Financial Uncertainty with Ambiguity and Learning

MS 2022.03

Hening Liu Yuzhao Zhang

汇报人：任宇卓

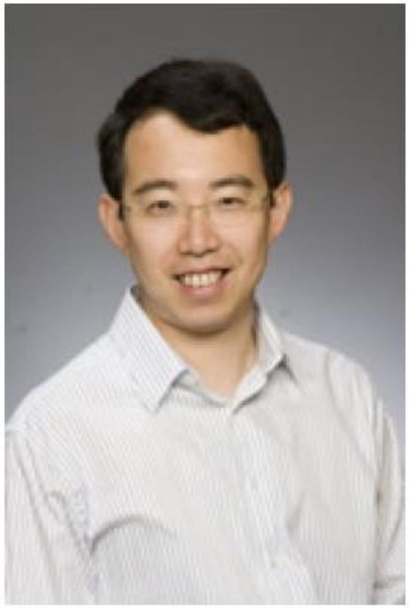
2022年12月28日

Hening Liu 刘赫宁
Professor of Finance， Alliance Manchester Business School
曼彻斯特商学院

教育背景：
北伊利诺伊大学：经济学硕士、博士学位。
暨南大学：经济学学士学位。

研究方向： Asset Pricing, Macro-Finance, Portfolio Choice

1. Liu H, Fulop A, Heng J, et al. Bayesian Estimation of Long-Run Risk Models Using Sequential Monte Carlo[J]. Journal of Econometrics, 2022, 228(1): 62-84.
2. Gallant A R, R Jahan-Parvar M, Liu H. Does smooth ambiguity matter for asset pricing?[J]. The Review of Financial Studies, 2019, 32(9): 3617-3666.
3. Guidolin M, Liu H. Ambiguity aversion and underdiversification[J]. Journal of Financial and Quantitative Analysis, 2016, 51(4): 1297-1323.



Yuzhao Zhang 张玉昭

罗格斯大学金融系助理教授

教育背景

加州大学洛杉矶分校：金融学博士

纽约大学：金融数学硕士

研究方向：资产定价，衍生证券等

1. Aretz K, Liu H, Yang S, et al. Consumption risks in option returns[J]. Journal of Empirical Finance, 2022.
2. Brennan M J, Zhang Y. Capital asset pricing with a stochastic horizon[J]. Journal of Financial and Quantitative Analysis, 2020, 55(3): 783-827.
3. Reeb D M, Zhang Y, Zhao W. Insider trading in supervised industries[J]. The Journal of Law and Economics, 2014, 57(3): 529-559.

Abstract

- We examine a production-based asset pricing model with regime-switching productivity growth, learning, and ambiguity.
- Both the mean and volatility of the growth rate of productivity are assumed to follow a Markov chain with an unobservable state. The agent's preferences are characterized by the generalized recursive smooth ambiguity utility function.
- Our calibrated benchmark model with modest risk aversion can match moments of the variance risk premium in the data and reconcile empirical relations between the riskneutral variance and macroeconomic quantities and their respective volatilities.
- We show that the interplay between productivity volatility risk and ambiguity aversion is important for pricing variance risk in returns.

2. Empirical Analysis

2.1. Risk-Neutral Variance and Variance Risk Premium

➢ **Variance risk premium：**

$$VRP_t \equiv \boxed{\mathrm{Var}_t^Q[r_{t+1}]} - \mathrm{Var}_t[r_{t+1}],$$

Risk-neutral variance：

$$VIX_t^2 = \frac{2}{T}\sum_i \frac{\Delta X_i}{X_i^2} e^{r_f T} Q(X_i, T) - \frac{1}{T}\left[\frac{F}{X_0} - 1\right]^2.$$

T : time to maturity.

X_i : the strike price of the *i*th out-of-the-money option.

$r_f{}^T$: the London Interbank Offered Rate (LIBOR) rate for maturity T.

$Q(Xi, T)$: the midpoint of bid and ask prices for an option with maturity T and strike X_i.

F : the forward index level derived from the put-call parity using index option prices.

X_0 : the first strike below F.

Objective variance:

First: computethe sum of squared daily log returns of the quarter leading up to time t.

$$RV_t = \sum_{j=0}^{T} r_{t-j}^2.$$

Then: regress RV_{t+1} on its on its own lag, RV_t, and lagged risk-neutral variance, VIX_t^2 , and use the fitted value as the conditional variance under the physical measure, denoted as VOL_t^2 .

- **Variance risk premium:**

$$VRP_t \equiv VIX_t^2 - VOL_t^2$$

2.2. Markov-Switching Models and the VAR Analysis

We empirically examine

1. the link between time-varying volatility of productivity growth and the risk-neutral variance，

2. financial uncertainty has negative impacts on aggregate quantities.

We assume that productivity growth follows a Markov-switching (MS) process

$$\Delta a_t = \mu(s_t) + \sigma(s_t)\epsilon_t, \quad \epsilon_t \sim \mathrm{N}(0,1), \tag{1}$$

Δa_t : the productivity growth rate, $\Delta a_t \equiv \ln(A_t/A_{t-1})$, A_t is the productivity level.

s_t : determines the regime of the conditional mean and volatility of the growth rate, evolves according to a Markov chain.

We denote by P the transition matrix of the Markov chain and π_t the posterior belief vector about the next period's states. The transition probability p_{ij} in matrix P is defined as $p_{ij} = \Pr(s_t = j \mid s_{t-1} = i)$. Given the prior belief vector π_0, and according to Bayes'rule, the posterior belief π_t is given by

$$\pi_t = P' \frac{\pi_{t-1} \odot f_t}{\mathbf{1}'(\pi_{t-1} \odot f_t)}$$

f_t : a vector of conditional Gaussian likelihood functions for $\mu(s_t)$ and $\sigma(s_t)$

$\odot$: element-wise multiplication, and **1** is a vector of ones.

Table 1. Parameter Estimates of the Markov-Switching Model

Panel A: Two-regime Markov-switching model					
μ_1	μ_2	σ_1	σ_2	p_{11}	p_{22}
–0.989 (0.100)	1.026 (0.112)	3.726 (1.320)	1.007 (0.140)	0.753 (0.003)	0.977 (0.037)
Panel B: Three-regime Markov-switching model					
μ_1	μ_2	μ_3	σ_1	$\sigma_2$0000	σ_3
–0.610 (0.004)	0.821 (0.035)	1.967 (0.001)	3.674 (0.006)	1.021 (0.001)	0.597 (0.030)
p_{11}	p_{13}	p_{21}	p_{22}	p_{31}	p_{33}
0.670 (0.004)	0.324 (0.004)	0.018 (0.037)	0.952 (0.037)	0.085 (0.007)	0.645 (0.007)

Panel C: Model selection	LL	BIC
Two-regime MS	1,040.242	–2,046.632
Three-regime MS	1,047.968	–2,028.233

We use macroeconomic data to construct Solow residuals and quarterly productivity growth rates.

