On the performance of volatility-managed portfolios

JFE 2020（10）

Scott Cederburg[a], Michael S. O'Doherty[b,*], Feifei Wang[c], Xuemin (Sterling) Yan[d]

Volatility-managed portfolios

JF 2017（10）

ALAN MOREIRA and TYLER MUIR*

汇报人：周洁

2021.10.13

1.Volatility-managed portfolios

Alan Moreira

Current Appointments

1.University of Rochester罗彻斯特大学, Simon Graduate School of Business，Associate Professor of Finance (without tenure) 2017-； Yale School

2.National Bureau of Economic Research (NBER)，
Faculty Research Fellow, Asset Pricing， 2021-

Education

Ph.D. Financial Economics, 2006-2011, University of Chicago, Booth School of Business and Department of Economics

M.A. Economics, Pontificia Universidade Catolica do Rio de Janeiro (天主教教皇大学PUC-RIO), Brazil

B.S. Industrial Engineering， Universidade Federal do Rio de Janeiro (里约热内卢联邦大学UFRJ), Brazil

- Should Long-Term Investors Time Volatility? (joint with Tyler Muir)， Journal of Financial Economics, March 2019
- Hedging Risk Factors (joint with Tyler Muir and Bernard Herskovic)
- Liquidity Creation as Volatility Risk (joint with Itamar Drechsler and Alexi Savov)
- Hedge funds, long-term opportunities, and optimal lockups锁仓 (with Juhani Linnainmaa)

Tyler Muir

Current Appointments

1. UCLA加州大学洛杉矶分校, Anderson School of Management Assistant Professor of Finance, 2016 – ； Yale School

2.National Bureau of Economic Research (NBER) ， Faculty Research Fellow, Asset Pricing， 2016-

Education

Ph.D. Financial , 2013, Northwestern University, Kellogg School of Management

B.A. Mathematics , University of California Berkeley加州大学伯克利分校

research interest

Asset Pricing, Financial Intermediation, Financial Crises

- “Financial Intermediaries and the Cross Section of Asset Returns” with Tobias Adrian and Erkko Etula. Journal of Finance 2014, 69(6): 2557-2596.
- “How Credit Cycles across a Financial Crisis” with Arvind Krishnamurthy (2017).
- “Do Intermediaries Matter for Aggregate Asset Prices?” with Valentin Haddad (2017).

Scott Cederburg

Current Appointments

1. Associate Professor of Finance, University of Arizona亚利桑那大学, (2011)2018–Present
2. Sheafe/Neill/Estes Fellow in Finance, University of Arizona, 2018–Present

Education

Ph.D. Financial , 2011, Business Administration (Finance), University of Iowa爱荷华大学, 2011
MBA, 2005 , University of Nebraska – Lincoln内布拉斯加大学 林肯分校
BSBA (Finance and Accounting),2003 , University of Nebraska – Lincoln

- Stocks for the Long Run? Evidence from a Broad Sample of Developed Markets (with Aizhan Anarkulova and Michael S. O'Doherty七八篇), forthcoming, Journal of Financial Economics.
- Is "Not Trading" Informative? Evidence from Corporate Insiders' Portfolios (with Luke DeVault and Kainan Wang), 2021
- On the Economic Significance of Stock Return Predictability (with Travis L. Johnson and Michael S. O'Doherty), 2021

Michael S. O’Doherty

Current Appointments

1.University of Missouri密苏里大学， Associate Professor of Finance， 2016-
Assistant Professor of Finance， 2011-2016

2. Charles Jones Russell Distinguished Professor of Finance， 2018-

Education

Ph.D. Financial , 2011, University of Iowa爱荷华大学

B.S. Mathematics , 2004， Chemical Engineering and Finance, Iowa State University

- Stocks for the long run? Evidence from a broad sample of developed markets,2021, Forthcoming in Journal of Financial Economics.
- Understanding the risk-return relation: The aggregate wealth proxy actually matters,2019 , Journal of Business & Economic Statistics 37 (4), 721-735.
- Does it pay to bet against beta? On the conditional performance of the beta anomaly,2016, Journal of Finance 71 (2), 737-774.

Feifei Wang

Current Appointments

Miami University, Oxford, OH迈阿密大学（Ohio牛津），Assistant Professor of Finance, 2017-

Education

- University of Missouri密苏里大学, Columbia, MO

Ph.D. in Finance, 2017；M.A. in Statistics, 2012；M.S. in Personal Financial Planning, 2010

- Beijing Forestry University, China

B.S. in Finance (Minor in Computer Science), 2008

- Feifei Wang, and Xuemin (Sterling) Yan, 2021, Downside Risk and the Performance of Volatility-Managed Portfolios, Forthcoming in Journal of Banking and Finance.
- Feifei Wang, Xuemin (Sterling) Yan, and Lingling Zheng, 2021, Should Mutual Fund Investors Time Volatility? Financial Analyst Journal 77(1), 30-42.
- Arbitrage Asymmetry, Mispricing, and the Illiquidity Premium (with Xuemin (Sterling) Yan and Lingling Zheng)
- Momentum Timing (with Xuemin (Sterling) Yan and Lingling Zheng)

XUEMIN (STERLING) YAN

Current Appointments

Lehigh University里海大学， Bethlehem, Perella Chair and Professor of Finance, 2019 -

University of Missouri密苏里大学， Columbia， Richard G. Miller Professor of Finance, 2008 -2019

Education

Ph.D. , Finance, 2001, University of Iowa爱荷华大学

Graduate studies in economics,1995-1996, Indiana State University

B.A., Finance,1991, Renmin University of China

research interest

asset pricing, institutional investors, mutual funds, hedge funds, short selling, and liquidity

- Financial Industry Affiliation and Hedge Fund Performance, (with Lingling Zheng), Forthcoming in Management Science, forthcoming in Management Science.
- Should Mutual Fund Investors Time Volatility?(with Feifei Wang, Lingling Zheng), Financial Analysts Journal, forthcoming in Financial Analysts Journal.

