

# Extrapolation and bubbles

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Barberis, Nicholas, et al. JFE, 2015, 115(1), 1-24.

**Using Neural Data to Test A Theory of Investor Behavior: An  
Application to Realization Utility,**Frydman, C., Barberis, N.,

Camerer, C., Bossaerts, P., and A. Rangel (2014), Journal of  
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**"Bubbles for Fama."** Greenwood, Robin, Andrei Shleifer,  
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### Publications

**X-CAPM: An Extrapolative Capital Asset Pricing Model**, (with Nicholas Barberis, Robin Greenwood, and Andrei Shleifer). *Journal of Financial Economics* (2015), 115 (1), 1-24. (Lead Article)

**“Realization Utility with Reference-Dependent Preferences.”** Ingersoll, Jonathan, and Lawrence Jin. 2013. *Review of Financial Studies* 26 (3), 723–767.

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**"Diagnostic Expectations and Credit Cycles."** Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer. *Journal of Finance* 73, no. 1 (February 2018): 199–227.

**"Bubbles for Fama."** Robin Greenwood, Andrei Shleifer, and Yang You. *Journal of Financial Economics* 131, no. 1 (January 2019): 20–43.



# Abstract

- We present an extrapolative model of bubbles.
- In the model, many investors form their demand for a risky asset by weighing two signals—an average of the asset's past price changes and the asset's degree of overvaluation—and “waver” over time in the relative weight they put on them.
- The model predicts that good news about fundamentals can trigger large price bubbles, that bubbles will be accompanied by high trading volume, and that volume increases with past asset returns.
- We present empirical evidence that bears on some of the model's distinctive predictions.



# Contents

1. Introduction
2. A model of bubbles
3. Asset prices in a bubble
4. Volume in a bubble
5. Negative bubbles
6. Comparison with other bubble models
7. Some evidence
8. Conclusion



# 1.Introduction

## A. Background

- Bubble episodes have fascinated economists and historians for centuries, in part because human behavior in bubbles is so hard to explain, and in part because of the devastating side effects of the crash.
- At the heart of the standard historical narratives of bubbles is the concept of extrapolation—the formation of expected returns by investors based on past returns. These historical narratives are supported by more recent research on investor expectations, using both survey data and lab experiments.





## B. The main work

In this paper, we present a new model of bubbles based on extrapolation. In doing so, we seek to shed light on two key features commonly associated with bubbles.

- We would like to understand which patterns of news are likely to generate the largest bubbles, and whether a bubble can survive once the good news comes to an end.
- Second, we would like to explain the crucial fact that bubbles feature very high trading volume.



## C. Contribution

- In a departure from prior models, extrapolators also put some weight on a “value signal” which measures the difference between the price and a rational valuation of the final cash flow.
- Our second departure from prior models is to assume that, at each date, and independently of other extrapolators, each extrapolator slightly but randomly shifts the relative weight he puts on the two signals.
- Indeed, in our model, volume during a bubble is predicted by past returns, a new prediction that other bubble models do not share.



## 2.A model of bubbles

We consider an economy with  $T + 1$  dates,  $t = 0, 1, \dots, T$ . There are two assets: one risk-free and one risky. The risk-free asset earns a constant return which we normalize to zero. The risky asset has a fixed supply of  $Q$  shares, and each share is a claim to a dividend  $\tilde{D}_T$  paid at the final date,  $T$ . The value of  $\tilde{D}_T$  is given by

$$\tilde{D}_T = D_0 + \tilde{\varepsilon}_1 + \dots + \tilde{\varepsilon}_T, \quad (1)$$

where

$$\tilde{\varepsilon}_t \sim N(0, \sigma_\varepsilon^2), \text{ i.i.d. over time.} \quad (2)$$

The value of  $D_0$  is public information at time 0, while the value of  $\tilde{\varepsilon}_t$  is realized and becomes public information at time  $t$ . The price of the risky asset,  $P_t$ , is determined endogenously.



There are two types of traders in the economy: fundamental traders and extrapolators. The time  $t$  per capita demand of fundamental traders for shares of the risky asset is

$$\frac{D_t - \gamma \sigma_\varepsilon^2 (T - t - 1) Q - P_t}{\gamma \sigma_\varepsilon^2}, \quad (3)$$

where  $D_t = D_0 + \sum_{j=1}^t \varepsilon_j$  and  $\gamma$  is fundamental traders' coefficient of absolute risk aversion.



If all investors in the economy were fundamental traders, then, setting the expression in (3) equal to the risky asset supply of  $Q$ , the equilibrium price of the risky asset would be

$$D_t - \gamma \sigma_\varepsilon^2 (T - t) Q. \quad (4)$$

We call this the “fundamental value” of the risky asset and denote it by  $P_t^F$ .



Extrapolators are the second type of trader in the economy. There are  $I$  types of extrapolators, indexed by  $i \in \{1, 2, \dots, I\}$ ; We build up our specification of extrapolator demand for the risky asset in three steps. An initial specification of per capita extrapolator share demand at time  $t$  is

$$\frac{X_t}{\gamma \sigma_\varepsilon^2}, \quad \text{where } X_t \equiv (1 - \theta) \sum_{k=1}^{t-1} \theta^{k-1} (P_{t-k} - P_{t-k-1}) + \theta^{t-1} X_1, \quad (5)$$

and where  $0 < \theta < 1$ .



First, we make extrapolators pay at least some attention to how the price of the risky asset compares to its fundamental value. Specifically, we change the demand function in (5) so that the demand of extrapolator  $i$  takes the form

$$w_i \left( \frac{D_t - \gamma \sigma_\varepsilon^2 (T - t - 1) Q - P_t}{\gamma \sigma_\varepsilon^2} \right) + (1 - w_i) \left( \frac{X_t}{\gamma \sigma_\varepsilon^2} \right). \quad (6)$$



Our second modification is to allow the weight  $\omega_i$  to vary slightly over time, and independently so for each extrapolator type, so that the demand function for extrapolator  $i$  becomes

$$w_{i,t} \left( \frac{D_t - \gamma \sigma_\varepsilon^2 (T - t - 1) Q - P_t}{\gamma \sigma_\varepsilon^2} \right) + (1 - w_{i,t}) \left( \frac{X_t}{\gamma \sigma_\varepsilon^2} \right), \quad (7)$$





To model wavering, we set

$$\begin{aligned} w_{i,t} &= \bar{w}_i + \tilde{u}_{i,t} \\ \tilde{u}_{i,t} &\sim N(0, \sigma_u^2), \text{ i.i.d. over time and across extrapolators.} \end{aligned} \tag{8}$$

Here,  $\omega_i \in (0, 1]$  is the average weight that extrapolator  $i$  puts on the value signal; in our numerical analysis, we set  $\omega_i = 0.1$  for all extrapolator types.



In Proposition 1 in Appendix A, we show that, in the economy described above, a unique market-clearing price always exists and is given by

$$P_t = D_t + \frac{\sum_{i \in I^*} \mu_i (1 - w_{i,t})}{\sum_{i \in I^*} \mu_i w_{i,t}} X_t - \gamma \sigma_\varepsilon^2 Q \frac{(\sum_{i \in I^*} \mu_i w_{i,t})(T - t - 1) + 1}{\sum_{i \in I^*} \mu_i w_{i,t}}, \quad (11)$$

where  $\mu_0$  and  $\mu_i$  are the fraction of fundamental traders and of extrapolators of type  $i$  in the population, respectively.