The sample period : Q1 1947 to Q1 2016.

We estimate two-regime, three-regime, and four-regime MS models using the expectation maximization algorithm developed by Hamilton (1990).

- **问题:**

  Does an increase in the conditional volatility of productivity growth lead to more financial uncertainty as proxied by the risk-neutral variance, which subsequently causes declines in the macroeconomic quantities and equity valuation?

- **方法:**

  Bayesian approach developed by Sims and Zha (1998) to estimate the VAR model.

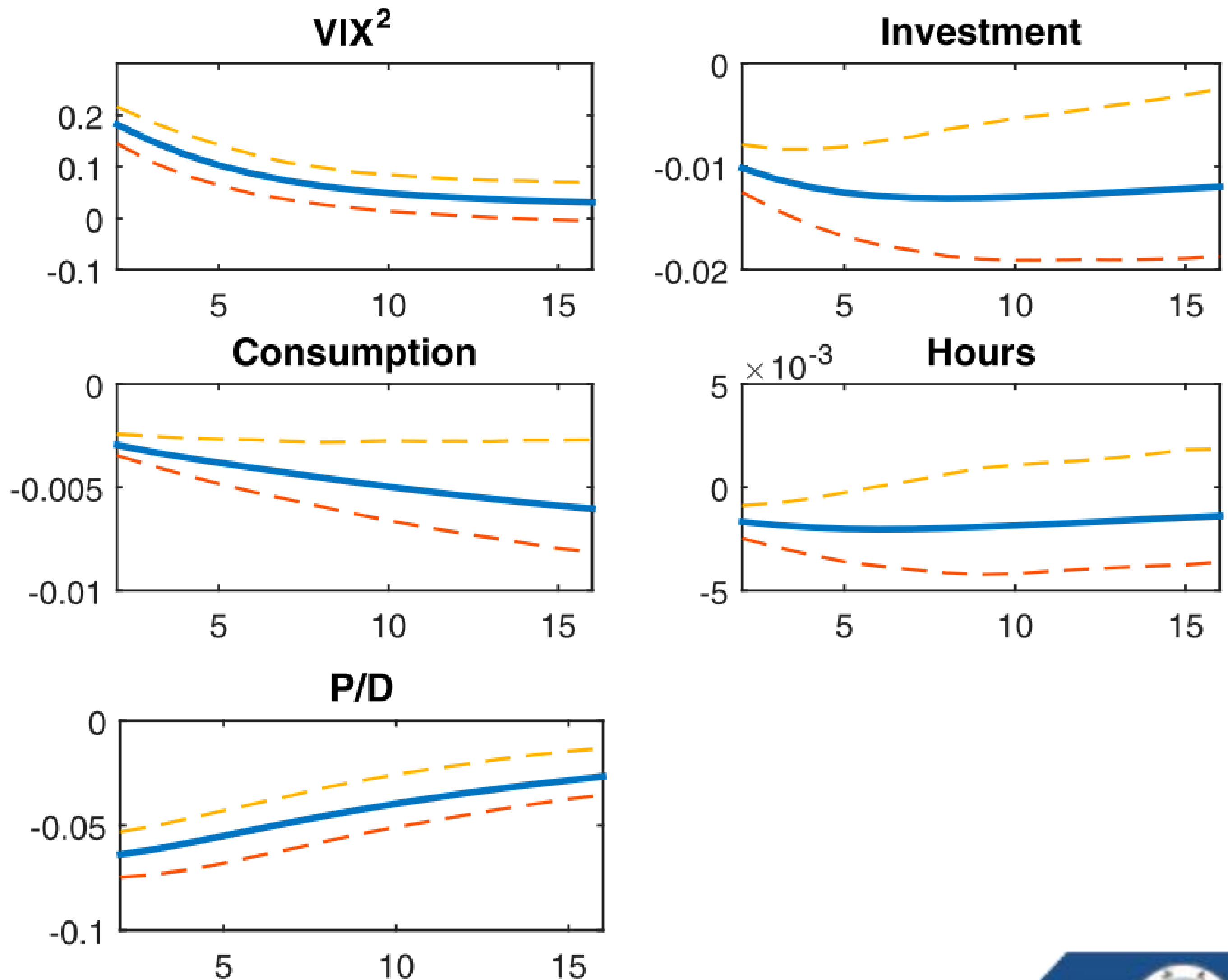
- **数据来源:**

  a. Macroeconomic data : National Income and Product Accounts.
  b. Consumption and investment data are deflated by the corresponding deflators.
  c. The price dividend ratio data are constructed from value weighted index returns that include and exclude distributions.
  d. Stock returns data are drawn from the Center for Research in Security Prices.

- **样本期:**

  Q1 1990 to Q1 2016

Figure 1. (Color online) VAR Impulse Responses: Historical Data



3. The Model

3.1. Preferences

- We assume that the representative agent cannot observe state s_t in the Markov-switching model (1) but can learn about it through observing past realizations of productivity growth.

The representative agent has generalized recursive smooth ambiguity preferences:

$$V_t = \left((1-\beta) U_t^{1-\frac{1}{\psi}} + \beta \left(\mathbb{E}_{\pi_t} \left[\left(\mathbb{E}_{\{s_{t+1},t\}} \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1-\eta}{1-\gamma}} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\eta}} \right)^{\frac{1}{1-\frac{1}{\psi}}},$$

$$U_t = C_t (1-N_t)^{\nu}, \qquad (3)$$

U_t : felicity function.

ν : leisure preference.

β : the subjective discount factor, $\beta \in (0, 1)$

$\widehat{\Psi}$: the elasticity of intertemporal substitution (EIS), $\widehat{\Psi} = (1-(1+\nu)(1-1/\psi))^{-1}$

η : the degree of ambiguity aversion, For the utility function (3), the agent is ambiguity averse if and only if $\eta > \gamma$.

$(\gamma-1) + 1/(1+\nu)$: degree of relative risk aversion .

Expressed differently from recursive preferences with ambiguity neutrality, the certainty equivalent of smooth ambiguity utility is defined as

$$\mathcal{R}_t(V_{t+1}) = \left(\mathbb{E}_{\pi_t}\left[\left(\mathbb{E}_{\{s_{t+1},t\}}\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1-\eta}{1-\gamma}}\right]\right)^{\frac{1}{1-\eta}}, \qquad (4)$$

$E\{s_{t+1},t\}[\cdot]$ denotes the expectation conditional on the history up to time t and a probability distribution of productivity growth given state s_{t+1}.