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1.Volatility-managed portfolios

Volatility-managed portfolios？？

$$f_{t+1}^{\sigma} = \frac{c}{\hat{\sigma}_t^2(f)} f_{t+1}, \tag{1}$$

$$\hat{\sigma}_t^2(f) = RV_t^2(f) = \sum_{d=1/22}^{1} \left(f_{t+d} - \frac{\sum_{d=1/22}^{1} f_{t+d}}{22} \right)^2. \tag{2}$$

$$f_{t+1}^{\sigma} = \alpha + \beta f_{t+1} + \epsilon_{t+1}. \tag{3}$$

Abstract

- Managed portfolios that take less risk when volatility is high produce large alphas, increase Sharpe ratios, and produce large utility gains for mean-variance investors.
- We document this for the market, value, momentum, profitability, return on equity, investment, and betting-against-beta factors, as well as the currency carry trade.
- Volatility timing increases Sharpe ratios because changes in volatility are not offset by proportional changes in expected returns.
- Our strategy is contrary to conventional wisdom because it takes relatively less risk in recessions. This rules out typical risk-based explanations and is a challenge to structural models of time-varying expected returns.

Abstract

- Managed portfolios that take less risk when volatility is high produce large alphas, increase Sharpe ratios, and produce large utility gains for mean-variance investors.
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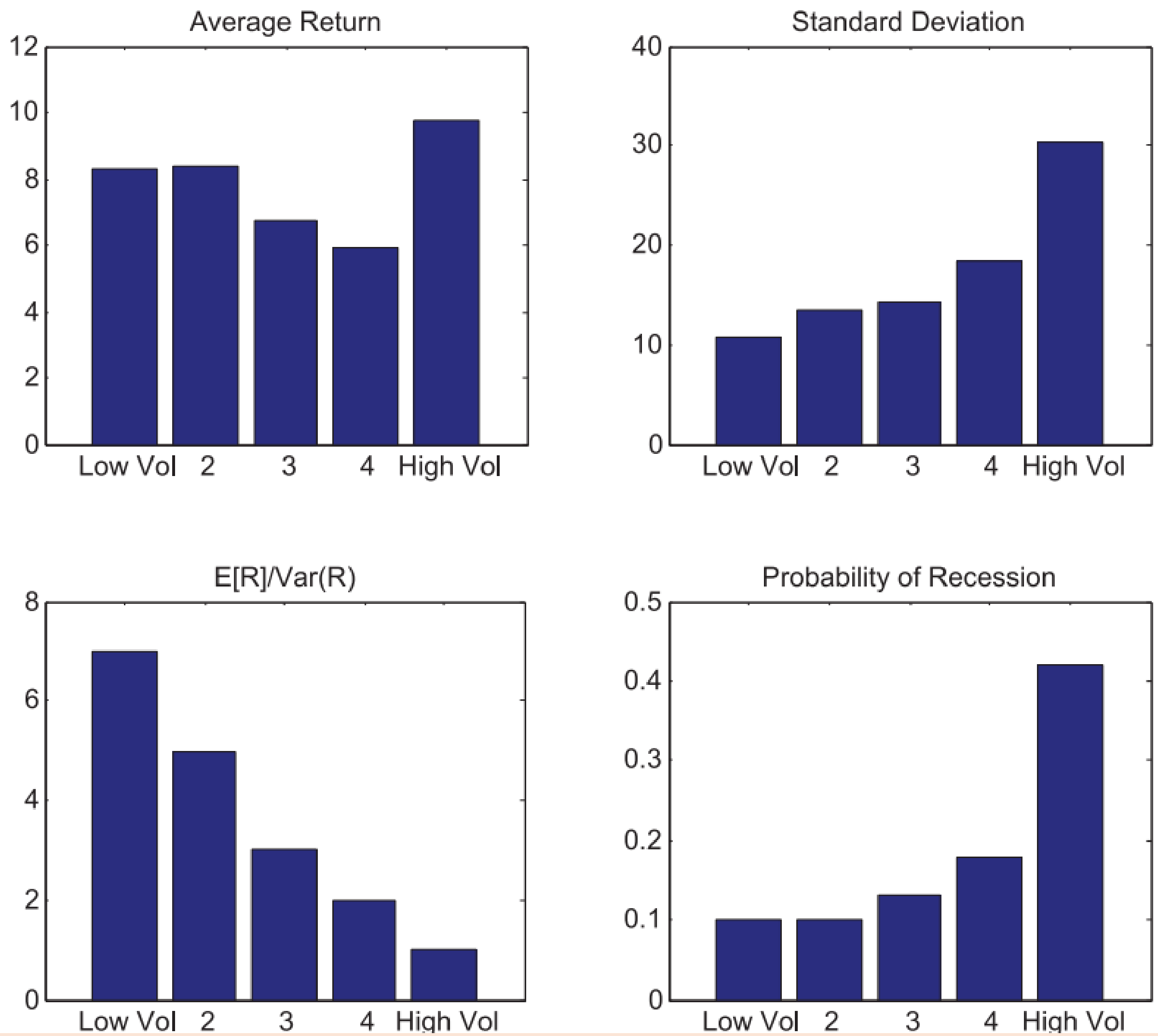
- Volatility timing increases Sharpe ratios because changes in volatility are not offset by proportional changes in expected returns.

Volatility-managed portfolios

- 波动管理后单因素组合
  volatility-managed Single-Factor Portfolios
- 波动管理后有效前沿组合
  volatility-managed mean-variance efficient MVE frontier (or multifactor)Portfolios
- 波动管理后组合
  volatility-managed “combination” strategies