## 2.1. Parameter values

The asset-level parameters

$$D_0 = 100$$

$$Q = 1$$

$$\sigma_\varepsilon = 3$$

$$T = 50$$



## The investor-level parameters

$$I = 50$$

$$\mu_0 = 0.3$$

$$\bar{\omega}_i = 0.1$$

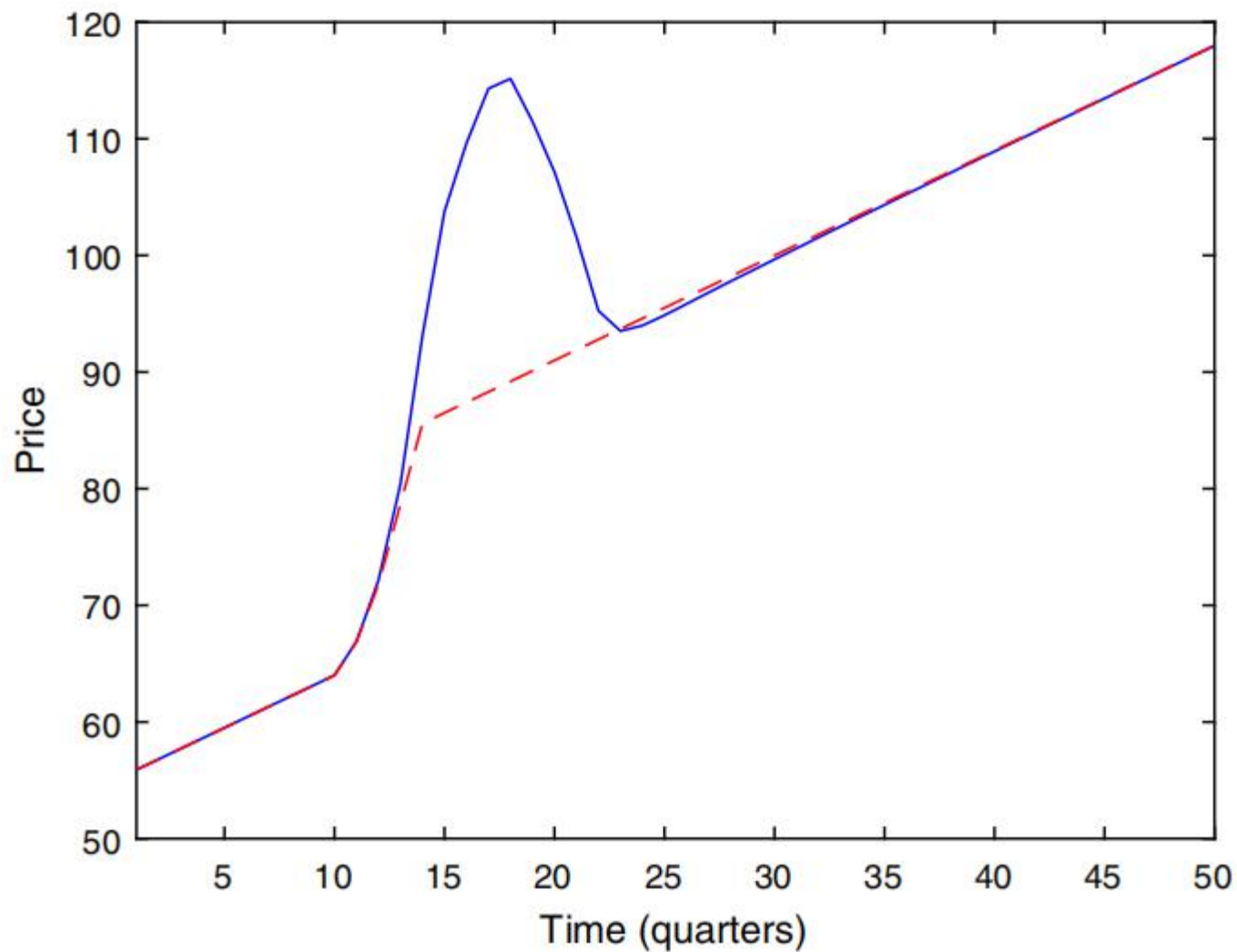
$$\gamma = 0.1$$

$$\theta \approx 0.9$$

$$\sigma_u = 0.03$$



### 3. Asset prices in a bubble



This bubble has three distinct stages defined by the composition of the investor base.