For comparison, the certainty equivalent under ambiguity neutrality is based on the predictive distribution of Δa_t+1 and expressed as

$$\mathcal{R}_t(V_{t+1}) = \left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}$$
$$= \left(\int V_{t+1}^{1-\gamma} p(\Delta a_{t+1}|\boldsymbol{\pi}_t) d(\Delta a_{t+1})\right)^{\frac{1}{1-\gamma}}$$

predictive density: $p(\Delta a_{t+1}|z_t) = \pi_t \odot f_{t+1}$.

The key property of the smooth ambiguity model is that it distinguishes ambiguity from ambiguity aversion.

3.2. Equilibrium Characterization

- Aggregate output (Y_t) is produced according to a standard constant-returns-to-scale, Cobb–Douglas production function:

$$Y_t = K_t^{\alpha}(A_t N_t)^{1-\alpha}, \ \alpha \in (0,1)$$

α is the capital share, and K_t denotes the capital stock.

- The law of motion for capital accumulation is:

$$K_{t+1} = (1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right)K_t, \qquad (5)$$

- adjustment cost function:

$$\phi\left(\frac{I_t}{K_t}\right) = a_1 + \frac{a_2}{1-1/\xi}\left(\frac{I_t}{K_t}\right)^{1-1/\xi}, \ a_2 > 0, \ \ \xi > 0$$

ξ is the elasticity of the investment rate to Tobin's q, and the parameters a_1 and a_2 are chosen such that there is no adjustment cost in the steady state.

3.3. Asset Prices

Stochastic discount factor (SDF) for the generalized recursive smooth ambiguity utility:

$$M_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{1-N_{t+1}}{1-N_t}\right)^{\left(1-\frac{1}{\psi}\right)\nu}\left(\frac{V_{t+1}}{\mathcal{R}_t(V_{t+1})}\right)^{\frac{1}{\psi}-\gamma} \times \left(\frac{\left(\mathbb{E}_{\{s_{t+1},t\}}\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}}{\mathcal{R}_t(V_{t+1})}\right)^{-(\eta-\gamma)}.$$

The risk-free rate, $R_{f,t}$, is the reciprocal of the expectation of the pricing kernel:

$$R_{f,t} = \frac{1}{\mathbb{E}_t\left[M_{t,t+1}\right]}$$

Following standard q-theory arguments, Tobin's q is expressed as

$$q_t = \frac{1}{\phi'\left(\frac{I_t}{K_t}\right)}.$$

In RBC models, the firm's payout in period *t* is expressed as

$$D_t^* = Y_t - w_t N_t - I_t = \alpha Y_t - I_t,$$

w_t :the equilibrium wage, $w_t = \partial Y_t / \partial N_t = (1-\alpha) A_t^{1-\alpha} K_t^{\alpha} N_t^{-\alpha}$.

The return on capital (investment) is:

$$R_{t+1}^K = \frac{1}{q_t}\left\{q_{t+1}\left[1 - \delta_K + \phi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right] + \frac{\alpha Y_{t+1} - I_{t+1}}{K_{t+1}}\right\}$$

In particular, we specify the dividend growth process as containing a component proportional to consumption growth and an independent component.

$$\Delta d_{t+1} \equiv \ln\left(\frac{D_{t+1}}{D_t}\right) = \lambda \Delta c_{t+1} + g_d + \sigma_d \varepsilon_{d,t+1}, \qquad (6)$$

for which $\varepsilon_{d,t+1}$ is an i.i.d. standard, normal random variable that is independent of all other shocks in the model. The parameter λ can be interpreted as the leverage ratio on expected consumption growth. The parameters g_d and σ_d are calibrated to match the first and second moments of dividend growth in the data.

Stock returns, R_{t+1}, are defined as

$$R_{t+1} \equiv \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{1 + P_{t+1}/D_{t+1}}{P_t/D_t} \frac{D_{t+1}}{D_t}$$

and satisfy the Euler equation:

$$\mathbb{E}_t\left[M_{t,t+1} R_{t+1}\right] = 1.$$

To solve for the price-dividend ratio, we rewrite the Euler equation as

$$\frac{P_t}{D_t} = \mathbb{E}_t\left[M_{t,t+1}\left(1 + \frac{P_{t+1}}{D_{t+1}}\right)\frac{D_{t+1}}{D_t}\right].$$

Using the state variables to express the price-dividend ratio and setting $\frac{P_t}{D_t} = \xi(\tilde{K}_t, \boldsymbol{\pi}_t)$, the Euler equation becomes

$$\xi(\tilde{K}_t, \boldsymbol{\pi}_t) = \mathbb{E}_t[M_{t,t+1}(1 + \xi(\tilde{K}_{t+1}, \boldsymbol{\pi}_t))\exp(\Delta d_{t+1})].$$

$$VIX_t^2 = \frac{\mathbb{E}_t[M_{t,t+1}R_{t+1}^2]}{\mathbb{E}_t[M_{t,t+1}]} - \left(\frac{\mathbb{E}_t[M_{t,t+1}R_{t+1}]}{\mathbb{E}_t[M_{t,t+1}]}\right)^2,$$

$$VOL_t^2 = \mathbb{E}_t[R_{t+1}^2] - (\mathbb{E}_t[R_{t+1}])^2.$$

$$VRP_t = VIX_t^2 - VOL_t^2,$$

4. Calibration

4.1. Parameter Choice

Table 2. Benchmark Parameter Choices

Parameter	Description	Value
β	Subjective discount factor	0.9975
γ	Risk aversion	5
$\hat{\psi}$	EIS	2.5
η	Ambiguity aversion	55
ν	Leisure preference	2
α	Capital share	0.36
δ	Depreciation rate	0.02
ξ	Adjustment cost parameter	6.5
λ	Leverage	2.75

4.2 Impulse Responses

Figure 2. (Color online) Impulse Responses for Filtered Probabilities and Conditional Mean and Volatility of Productivity Growth: Model AA3S