background

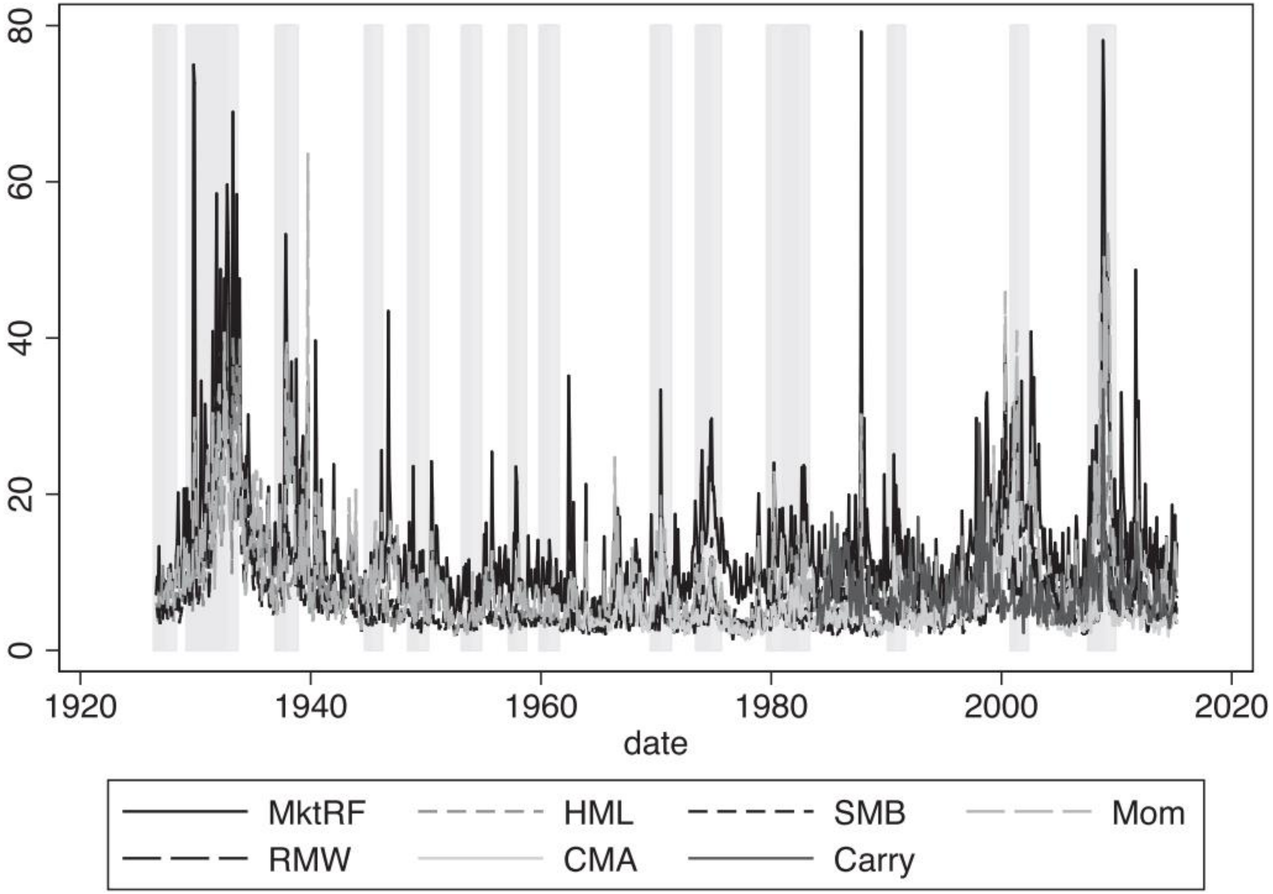


For the market portfolio， our strategy produces an alpha of 4.9%, and an overall 25% increase in the buy-and-hold Sharpe ratio.

We group months by the previous month’s realized volatility and plot average returns, volatility , and the mean-variance trade-off over the subsequent month

A mean-variance investor should time volatility, that is, take more risk when the mean-variance trade-off is attractive (volatility is low), and take less risk when

The motivation for this strategy comes from the portfolio problem of a mean-variance investor who is deciding how much to invest in a risky portfolio.


0
20
40
60
80
1920
1940
1960
1980
2000
2020
date
MktRF
HML
SMB
Mom
RMW
CMA
Carry

frame

- Section I Documents our main empirical results
- Section II Understanding the Profitability of Volatility Timing
- Section III Theoretical Framework
- Section IV General Equilibrium Implications
- Section V Concludes.

A. Data Description

daily and monthly data(Mkt, SMB, HML, MOM, RMW, and CMA)

- Kenneth French's website

IA and ROE

- Hou, Xue, and Zhang(2014)

BAB factor

- Frazzini and Pedersen (2014)

currency returns

- Lustig, Roussanov, and Verdelhan (2011)

B. Portfolio Formation

$$f_{t+1}^{\sigma} = \frac{c}{\hat{\sigma}_t^2(f)} f_{t+1}, \quad (1)$$

$$\hat{\sigma}_t^2(f) = RV_t^2(f) = \sum_{d=1/22}^{1} \left(f_{t+d} - \frac{\sum_{d=1/22}^{1} f_{t+d}}{22} \right)^2 . \quad (2)$$

C. Empirical Methodology

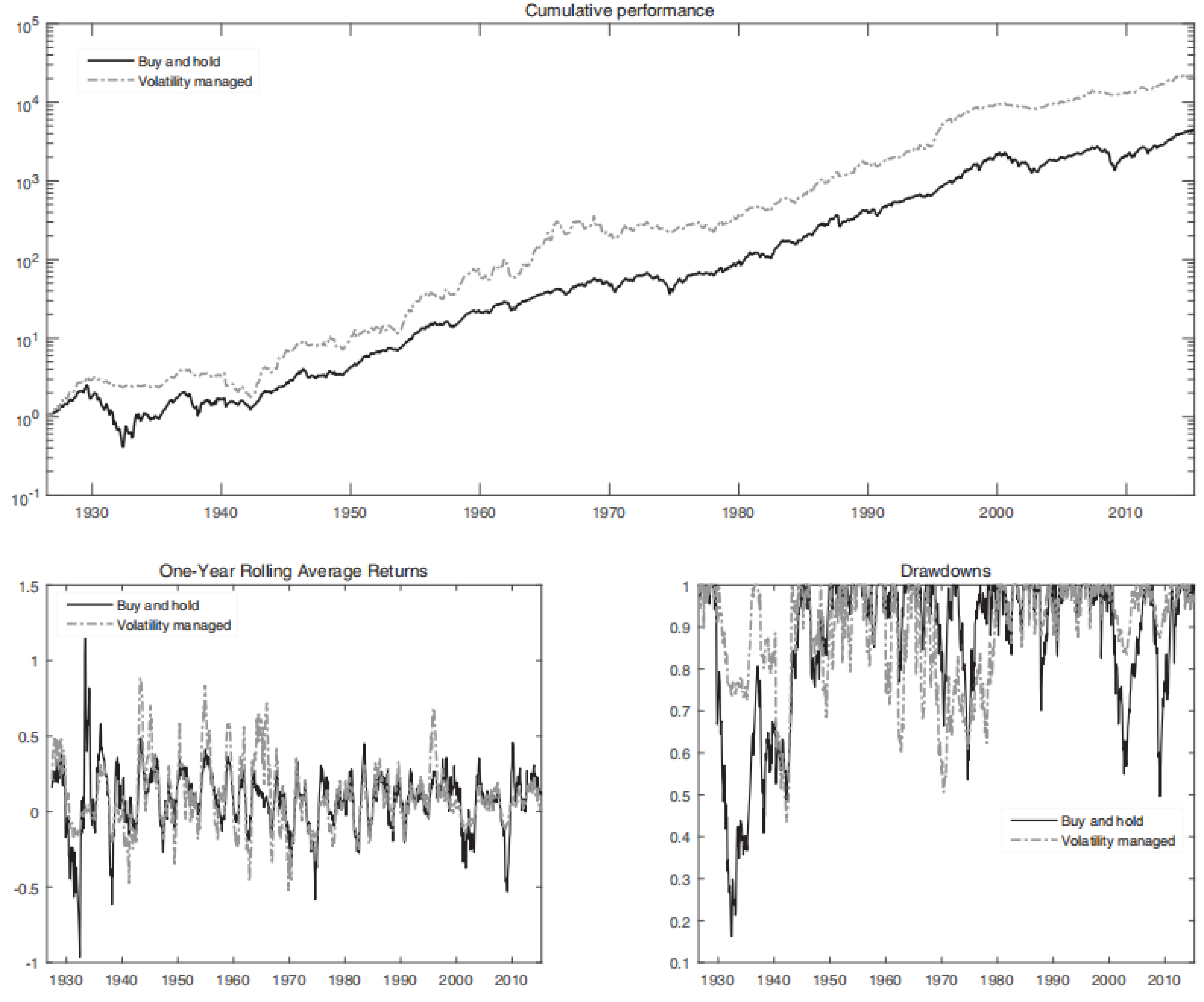
$$f_{t+1}^{\sigma} = \alpha + \beta f_{t+1} + \epsilon_{t+1}. \quad (3)$$

