bad) \frac{v'(d)}{u'(c_1)}. \quad (25)$$



## ➤ C6. Capital Structure Choices of Nonfinancial Firms (cont.)

To compute the optimal face value  $F_i$  of risky debt, note that except on an event of probability 0,<sup>16</sup>

$$\frac{\partial \min(\delta_i^* - \bar{\delta}_i^*, F_i)}{\partial F_i} = -\frac{\partial \max(\delta_i^* - \bar{\delta}_i^* - F_i, 0)}{\partial F_i} = \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}. \quad (26)$$

The first-order condition for the optimal face value  $F_i$  of risky debt is therefore

$$\begin{aligned} & E\left(\frac{u'(c_2)}{u'(c_1)} C' \left( \int q_I(s) x_s ds \right) \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}\right) \\ &= P' \left( \int q_I(s) E(x_s | bad) ds \right) \Pr(\delta_i^* - \bar{\delta}_i^* \geq F_i | bad) \frac{v'(d)}{u'(c_1)}. \end{aligned} \quad (27)$$



## ➤ C7. Equilibrium

**DEFINITION 1:** An equilibrium is a set of asset prices, portfolio and leverage choices, and consumption allocations that satisfies the following conditions. (1) The household, the intermediary, and nonfinancial firms behave optimally, solving maximization problems (11), (16), and (17). In addition, managers optimize, so they transfer all seized consumption resources in exchange for utility transfers. (2) Resource constraints are satisfied, so  $c_1 = Y_1$  and  $c_2 = \int_0^1 \delta_i di$ . (3) Asset markets clear, so  $q_I + q_H = 1$ .

Plugging in  $c_2 = \int_0^1 \delta_i di$  and using the fact that the intermediary's portfolio consists of all risky debt issued by the nonfinancial sector, so  $\delta_I = \int_0^1 \min(\delta_i^* - \bar{\delta}_i^*, F_i) di$ , equation (27) becomes

$$\begin{aligned} & E \left[ u' \left( \int_0^1 \delta_i di \right) C' \left( \int_0^1 \min(\delta_i^* - \bar{\delta}_i^*, F_i) di \right) \mathbb{1}_{\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}} \right] \\ &= v' \left( \mu + P \left( \int_0^1 E(\min(\delta_i^* - \bar{\delta}_i^*, F_i) | bad) di \right) \right) P' \left( \int_0^1 E(\min(\delta_i^* - \bar{\delta}_i^*, F_i) | bad) di \right) * \\ & \Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i\} | bad). \end{aligned} \tag{28}$$



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