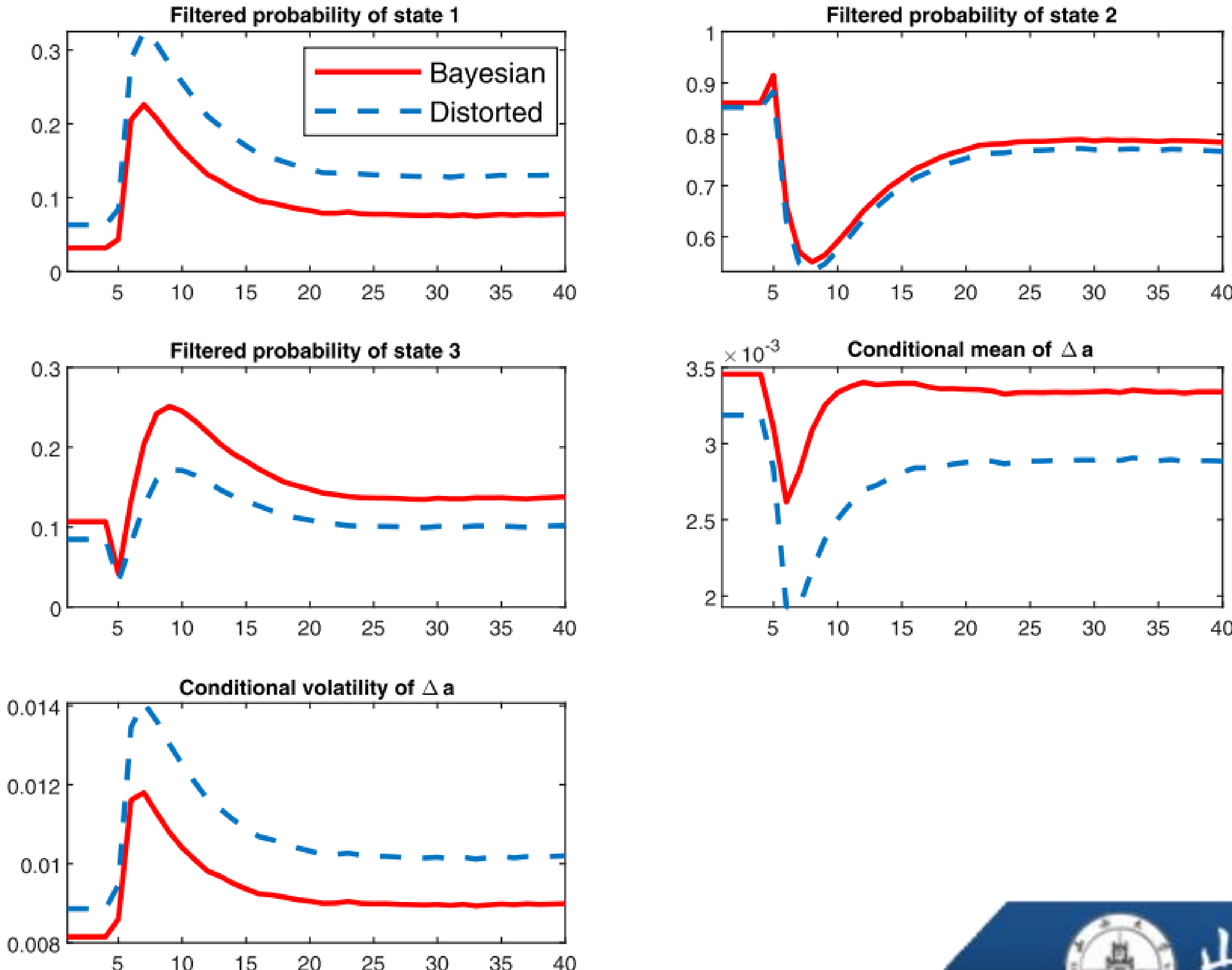
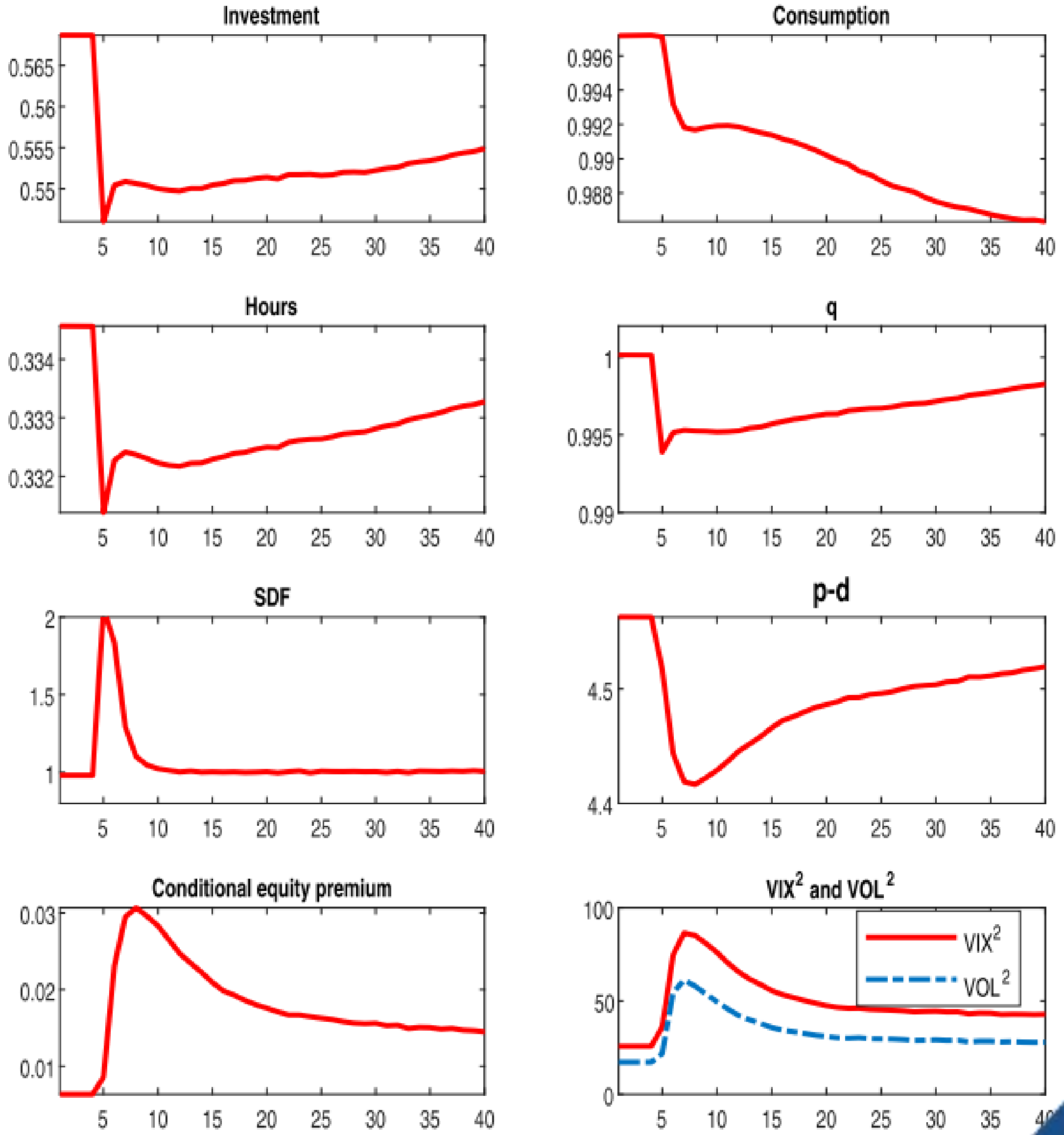