$$SR_{new} = \sqrt{SR_{old}^2 + (\frac{\alpha}{\sigma_\varepsilon})^2},$$ σ_ε为RMSE，均方根误差

	(1) Mkt^{σ}
MktRF	0.61
	(0.05)
SMB	
HML	
Mom	
RMW	
CMA	
Carry	
ROE	
IA	
BAB	
Alpha (α)	4.86
	(1.56)
N	1,065
R^2	0.37
RMSE	51.39

Pa

Alpha (α)	5.45
	(1.56)



D. Single-Factor Portfolios

The worst time periods for our strategy do not overlap much with the worst market crashes.

standard error

Panel A: Mean-Variance Efficient Portfolios (Full Sample)

	(1) Mkt	(2) FF3	(3) FF3 Mom	(4) FF5	(5) FF5 Mom	(6) HXZ	(7) HXZ Mom
Alpha (α)	4.86	4.99	4.04	1.34	2.01	2.32	2.51
	(1.56)	(1.00)	(0.57)	(0.32)	(0.39)	(0.38)	(0.44)
Observations	1,065	1,065	1,060	621	621	575	575
R^2	0.37	0.22	0.25	0.42	0.40	0.46	0.43
RMSE	51.39	34.50	20.27	8.28	9.11	8.80	9.55
Original Sharpe	0.42	0.52	0.98	1.19	1.34	1.57	1.57
Vol-Managed Sharpe	0.51	0.69	1.09	1.20	1.42	1.69	1.73
Appraisal Ratio	0.33	0.50	0.69	0.56	0.77	0.91	0.91

Panel B: Subsample Analysis

	(1) Mkt	(2) FF3	(3) FF3 Mom	(4) FF5	(5) FF5 Mom	(6) HXZ	(7) HXZ Mom
α: 1926–1955	8.11	1.94	2.45				
	(3.09)	(0.92)	(0.74)				
α: 1956–1985	2.06	0.99	2.54	0.13	0.71	0.77	1.00
	(2.82)	(1.43)	(1.16)	(0.43)	(0.67)	(0.39)	(0.51)
α: 1986–2015	4.22	5.66	4.98	1.88	2.65	3.03	3.24
	(1.66)	(1.74)	(0.95)	(0.41)	(0.47)	(0.50)	(0.57)

E. Multifactor Portfolios

$$f_{t+1}^{MVE,\sigma} = \frac{c}{\hat{\sigma}_t^2\left(f_{t+1}^{MVE}\right)} f_{t+1}^{MVE}, \tag{5}$$

MVE alpha is the right measure of expansion in the mean-variance frontier.

if volatility were constant over a particular period, our strategy would be identical to the buy-and-hold strategy and alphas would be zero

$$\alpha = -cov\left(\frac{\mu_t}{\sigma_t^2}, \sigma_t^2\right)\frac{c}{E\left[\sigma_t^2\right]}$$

standard error

A. Business Cycle Risk

B. Transaction Costs

C. Leverage Constraints

D. Contrasting with Cross-Sectional Low-Risk Anomalies

E. Volatility Comovement

F. Horizon Effects

A. Business Cycle Risk

	(1) Mkt$^\sigma$	(2) HML$^\sigma$	(3) Mom$^\sigma$	(4) RMW$^\sigma$	(5) CMA$^\sigma$	(6) FX$^\sigma$	(7) ROE$^\sigma$	(8) IA$^\sigma$
MktRF	0.83							
	(0.08)							
MktRF $\times 1_{rec}$	−0.51							
	(0.10)							
HML		0.73						
		(0.06)						
HML $\times 1_{rec}$		−0.43						
		(0.11)						
Mom			0.74					
			(0.06)					
Mom $\times 1_{rec}$			−0.53					
			(0.08)					
RMW				0.63				
				(0.10)				
RMW $\times 1_{rec}$				−0.08				
				(0.13)				
CMA					0.77			
					(0.06)			
CMA $\times 1_{rec}$					−0.41			
					(0.07)			
Carry						0.73		
						(0.09)		
Carry $\times 1_{rec}$						−0.26		
						(0.15)		
ROE							0.74	
							(0.08)	
ROE $\times 1_{rec}$							−0.42	
							(0.11)	
IA								0.77
								(0.07)
IA $\times 1_{rec}$								−0.39
								(0.08)
Observations	1,065	1,065	1,060	621	621	362	575	575
R^2	0.43	0.37	0.29	0.38	0.49	0.51	0.43	0.49

$$f_t^\sigma = \alpha_0 + \alpha_1 1_{rec,t} + \beta_0 f_t + \beta_1 1_{rec,t} \times f_t + \varepsilon_t$$

- The results in the table show that, across the board for all factors, our strategies take less risk during recessions and thus have lower betas during recessions.(如图3)
- Thus, our strategies decrease risk exposure in NBER recessions.