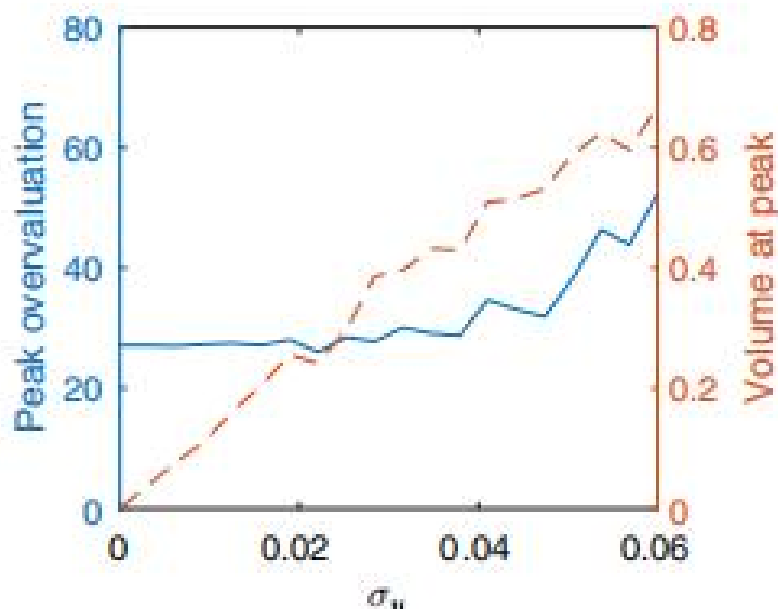
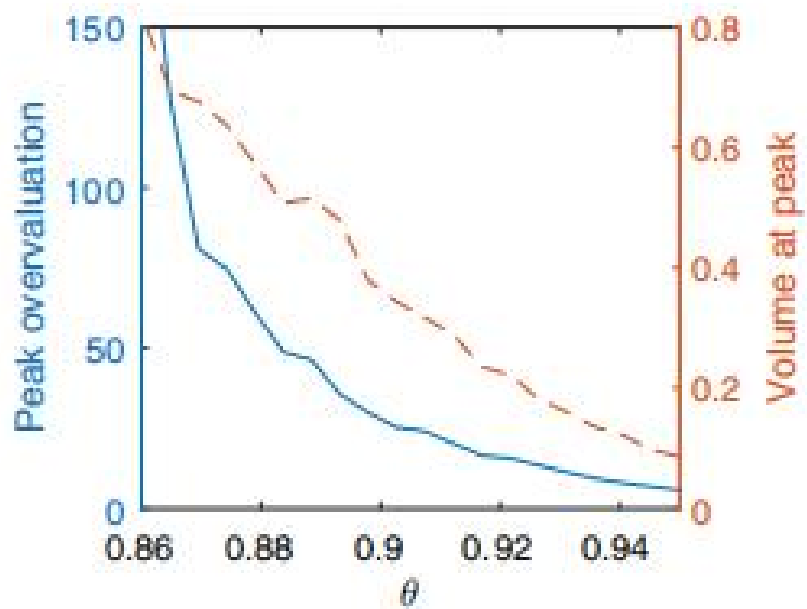
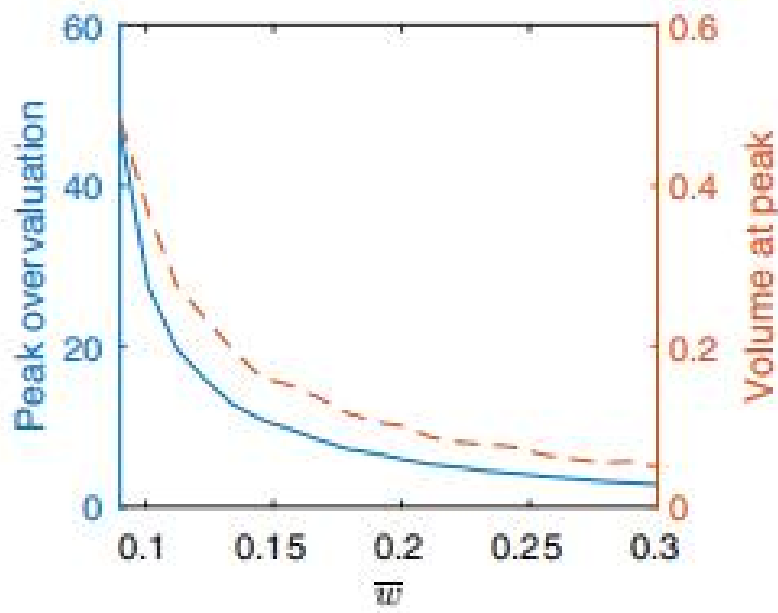
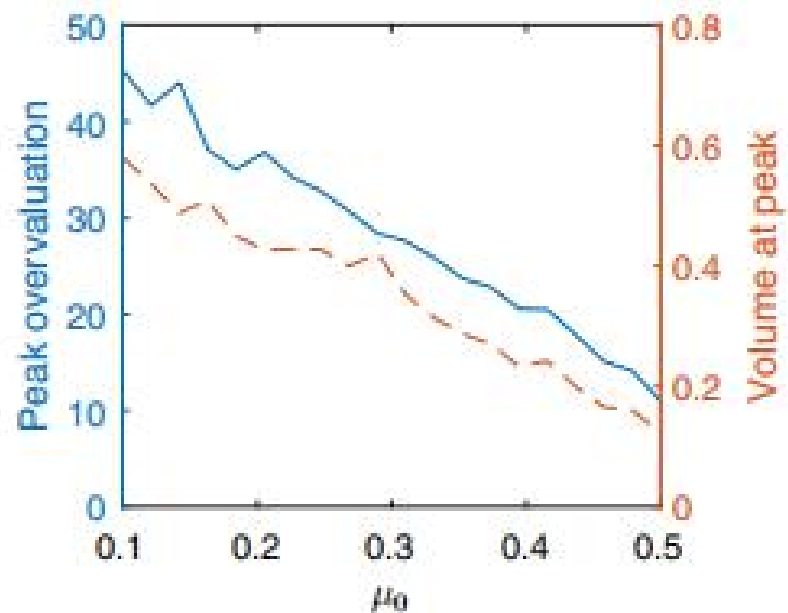
- In the first stage, the fundamental traders are still in the market: even though the risky asset is overvalued, the overvaluation is sufficiently mild.
- The second stage of the bubble begins when the risky asset becomes so overvalued that the fundamental traders exit the market.
- The third stage of the bubble begins when the bubble has deflated to such an extent that the fundamental traders re-enter the market.



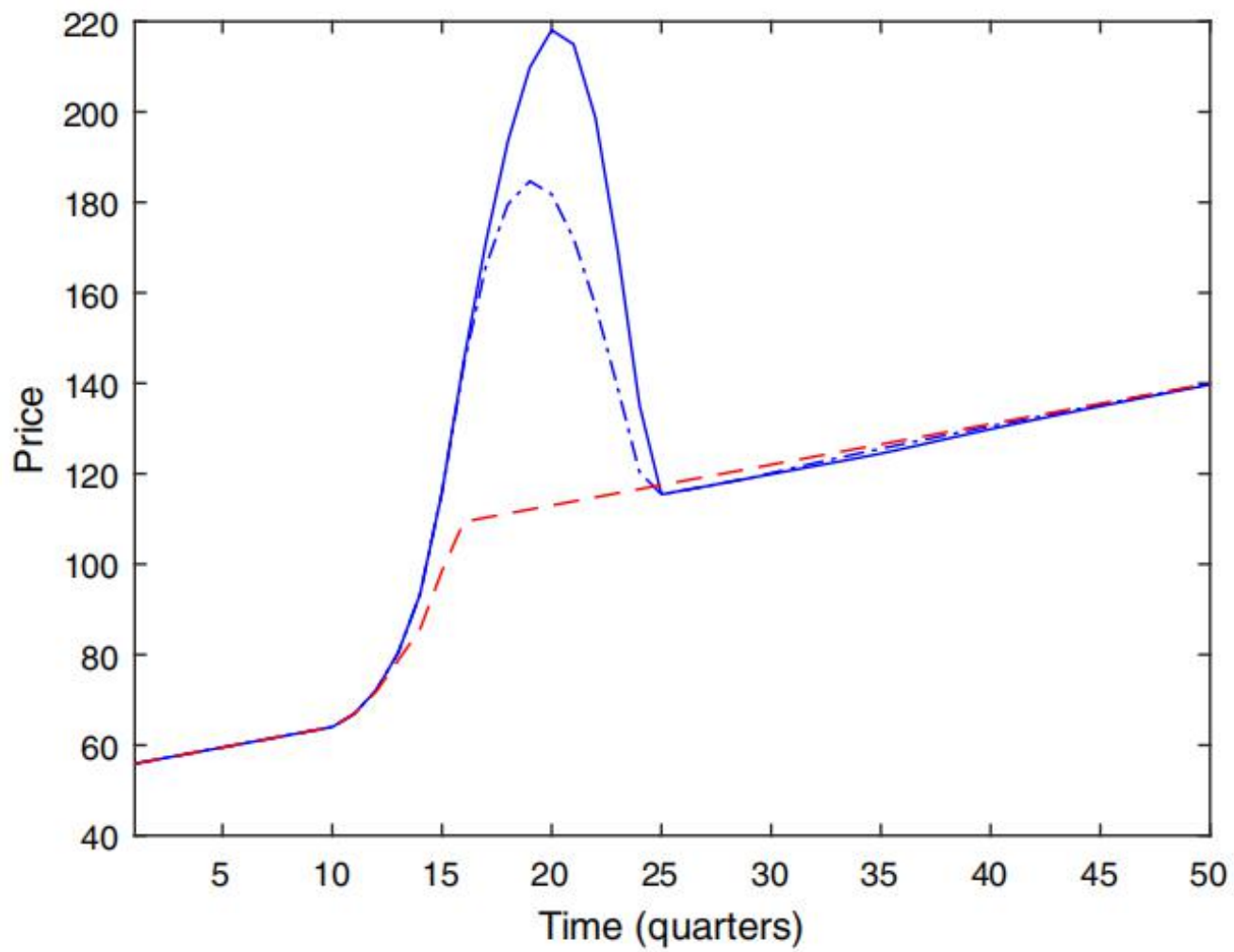
To see how the bubble in Fig. 1 bursts, note that, from Eq. (11), the size of the bubble depends on the magnitude of the growth signal  $X_t$ , itself a measure of extrapolator enthusiasm. From Eq. (5), this signal evolves as