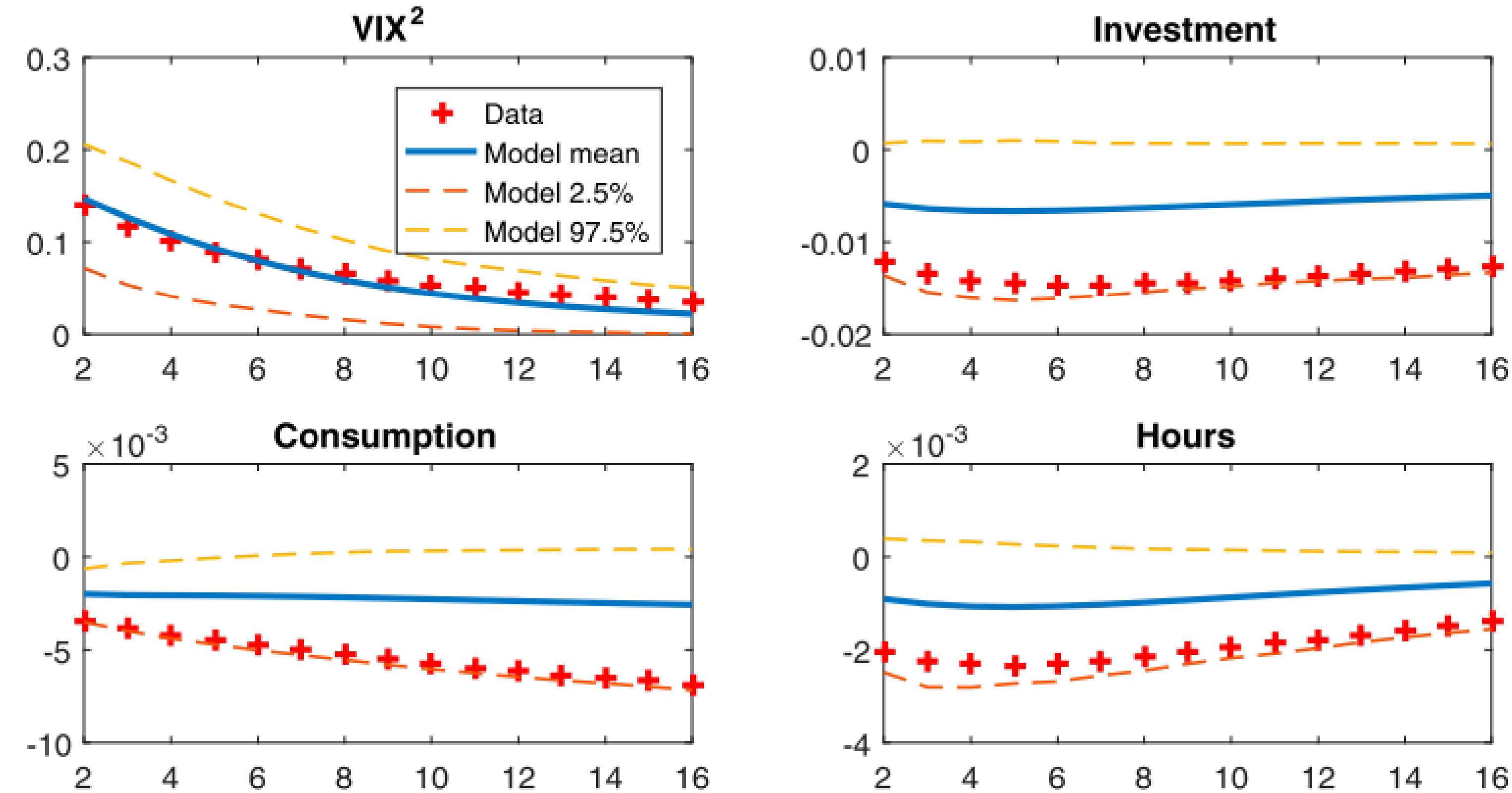