- This makes it difficult for a business cycle risk story to explain our facts. However, we still review several specific risk-based stories below.

w	Description	月度 $\vert \Delta w \vert$	$E[R]$	α	α After Trading Costs 年度 1bps	10bps	14bps	Break Even
$\frac{1}{RV_t^2}$	Realized variance	0.73	9.47%	4.86%	4.77%	3.98%	3.63%	56bps
$\frac{1}{RV_t}$	Realized vol	0.38	9.84%	3.85%	3.80%	3.39%	3.21%	84bps
$\frac{1}{E_t[RV_{t+1}^2]}$	Expected variance	0.37	9.47%	3.30%	3.26%	2.86%	2.68%	74bps
$min(\frac{c}{RV_t^2}, 1)$	No leverage	0.16	5.61%	2.12%	2.10%	1.93%	1.85%	110bps
$min(\frac{c}{RV_t^2}, 1.5)$	50% leverage	0.16	7.18%	3.10%	3.08%	2.91%	2.83%	161bps

We report the average absolute change in monthly weights, expected return, and alpha of each strategy before transaction costs. We then report the alpha when including various transaction cost assumptions.

We do not report results for all factors, since we do not have good measures of transaction costs for implementing the original factors, much less their volatility-managed portfolios.

Panel A: Weights and Performance for Alternative Volatility-Managed Portfolios

					Distribution of Weights w			
w_t	Description	α	Sharpe	Appraisal	P50	P75	P90	P99
$\frac{1}{RV_t^2}$	Realized variance	4.86 (1.56)	0.52	0.34	0.93	1.59	2.64	6.39
$\frac{1}{RV_t}$	Realized volatility	3.30 (1.02)	0.53	0.33	1.23	1.61	2.08	3.36
$\frac{1}{E_t[RV_{t+1}^2]}$	Expected variance	3.85 (1.36)	0.51	0.30	1.11	1.71	2.38	4.58
$min(\frac{c}{RV_t^2}, 1)$	No leverage	2.12 (0.71)	0.52	0.30	0.93	1	1	1
$min(\frac{c}{RV_t^2}, 1.5)$	50% leverage	3.10 (0.98)	0.53	0.33	0.93	1.5	1.5	1.5

Panel B: Embedded Leverage Using Options: 1986–2012

			Vol Timing with Embedded Leverage	
	Buy and Hold	Vol Timing	Calls	Calls + Puts
Sharpe Ratio	0.39	0.59	0.54	0.60
α	–	4.03	5.90	6.67
$s.e.(\alpha)$	–	(1.81)	(3.01)	(2.86)
β	–	0.53	0.59	0.59
Appraisal Ratio	–	0.44	0.39	0.46

We consider various strategies that capture volatility timing but reduce trading activity, including using standard deviation instead of variance, using expected rather than realized variance, and two strategies that cap the strategy’s leverage at 1 and 1.5.

In Panel B of Table V, we compare the strategy implemented with options to the strategy implemented with leverage.

The Alphas and SR are very similar, which suggests that our results are not due to leverage constraints, even for investors with relatively low risk-aversion.

- The first strategy is risk parity, we keep the relative weights of all factors constant and only increase or decrease overall risk exposure based on total volatility.
- The second strategy is the betting-against-beta (BAB) factor

Table VI

Time-Series Alphas Controlling for Risk Parity Factors

In this table, we run time-series regressions of each volatility-managed factor on the nonmanaged factor plus a risk parity factor based on Asness, Frazzini, and Pedersen (2012). The risk parity factor is given by $RP_{t+1} = b_t' f_{t+1}$, where $b_{i,t} = \frac{1/\tilde{\sigma}_t^i}{\sum_i 1/\tilde{\sigma}_t^i}$ and f is a vector of pricing factors. Volatil-

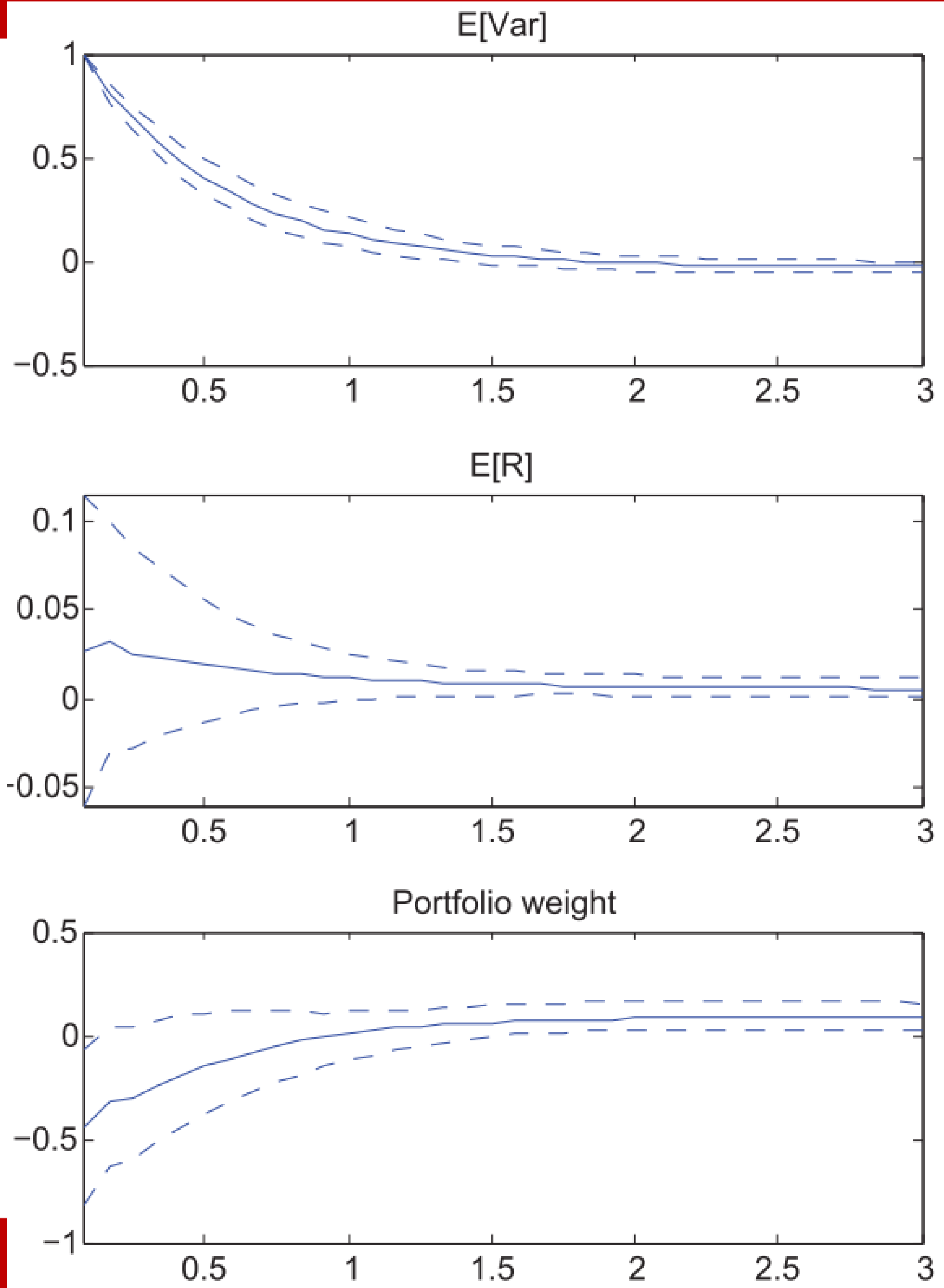
	(1) Mkt	(2) FF3	(3) FF3 Mom	(4) FF5	(5) FF5 Mom	(6) HXZ	(7) HXZ Mom	(8) BAB^{σ}
Alpha (α)	4.86	5.00	4.09	1.32	1.97	2.03	2.38	5.67
	(1.56)	(1.00)	(0.57)	(0.31)	(0.40)	(0.32)	(0.44)	(0.98)
N	1,065	1,065	1,060	621	621	575	575	996
R^2	0.37	0.23	0.26	0.42	0.40	0.50	0.44	0.33
RMSE	51.39	34.30	20.25	8.279	9.108	8.497	9.455	29.73