$$X_{t+1} = \theta X_t + (1 - \theta)(P_t - P_{t-1}). \quad (12)$$









Suppose that the economy has been in its steady state up to time  $l - 1$  and that there is then a sequence of positive shocks  $\varepsilon_l, \varepsilon_{l+1}, \dots, \varepsilon_n$  that move the economy from the first stage of the bubble to the second stage of the bubble at some intermediate date  $j$  with  $l < j < n$ . Suppose also that the bubble remains in the second stage through at least date  $N > n$ , in this case, the overvaluation at time  $t$  in the second stage, where  $j \leq t \leq N$ , is approximately equal to

$$\sum_{m=j}^{t-1} \mathcal{L}_2(t-m)\varepsilon_m, \quad (13)$$



For example, if there have been eight cash-flow shocks during the second stage of the bubble, namely,  $\varepsilon_{t-8}$ ,  $\varepsilon_{t-7}$ , ...,  $\varepsilon_{t-1}$ , then, for the parameter values we are using, the degree of overvaluation at time  $t$  is approximately

$$\begin{aligned} & \mathcal{L}_2(1)\varepsilon_{t-1} + \mathcal{L}_2(2)\varepsilon_{t-2} + \cdots + \mathcal{L}_2(7)\varepsilon_{t-7} + \mathcal{L}_2(8)\varepsilon_{t-8} \\ &= 0.9\varepsilon_{t-1} + 1.62\varepsilon_{t-2} + 2.11\varepsilon_{t-3} + 2.33\varepsilon_{t-4} + 2.3\varepsilon_{t-5} \\ & \quad + 2.05\varepsilon_{t-6} + 1.61\varepsilon_{t-7} + 1.06\varepsilon_{t-8}. \end{aligned} \tag{14}$$

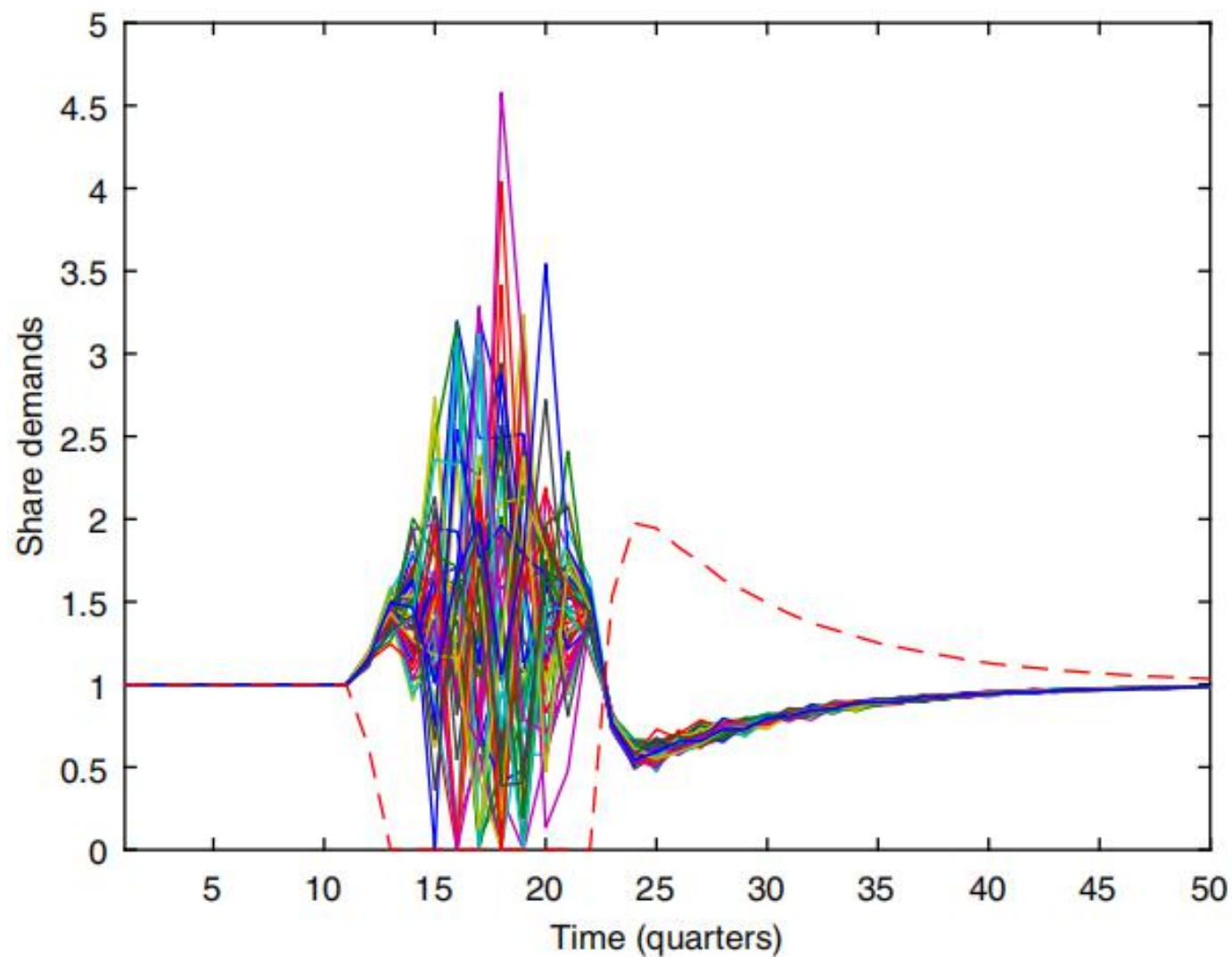


To compute the frequency of large bubbles in our model, we use the cash-flow process in (1) and the price process in (11) to simulate a  $T = 40,000$ -period price sequence and record the number of bubbles for which the level of overvaluation exceeds a threshold such as 10 or 20, and also the length of time for which this threshold is exceeded.