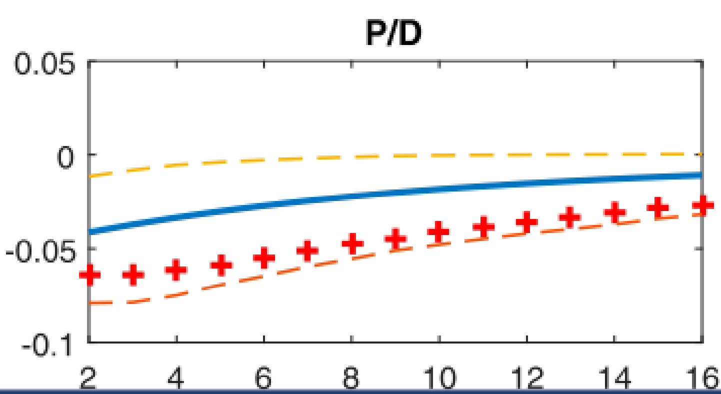
Figure 3. (Color online) Impulse Responses for Quantities and Financial Market Variables: Model AA3S



由于制度转变导致了生产力水平的长期变化，消费和投资会下降，趋于较低的稳态。相应工作时间也减少。有与预期消费增长率的降低，导致了资产价格的长期风险。

Figure 4. (Color online) VAR Impulse Responses: Simulated Data and Historical Data





我们应用第2节中的经验方法，使用基准模型AA3S模拟的数据来估计VAR模型。在图4中，模型模拟产生的VAR脉冲响应的平均水平与经验数据的结果接近，而且经验数据的图在基准模型模拟脉冲响应的2.5%和97.5%范围内。
可以解释由风险中性方差衡量的不确定性抑制了实际经济活动

Table 3. Calibration Results: Models with the Three-Regime MS Process

	Data	AA3S	$\overline{AA3S}$	EZ3S	$\widehat{EZ3S}$
Panel A: Macroeconomic moments					
$\sigma_{\Delta c}$ (%)	1.06	0.96	0.93	0.92	0.96
$\sigma_{\Delta i}$ (%)	4.87	4.20	3.09	4.13	4.28
$\sigma_{\Delta y}$ (%)	2.56	1.85	1.51	1.84	1.86
$\rho(\Delta i, \Delta y)$	0.71	0.95	0.94	0.96	0.95
$\rho(\Delta c, \Delta y)$	0.46	0.69	0.73	0.74	0.67
$\rho(\Delta c, \Delta i)$	0.42	0.45	0.46	0.53	0.42
$\rho(\Delta c_t, \Delta c_{t+1})$	0.29	0.19	0.13	0.20	0.19
Panel B: Financial moments					
$\mathbb{E}[R_f] - 1$ (%)	1.04	1.74	1.93	1.96	1.80
$\sigma(R_f)$ (%)	0.78	0.22	0.19	0.20	0.23
$\mathbb{E}(R - R_f)$ (%)	6.23	5.67	2.68	1.38	3.29
$\sigma(R - R_f)$ (%)	15.26	17.39	16.11	14.15	15.95
$\mathbb{E}(VRP)$	11.08	14.56	0.26	1.06	5.19
$\sigma(VRP)$	23.62	13.34	0.39	2.35	8.17
Skewness (VRP)	2.33	3.36	0.62	5.89	4.62
Kurtosis (VRP)	10.23	15.23	11.22	44.55	26.88
$\sigma(M)/\mathbb{E}(M)$	n.a.	0.54	0.27	0.10	0.23