Thus, our time-series volatility-managed portfolios are distinct from the low-beta anomaly documented in the cross-section.

Normalizing by Common Volatility

In this table, we construct volatility-managed strategies for each factor using the first principal component of realized variance across all factors. Each factor is thus normalized by the same variable, in contrast to our main results, where each factor is normalized by that factor's past realized variance. We run time-series regressions of each managed factor on the nonmanaged factor. Standard errors are in parentheses and adjust for heteroskedastiticy.

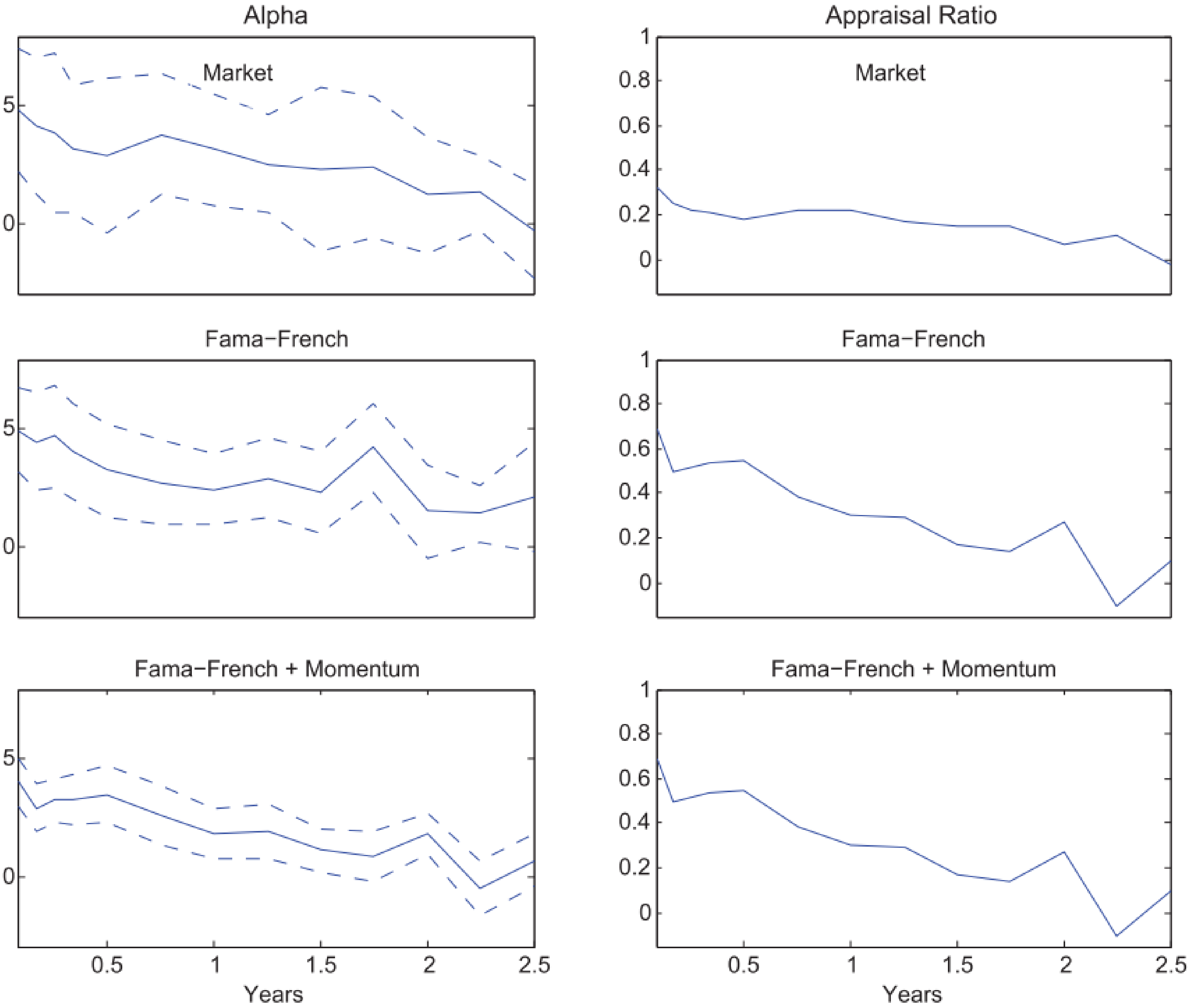
	(1) Mkt^{σ}	(2) SMB^{σ}	(3) HML^{σ}	(4) Mom^{σ}	(5) RMW^{σ}	(6) CMA^{σ}	(7) FX^{σ}	(8) ROE^{σ}	(9) IA^{σ}
Alpha (α)	4.22	0.24	3.09	11.00	1.16	−0.22	−1.28	4.21	1.24
	(1.49)	(0.83)	(0.96)	(1.70)	(0.81)	(0.66)	(1.21)	(1.00)	(0.61)
N	1,061	1,061	1,061	1,060	622	622	362	576	576
R^2	0.42	0.45	0.36	0.33	0.44	0.51	0.64	0.47	0.56
RMSE	49.31	28.74	33.87	46.57	19.11	16.67	18.49	22.13	15.06