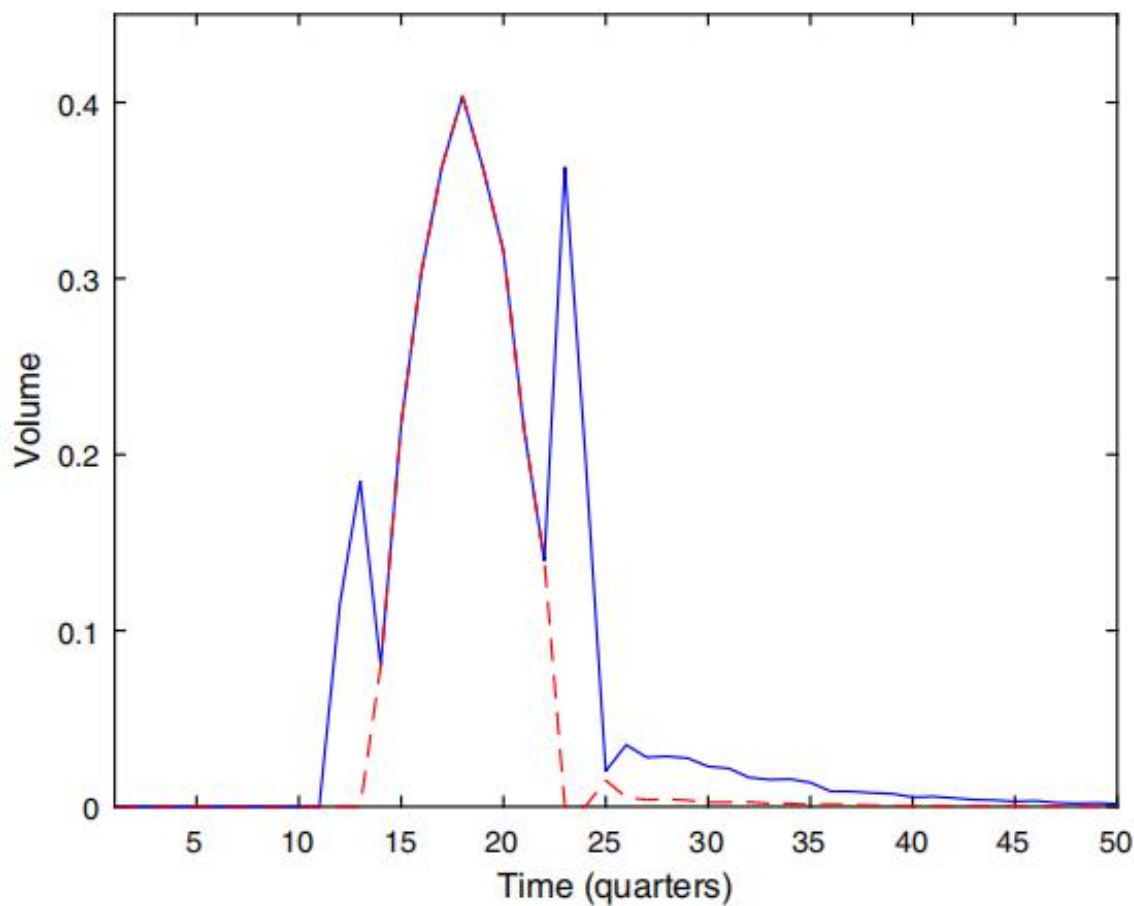
In our model, large bubbles are rare. For our benchmark parameter values, a bubble whose size exceeds 10 occurs once every 17 years, on average, with the overvaluation exceeding 10 for approximately one year. A bubble of size 20 occurs just once every 50 years, on average, and maintains this size for approximately three quarters.



## 4. Volume in a bubble



$$0.5 \left( \mu_0 |N_t^F - N_{t-1}^F| + \sum_{i=1}^I \mu_i |N_t^{E,i} - N_{t-1}^{E,i}| \right). \quad (15)$$



To understand this, we write the share demand of extrapolator  $i$  in Eq. (10) more simply as

$$\omega_{i,t} V_t + (1 - \omega_{i,t}) G_t$$

where  $V_t$  and  $G_t = X_t / \gamma \sigma 2\varepsilon$  are the value and growth signals, respectively, at time  $t$ .



Proposition 3. Suppose that there is a continuum of extrapolators and that each extrapolator draws an independent weight  $\omega_{i,t}$  at time  $t$  from a bounded and continuous density function  $g(\omega), \omega \in [\omega_l, \omega_h]$ , with mean  $\bar{\omega}$  and with  $0 < \omega_l < \omega_h < 1$ . The sensitivity of per capita wavering-induced trading volume  $V^W(X_t)$  to the growth signal  $X_t$ , denoted by  $\partial V^W(X_t)/\partial X_t$ , is given by

$$\frac{\partial V^W(X_t)}{\partial X_t} = \begin{cases} \frac{\text{sign}(X_t - \gamma\sigma_\varepsilon^2 Q)\bar{\Delta}_0}{(\mu_0 + (1 - \mu_0)\bar{\omega})\gamma\sigma_\varepsilon^2} & \text{for} \\ \quad - \frac{w_l\gamma\sigma_\varepsilon^2 Q}{\mu_0(1 - w_l) + (1 - \mu_0)(\bar{\omega} - w_l)} \\ \quad \leq X_t < \frac{\gamma\sigma_\varepsilon^2 Q}{(1 - \mu_0)(1 - \bar{\omega})} \\ \frac{\bar{\Delta}_0}{\bar{\omega}\gamma\sigma_\varepsilon^2} & \text{for} \\ \quad \frac{\gamma\sigma_\varepsilon^2 Q}{(1 - \mu_0)(1 - \bar{\omega})} \leq X_t \\ \quad \leq \frac{w_h\gamma\sigma_\varepsilon^2 Q}{(w_h - \bar{\omega})(1 - \mu_0)} \end{cases}$$

$$\bar{\Delta}_0 \equiv \int_{w_l}^{w_h} \int_{w_l}^{w_h} |w_1 - w_2| g(w_1)g(w_2) dw_1 dw_2. \quad (17)$$





we simulate a 40,000-period price sequence from the model and extract three subsamples—the subsample where the asset price differs from fundamental value by less than  $\gamma\sigma_\varepsilon^2 Q = 0.9$ ; the subsample where the asset is overvalued by at least  $\gamma\sigma_\varepsilon^2 Q = 0.9$ ; and the subsample where it is overvalued by at least  $10\gamma\sigma_\varepsilon^2 Q = 9$ . We find that in these three subsamples, the correlation between volume at time  $t + 1$  and the price change between  $t - 4$  and  $t$  is  $-0.22$ ,  $0.41$ , and  $0.6$ , respectively.



## 5. Negative bubbles

### ◆ Price

First, our model does not generate “negative” bubbles.

There is no significant undervaluation.

For the fundamental traders to be willing to hold the entire supply of the risky asset, the price has to be lower than the fundamental value in (4).

### ◆ Volume

Our model predicts heavy trading during bubbles, but little trading during severe downturns.

Once the extrapolators leave the market, however, the asset is held only by fundamental traders, a homogeneous group. There is no more trading until the market recovers and extrapolators re-enter.



## 6. Comparison with other bubble models

### 6.1. Rational bubble models

In models of rational bubbles, the price of a risky asset is given by

$$P_t = P_{D,t} + B_t, \quad (18)$$

where  $P_{D,t}$  is the present value of the asset's future cash flows and where  $B_t$ , the bubble component, satisfies

$$B_t = \frac{E(B_{t+1})}{1+r}, \quad (19)$$

where  $r$  is the expected return.



We note four points.

- First, the rational bubble model does not explain how a bubble gets started in the first place.
- Second, the rational bubble model has nothing to say about trading volume.
- Third, the rational bubble model does not capture the extrapolative expectations that are often observed during bubbles.
- Finally, direct tests of the key prediction of rational bubble models—that payoffs in the infinite future have positive present value—reject it.



## 6.2. Disagreement-based models

Scheinkman and Xiong (2003) present a model in which two risk-neutral investors observe two signals about the fundamental value of a risky asset, but disagree about how useful each signal is.

- In Scheinkman and Xiong (2003), this increase in disagreement is exogenous. In our model, disagreement grows endogenously over the course of the bubble.
- In our model, many investors hold expectations that depend positively on past returns. In Scheinkman and Xiong (2003), however, the holder of the asset has constant expectations about the asset's future return.
- Our framework also predicts a strongly positive correlation between volume and past returns during bubble episodes. In Scheinkman and Xiong (2003), this correlation is close to zero.