这些结果表明，在具有时变波动率和模糊性规避的基准模型中，方差风险的定价是正确的。时变的生产率波动导致了股票回报中大量的方差风险。此外，歧义厌恶扭曲了风险中性度量，使代理更加关注方差风险。因此，风险中性方差在模型中被大大放大。

我们发现，抑制时变生产率波动对股权溢价和方差风险溢价有重大影响。尽管两种模型（AA3S和AA3S）的回报的无条件波动率大致相同，但这两种模型的波动率风险定价显著不同。由于模型AA3S缺乏时变的生产力波动，因此对SDF和回报波动的补偿不足。因此，在这个模型中，回报中的波动性风险没有被充分地定价。

Figure 5. (Color online) Conditional Moments

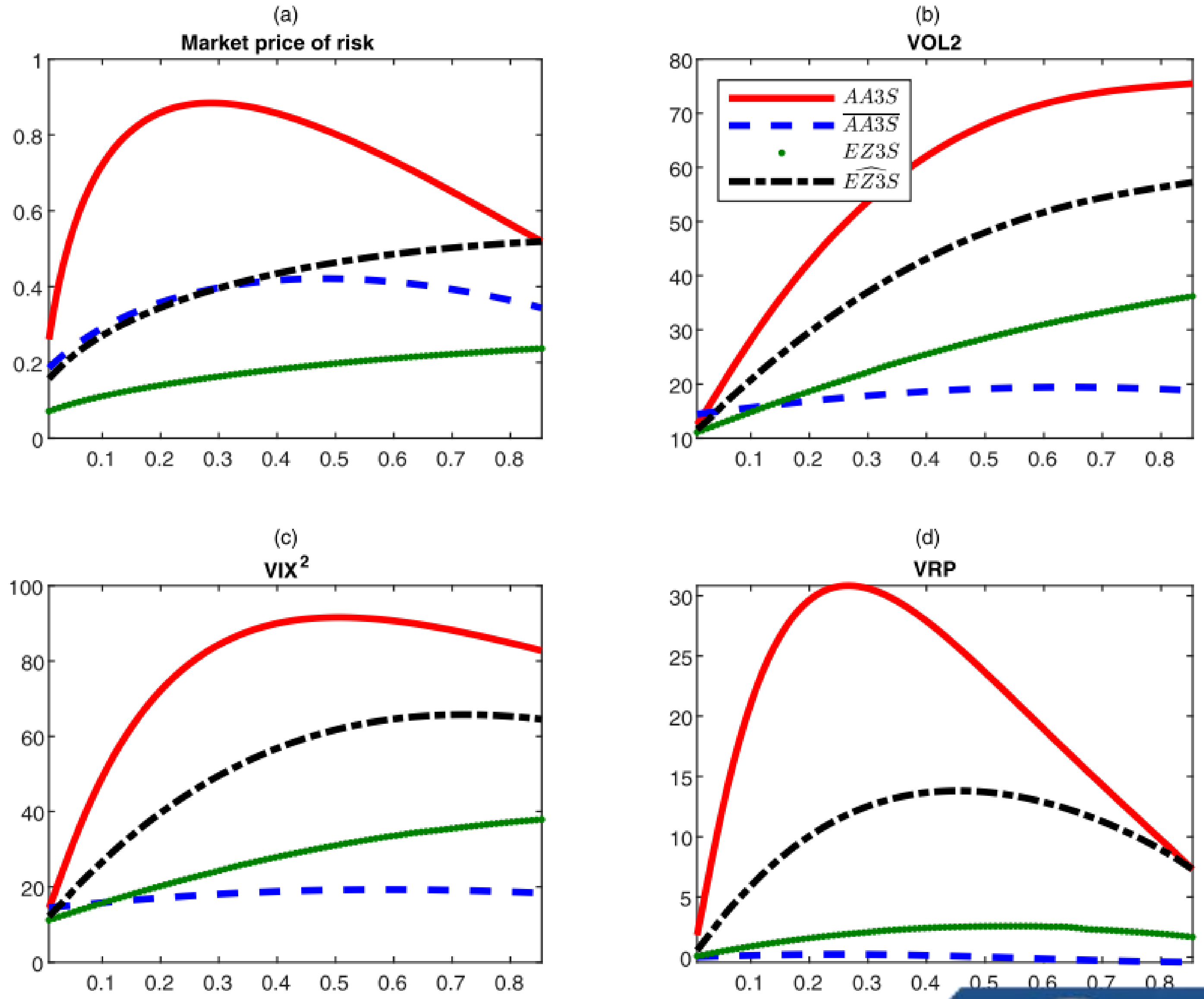


Figure 6. (Color online) The Term Structure of Implied Volatility

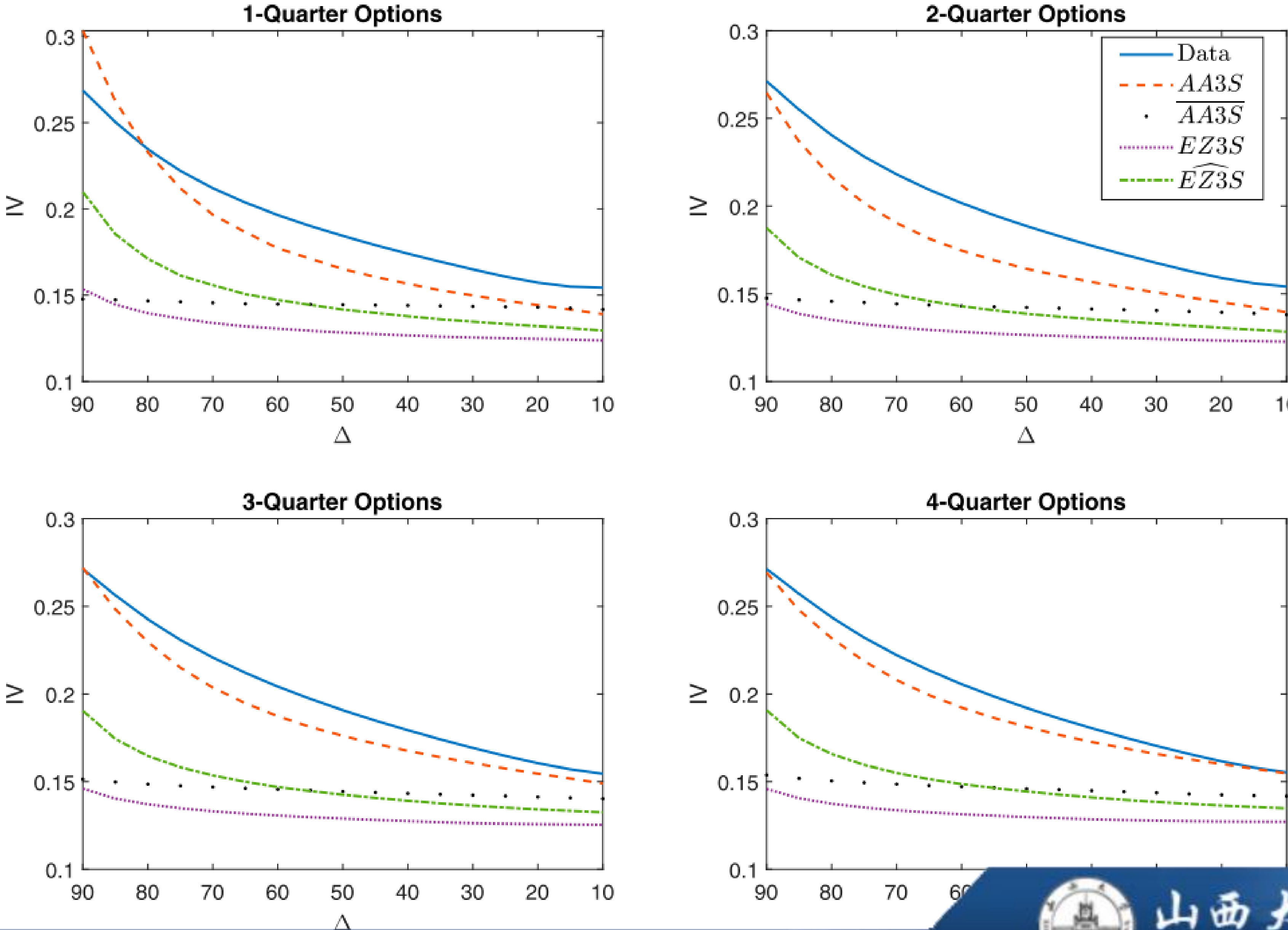


Table 4. The Term Structure of Implied Volatility

	1-Quarter	2-Quarter	3-Quarter	4-Quarter
Panel A: Model AA3S				
Constant	0.0588	0.0212	0.0123	−0.0028
t-stat	(8.4232)	(2.6178)	(1.7177)	(−0.3960)
β	0.7348	0.9944	0.9925	1.0560
t-stat	(19.7565)	(22.0294)	(26.3220)	(29.4880)
t-stat (β=1)	(−7.1314)	(−0.1247)	(−0.1977)	(1.5632)
R^2	0.9605	0.9680	0.9774	0.9819
Panel B: Model $\overline{AA3S}$				
Constant	−2.7353	−1.6405	−1.4029	−1.2749
t-stat	(−16.8194)	(−21.9056)	(−47.0844)	(−51.6707)
β	20.2405	12.8948	11.0568	10.0689
t-stat	(18.0060)	(24.5334)	(53.7508)	(59.7674)
t-stat (β=1)	(17.1164)	(22.6308)	(48.8894)	(53.8315)
R^2	0.9528	0.9741	0.9945	0.9955
Panel C: Model EZ3S				
Constant	−0.3545	−0.5530	−0.5855	−0.6549
t-stat	(−11.7199)	(−14.1903)	(−13.0367)	(−11.3762)
β	4.1682	5.8270	5.9904	6.4843
t-stat	(18.1259)	(19.2514)	(17.4674)	(14.8510)
t-stat (β=1)	(13.7773)	(15.9476)	(14.5515)	(12.5606)
R^2	0.9534	0.9585	0.9500	0.9321
Panel D: Model $\widehat{EZ3S}$				
Constant	−0.0389	−0.1148	−0.1228	−0.1351
t-stat	(−2.6843)	(−5.8794)	(−5.9739)	(−6.0245)
β	1.5497	2.1593	2.1703	2.2362
t-stat	(16.1418)	(16.0340)	(15.7078)	(14.9879)
t-stat (β=1)	(5.7255)	(8.6085)	(8.4701)	(8.2854)
R^2	0.9419	0.9412	0.9389	0.9332

Figure 7. (Color online) Cross-Correlations with Quantities and Equity Return $\mathrm{Corr}(\mathrm{Var}_t^Q[R_{t+1}], x_{t+k})$ for $k = -6$ to 6