- Given the increase in variance but only small and persistent increase in expected return,
- The lower panel shows that it is optimal for the investor to reduce his portfolio exposure by 50% on impact because of an unfavorable risk-return trade-off.
- The portfolio share is consistently below 0 for roughly 12 months after the shock.

F. Horizon Effects

$$SR_{new} = \sqrt{SR^2_{old} + (\frac{\alpha}{\sigma_\varepsilon})^2},$$



$$\frac{c}{\hat{\sigma}^2_t(f_{t+1})} f_{t\to t+T} = \alpha + \beta f_{t\to t+T} + \epsilon_{t+T}$$

Letting $f_{t\to t+T}$ be the cumulative factor excess returns from buying at the end of month T and holding until the end of month t + T

- Alphas are statistically significant for longer holding periods but gradually decline in magnitude.
- For example, for the market portfolio, alphas are statistically different from zero (at the 10% confidence level) for up to 18 months. This same pattern holds for the two MVE portfolios we consider.

1.We start by making the intuitive point that our alphas are proportional to the covariance between variance and the factor price of risk.
2.We then impose more structure to derive aggregate pricing implications.

$$dR_t = (r_t + \mu_t)dt + \sigma_t dB_t$$

$$dR_t^\sigma = r_t dt + \frac{c}{\sigma_t^2}(dR_t - r_t dt)$$

α of a time-series regression of the volatility-managed portfolio excess return $dR_t^\sigma - r_t dt$ on the original portfolio excess return $dR_t - r_t dt$ is given by

$$\alpha = E[dR_t^\sigma - r_t dt]/dt - \beta E[dR_t - r_t dt]/dt.$$

$$f_{t+1}^\sigma = \alpha + \beta f_{t+1} + \epsilon_{t+1}.$$

Using the fact that $E[dR_t^\sigma - r_t dt]/dt = cE[\frac{\mu_t}{\sigma_t^2}]$, $\beta = \frac{c}{E[\sigma_t^2]}$, and $cov(\frac{\mu_t}{\sigma_t^2}, \sigma_t^2) = E[\mu_t] - E[\frac{\mu_t}{\sigma_t^2}]E[\sigma_t^2]$, we obtain a relation between alpha and the dynamics of the price of risk μ_t/σ_t^2,[16]

$$\alpha = -cov\left(\frac{\mu_t}{\sigma_t^2}, \sigma_t^2\right)\frac{c}{E[\sigma_t^2]}.$$

$$f_{t+1}^\sigma = \frac{c}{\hat{\sigma}_t^2(f)} f_{t+1},$$

$$\alpha = -cov\left(\frac{\mu_t}{\sigma_t^2}, \sigma_t^2\right)\frac{c}{\boldsymbol{E}\left[\sigma_t^2\right]}$$

$$\hat{\alpha} = \left(1 + \frac{\mathrm{E}^2(f)}{\mathrm{Var}(f_t)}\right)\mathrm{cov}(w_t, f_t) - \frac{\mathrm{E}(f_t)}{\mathrm{Var}(f_t)}\mathrm{cov}(w_t, f_t^2)$$

山西财经大学 Shanxi University of Finance and Economics

- In this section we show how our vo to systematic risk factors, recover t variation driven by volatility .

This assumption says that unconditional exposures to these factors contain all relevant information to price the static portfolios R, but one may also need information on the price of risk dynamics to properly price dynamic strategies of these assets

Let $dR = [dR_1, \dots, dR_N]'$ be a vector o μ_t^R and covariance matrix Σ_t^R. The em that exposures to a few factors summarize expected return variation for a larger cross-section of assets and strategies captured by dR_t. We formalize our interpretation of this literature as follows:

ASSUMPTION 1: *Let return factors $dF = [dF_1, \dots, dF_I]$, with dynamics given by μ_t and Σ_t, span the unconditional mean-variance frontier for static portfolios of $d\widetilde{R} = [dR; dF_t]$ and the conditional mean-variance frontier for dynamic portfolios of $d\widetilde{R}$. Define the process $\Pi_t(\gamma_t)$ as*

$$\frac{d\Pi_t(\gamma_t)}{\Pi_t(\gamma_t)} = -r_t dt - \gamma_t'(dF_t - E_t[dF_t]). \quad (9)$$

Then there exists a constant price of risk vector γ^u such that $E[d(\Pi_t(\gamma^u)w\widetilde{R})] = 0$ holds for any static weights w, and there is a γ_t^* process for which $E[d(\Pi_t(\gamma_t^*)w_t\widetilde{R})] = 0$ holds for any dynamic weights w_t.

2.We then impose more structure to derive aggregate

- In this section we show how our vol[a…] to systematic risk factors, recover th[e…] variation driven by volatility .

Let $dR = [dR_1, \ldots, dR_N]'$ be a vector of [...] μ_t^R and covariance matrix Σ_t^R. The emp[...] that exposures to a few factors summa[...] larger cross-section of assets and strategi[...] interpretation of this literature as follows[...]

ASSUMPTION 1: *Let return factors $dF =$ [...] by μ_t and Σ_t, span the unconditional me[...] lios of $d\widetilde{R} = [dR; dF_t]$ and the conditional mean-variance frontier for dynamic portfolios of $d\widetilde{R}$. Define the process $\Pi_t(\gamma_t)$ as*

$$\frac{d\Pi_t(\gamma_t)}{\Pi_t(\gamma_t)} = -r_t dt - \gamma_t'(dF_t - E_t[dF_t]). \quad (9)$$

Then there exists a constant price of risk vector γ^u such that $E[d(\Pi_t(\gamma^u)w\widetilde{R})] = 0$ holds for any static weights w, and there is a γ_t^* process for which $E[d(\Pi_t(\gamma_t^*)w_t\widetilde{R})] = 0$ holds for any dynamic weights w_t.