# 7. Some evidence

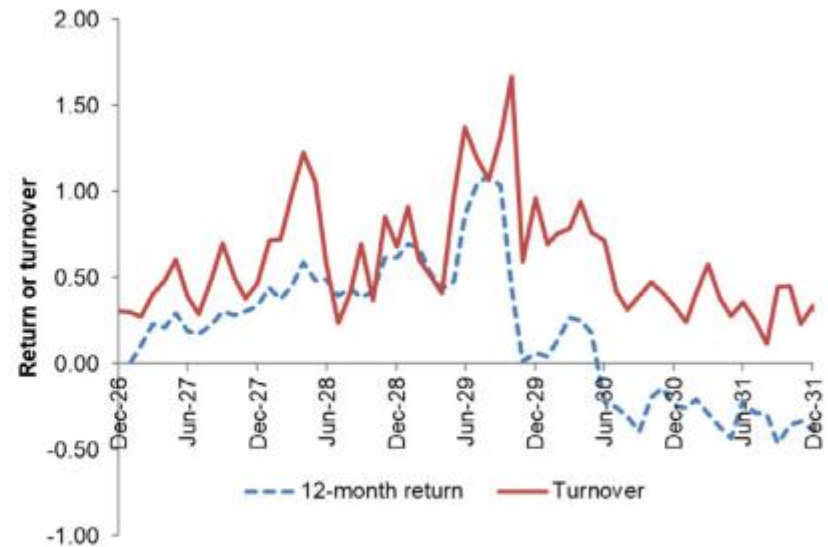
## 7.1. Volume and past returns

For four bubble episodes—the U.S. stock market in 1929, technology stocks in 1998–2000, U.S. housing in 2004–2006, and commodities in 2007–2008—we check whether, as predicted by our model, the correlation between volume and past return for the asset in question is higher during the bubble period than during the two-year period that follows the bubble’s collapse.





Panel A: Bubble of 1929, stock prices of utilities



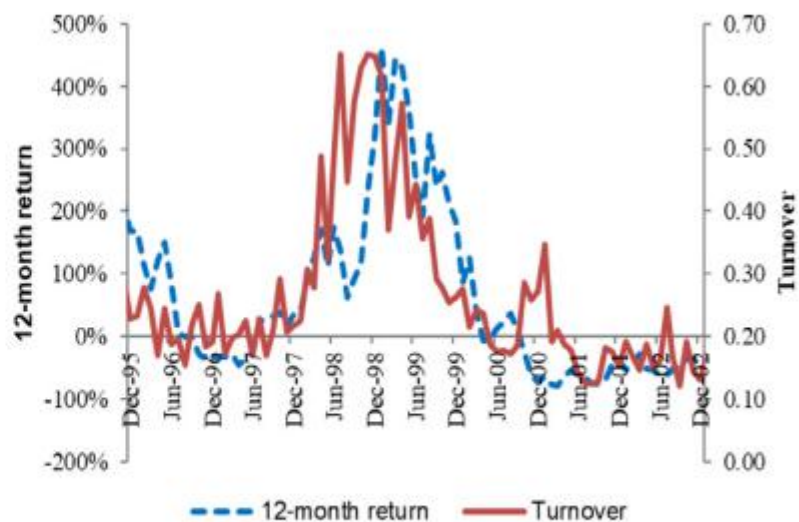
Panel B: Bubble of 1929, 12-month returns and turnover of utilities

- It compares the value-weighted cumulative return of public utilities listed in the CRSP data with the cumulative return of the broader stock market. Utilities outperformed the broader stock market by more than 80% in the March 1928–September 1929 period.
- Panel B of Fig. 6 plots the value-weighted monthly turnover of utility stocks over this period alongside their value-weighted 12-month past return. From January 1927 to December 1930, the correlation between turnover and the 12-month past return is 0.59. Over the two-year period after the bubble ends—from January 1931 to December 1932—the correlation is  $-0.03$ .





Panel C: Technology stock bubble, stock prices

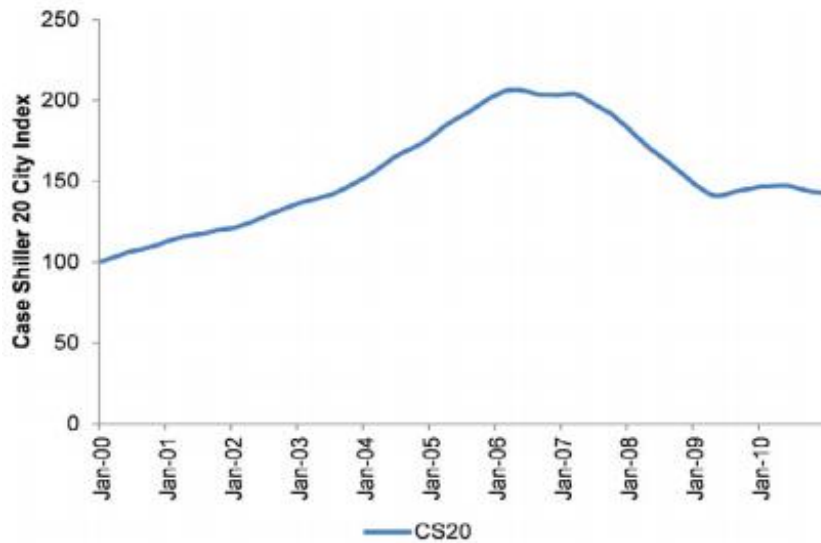


Panel D: Technology stock bubble, 12-month returns and turnover

- Panel C plots value-weighted monthly cumulative returns for the sample of .com stocks used by Ofek and Richardson (2003) and compares them to the cumulative returns of the CRSP value-weighted stock market index.
- The figure shows that a time-series correlation between the two of 0.73 between January 1998 and December 2002. In the 24-month post-bubble period from January 2003 to December 2004, the correlation is  $-0.14$ .







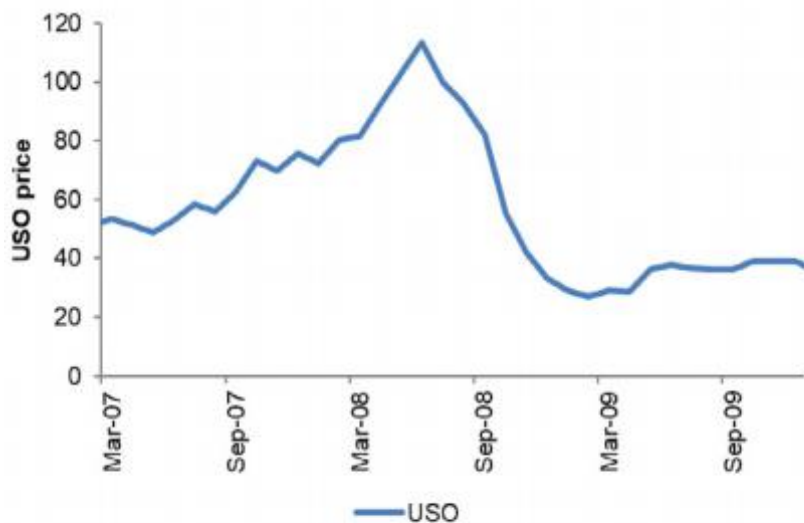
Panel E: Housing bubble of 2004–2006, price index