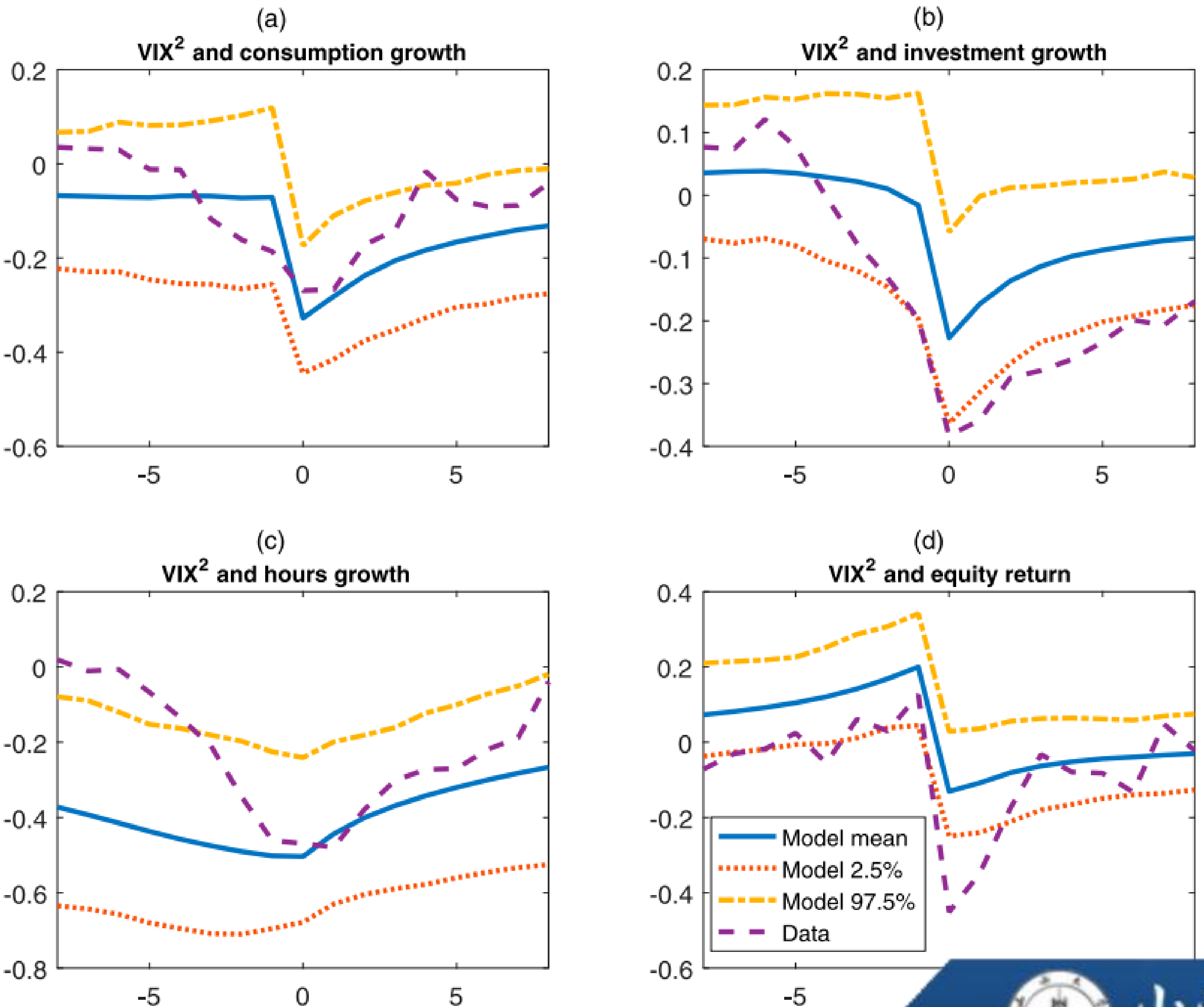
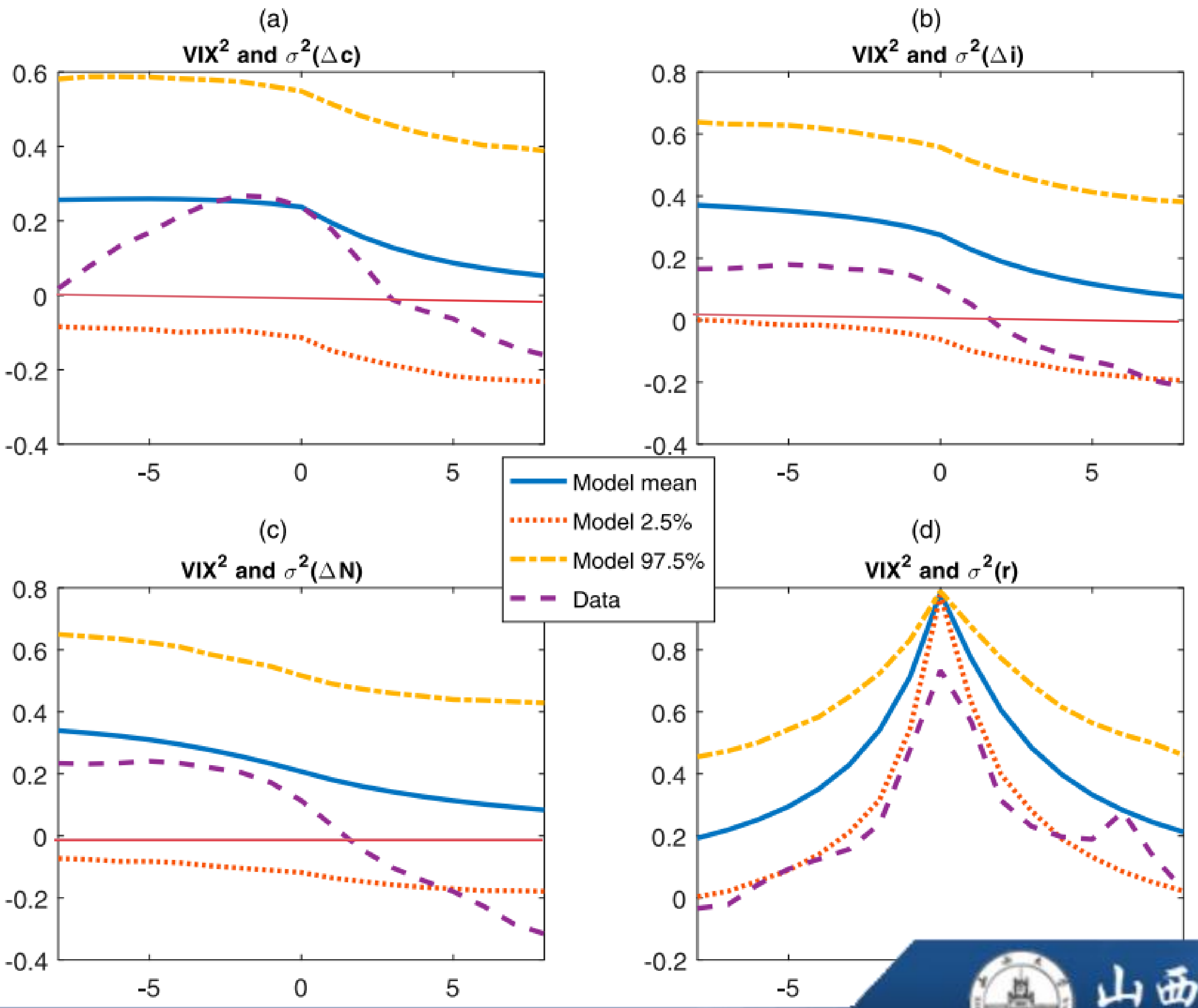


Figure 8. (Color online) Cross-Correlations with Volatilities of Quantities and Equity Return $\text{Corr}(VIX_t^2, \sigma_{x,t+k})$ for $k = -6$ to 6)



Limitations and Future Research

Our model is successful in matching the risk-neutral variance, the VRP, the term structure of variance risk, and other empirical regularities.

- Limitations: The time variation in the conditional mean or volatility of productivity growth can only partially explain the variation in the historical VIX^2 and VRP.

- Future research could explore the variance risk premium in production economies along potentially fruitful avenues such as labor market frictions, irreversible investment, parameter learning, or macroeconomic announcements.

Conclusion

- We have studied a production-based asset pricing model with regime-switching productivity growth, learning, and ambiguity. Compare it with alternative models .
- Our benchmark model with modest risk aversion can match both macroeconomic and financial moments well. It is important to note that, the model generates moments of the variance risk premium close to the data.
- The interplay between productivity volatility risk and ambiguity aversion is important in explaining these stylized facts and pricing variance risk in returns.

THANKS!