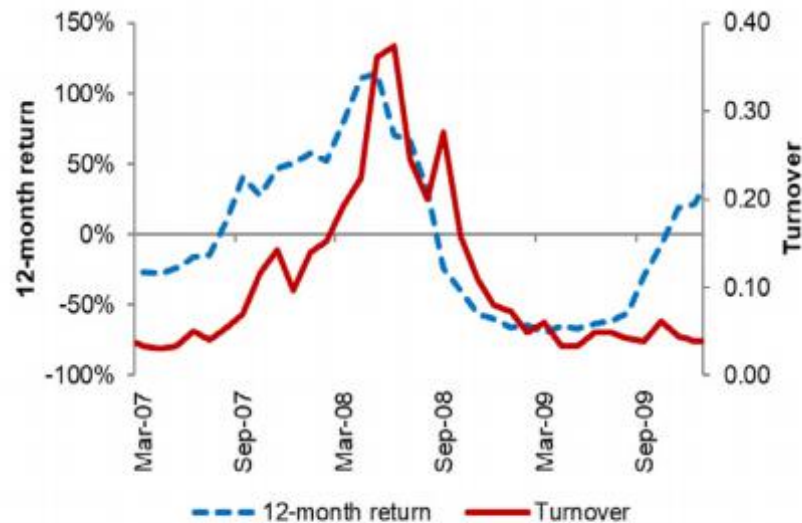
Panel F: Housing bubble of 2004–2006, 12-month returns and volume

- In Panel E, we plot the Case–Shiller 20-City Composite Home Price Index. The Case–Shiller Index rises from a base value of 100 in January 2000 to a peak of 206.61 in April 2006.
- In Panel F, we show the relationship between 12-month past returns and volume for the U.S. housing market. The figure shows that their time series correlation in monthly data from January 2003 to December 2008 is 0.96. This is higher than the correlation in the two-year post-bubble period from January 2009 to December 2010, namely, 0.2.





Panel G: 2007–2008 Commodity bubble, USO price



Panel H: 2007–2008 Commodity bubble, 12-month returns and turnover of USO

- Panel G shows the run-up in oil prices as reflected in the share price of United States Oil (USO) Fund. USO more than doubled between December 2006 and June 2008.
- In Panel H, we plot the monthly turnover and 12-month past return of this ETF. The turnover of USO closely tracks the past return; their time series correlation between April 2007 and December 2009 is 0.83. During the two-year post-bubble period, the correlation is 0.15.



## 7.2. The source of trading volume in a bubble

For technology stock  $i$  in quarter  $t$ , we compute a measure of extrapolator-weighted trading volume, namely,

$$\begin{aligned} & \text{Volume\_Mom}_{i,t} \\ &= \frac{\sum_j \text{Buys}_{i,j,t} \text{Fundmom}_{j,t-2} + \sum_j \text{Sells}_{i,j,t} \text{Fundmom}_{j,t-2}}{\sum_j \text{Buys}_{i,j,t} + \sum_j \text{Sells}_{i,j,t}}, \end{aligned} \quad (20)$$

where  $j$  indexes the mutual funds trading stock  $i$  in quarter  $t$ , so that  $\text{Buys}_{i,j,t}$  and  $\text{Sells}_{i,j,t}$  are the dollar buys and sells, respectively, of stock  $i$  by fund  $j$  in quarter  $t$ .

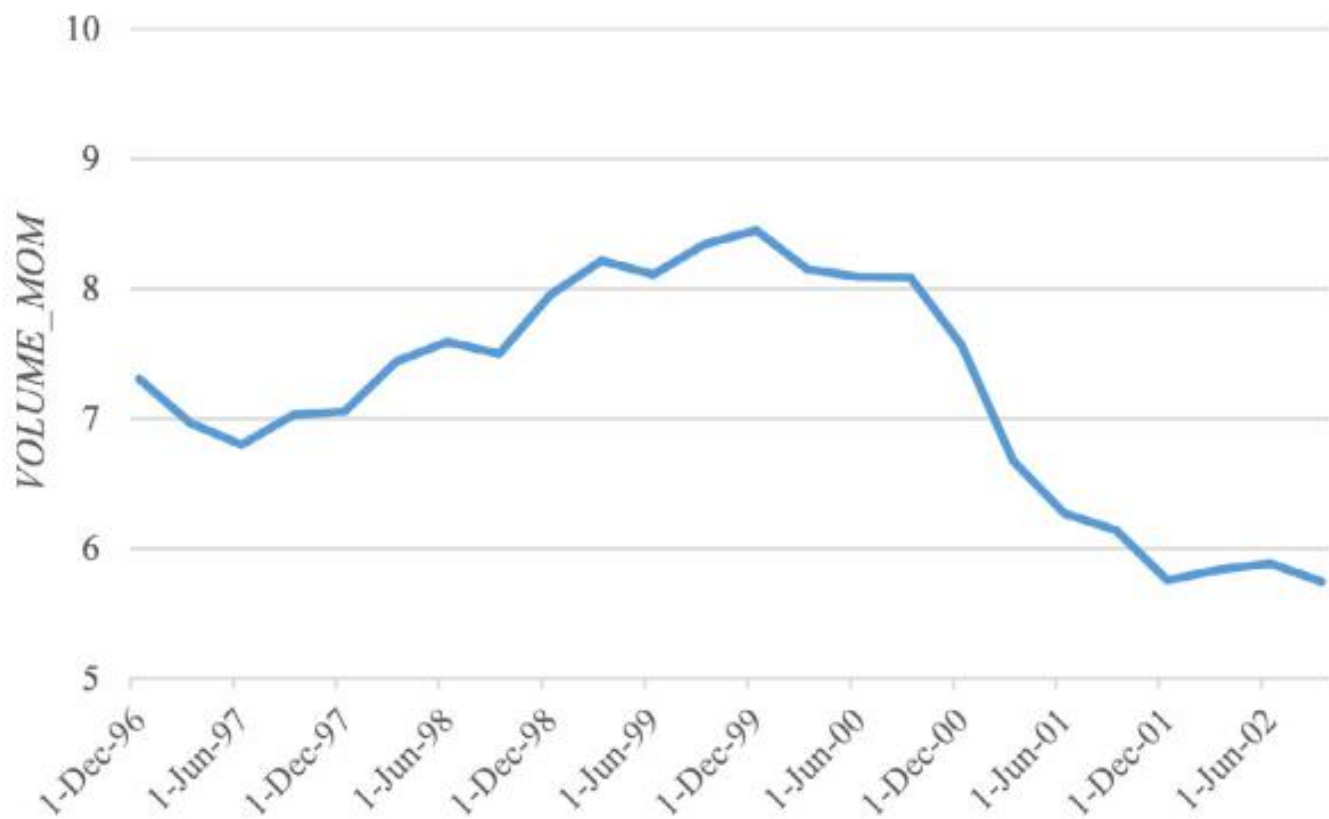


The growthiness of the fund's portfolio at time  $t - 2$ ,  $\text{Fundmom}_{j,t-2}$ , is then measured as the position-weighted past-return decile of the stocks in the portfolio:

$$\text{Fundmom}_{j,t-2} = \sum_i w_{i,j,t-2} \text{stockmom}_{i,t-2}, \quad (21)$$

where  $\text{stockmom}$  takes an integer value between 1 and 10.





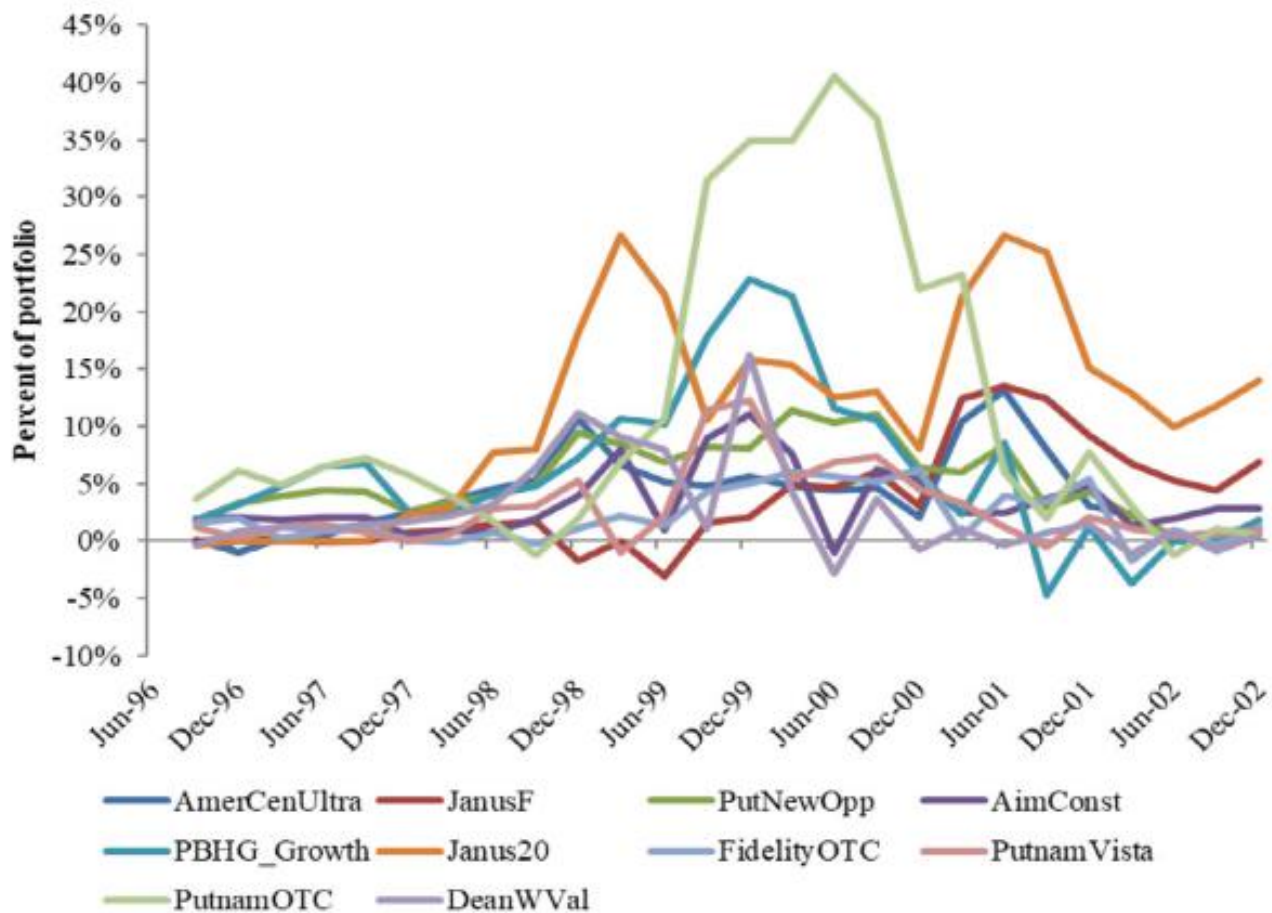
### 7.3. Evidence of wavering

For each mutual fund in our sample, we compute its maximum dollar exposure to the Internet sector between 1996 and 2000, and focus on the ten funds with the highest maximum exposure.

for each of the fund's positions in each quarter, we compute the change in the value of the position due to trading as a percentage of the total value of the fund's long positions that quarter; we denote this by  $\Delta w$ .

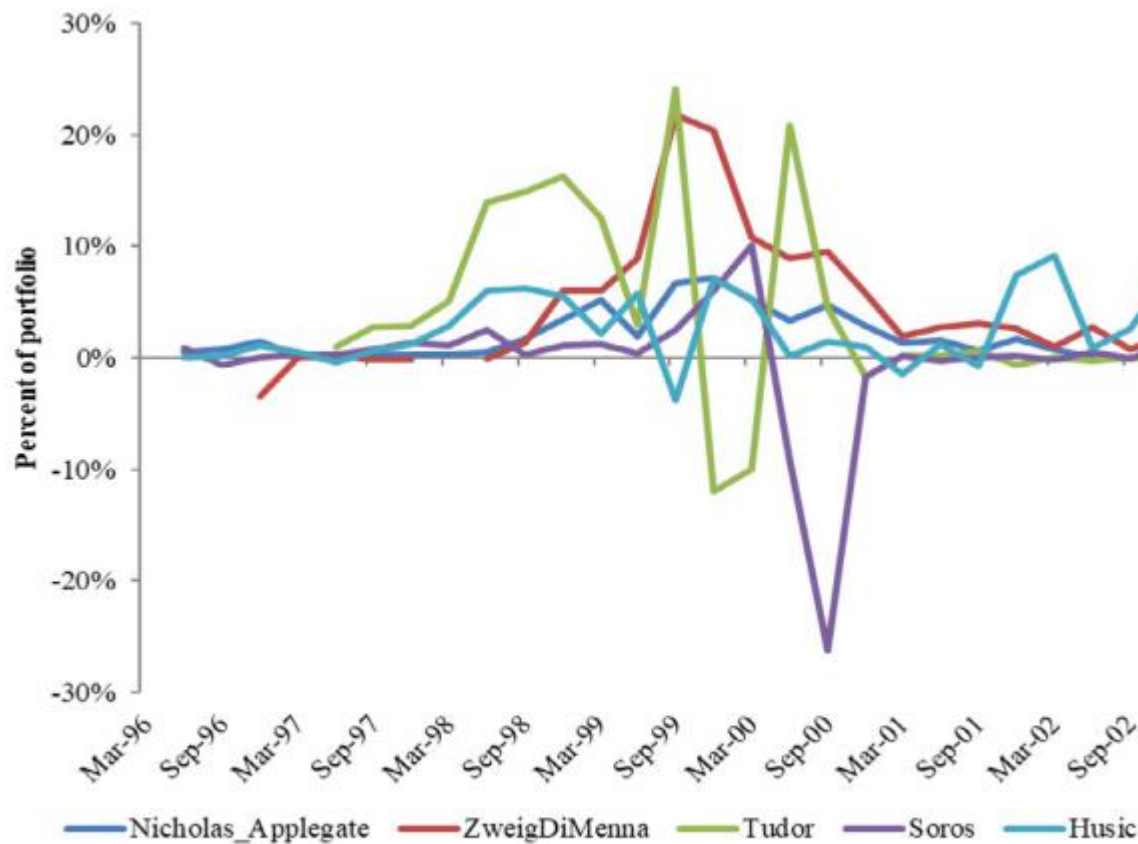
In each quarter, we sum  $\Delta w$  up over all Internet stocks held by the fund, with positive values indicating an active increase in exposure to these stocks and negative values an active decrease in exposure.





Panel A: Mutual fund net purchases





Panel B: Hedge fund net purchases

We take the five hedge funds in the Brunnermeier and Nagel (2004) sample with the highest maximum dollar exposure to the Internet sector between 1996 and 2000. For each of the five funds, we compute the time series of  $\Delta w$ .





## 8. Conclusion

- Historical accounts of price bubbles typically emphasize extrapolative expectations. In this paper, we embrace it.
- In our model, some investors hold extrapolative expectations, but also waver in their convictions in that they worry more or less about the possible overvaluation of the asset.
- The model generates occasional bubbles in asset prices. Such bubbles occur in response to particular patterns of good news.
- They are characterized by very high trading volume documented in earlier literature, which to a significant extent comes from trading between the wavering extrapolators.
- The model generates a new prediction that trading volume is driven by high past returns.