B. *Individual Stocks*

Consider a simple example in which the CAPM holds, and the market portfolio return given by dF_t has constant expected returns and variance. Consider a individual stock with returns $dR_t = (r_t dt + \mu_{R,t})dt + \beta_R(dF_t - E_t[dF_t]) + \sigma_{R,t} dB_{R,t}$, where $dB_{R,t}$ shocks are not priced. We have that the volatility-managed alpha is

$$\alpha_R \propto -cov\left(\sigma^2_{R,t}, \frac{1}{\beta_R^2\sigma_F^2 + \sigma^2_{R,t}}\right), \quad \text{(IA. 2)}$$

which is positive (negative) if $\beta_R > 0$ ($\beta_R < 0$), but CAPM alphas are always zero.

While volatility timing can "work" for any asset with positive expected returns for which volatility is forecastable but does not predict returns, the alphas become economically interesting when studying systematic factors.

We focus on the case in which the factor covariance matrix is diagonal, $\Sigma_t = diag\left([\sigma_{1,t}...\sigma_{I,t}]\right)$ (i.e., factors are uncorrelated), which empirically is a good approximation of the factors we study.[19] In fact, many of the factors are constructed to be nearly orthogonal through double-sorting procedures. Given this structure, we can show how our strategy alphas allow one to recover the component of the price of risk variation driven by volatility.

IMPLICATION 1: *The factor i price of risk is* $\gamma^*_{i,t} = \frac{\mu_{i,t}}{\sigma^2_{i,t}}$ *and* $\gamma^u_i = \frac{E[\mu_{i,t}]}{E[\sigma^2_{i,t}]}$. *Decompose factor excess returns as* $\mu_t = b\Sigma_t + \zeta_t$, *where we assume* $E[\zeta_t|\Sigma_t] = \zeta_t$. *Let* $\gamma^\sigma_{i,t} = E[\gamma^*_{i,t}|\sigma^2_{i,t}]$ *be the component of the price of risk variation driven by volatility, and* α_i *be factor i's volatility-managed alpha. Then*

$J_\sigma = (E[\sigma_t^2]E[\frac{1}{\sigma_t^2}] - 1)$

$$\gamma^\sigma_{i,t} = \gamma^u_i + \frac{\alpha_i}{c_i} J^{-1}_{\sigma,i}\left(\frac{E\left[\sigma^2_{i,t}\right]}{\sigma^2_{i,t}} - 1\right), \qquad (10)$$

and the process $\Pi_t(\gamma^\sigma_t)$ *is a valid Stochastic Discount Factor (SDF) for* $d\widetilde{R}_t$ *and volatility-managed strategies* $w(\Sigma_t)$, *that is,* $E[d(\Pi_t(\gamma^\sigma_t)w(\Sigma_t)\widetilde{R}_t)] = 0.$[20]

[20]Formally, $\gamma^\sigma_t = [\gamma^\sigma_{1,t}...\gamma^\sigma_{I,t}]$, and the strategies $w(\Sigma_t)$ must be adapted to the filtration generated by Σ_t, self-financing, and satisfy $E[\int_0^T ||w(\Sigma_t)\Sigma_t||^2 dt] < \infty$ (see Duffie (2010)).

Now we show that the SDF $\Pi(\gamma_t^\sigma)$ prices all volatility-based strategies. We need to show that $E\left[d\left(\Pi_t(\gamma_t^\sigma)w(\Sigma_t)\widetilde{R}_t\right)\right] = 0$, which is equivalent to

$$\frac{d\Pi_t(\gamma_t)}{\Pi_t(\gamma_t)} = -r_t dt - \gamma_t'(dF_t - E_t[dF_t]).$$

$$E\left[d\left(\Pi_t(\gamma_t^\sigma)w(\Sigma_t)\widetilde{R}_t\right)\right] = E[w(\Sigma_t)\mu_t] - E[\gamma_t^\sigma(dF_t - E_t[])w(\Sigma_t)\widetilde{R}_t] \quad \text{(IA. 5)}$$

Using the fact that factors are on the conditional mean-variance frontier, it is sufficient to show that the expression holds for the factors themselves. Furthermore, it is sufficient to show that the pricing equation holds for each portfolio conditional on Σ_t information.

This yields

$$\begin{aligned} E\left[d\left(\Pi_t(\gamma_t^\sigma)F_t\right)|\Sigma_t\right] &= E[\mu_t|\Sigma_t] - E[\gamma_t^\sigma(dF_t - E_t[dF_t])dF_t|\Sigma_t] && \text{(IA. 6)} \\ &= b\Sigma_t + E[\zeta_t] - \gamma_t^\sigma\Sigma_t && \text{(IA. 7)} \\ &= b\Sigma_t + E[\zeta_t] - (b + E[\zeta_t]\Sigma_t^{-1})\Sigma_t && \text{(IA. 8)} \\ &= 0, && \text{(IA. 9)} \end{aligned}$$

where in the last line we use the fact that $\gamma_{i,t}^\sigma = E[\gamma_{i,t}^*|\Sigma_t] = b + E[\zeta_t]/\sigma_{i,t}$. This proves Implication 1.

IMPLICATION 1: *The factor i price of risk is* $\gamma^*_{i,t} = \frac{\mu_{i,t}}{\sigma^2_{i,t}}$ *and* $\gamma^u_i = \frac{E[\mu_{i,t}]}{E[\sigma^2_{i,t}]}$. *Decompose factor excess returns as* $\mu_t = b\Sigma_t + \zeta_t$, *where we assume* $E[\zeta_t|\Sigma_t] = \zeta_t$. *Let* $\gamma^\sigma_{i,t} = E[\gamma^*_{i,t}|\sigma^2_{i,t}]$ *be the component of the price of risk variation driven by volatility, and* α_i *be factor i's volatility-managed alpha. Then*

$$\gamma^\sigma_{i,t} = \gamma^u_i + \frac{\alpha_i}{c_i} J^{-1}_{\sigma,i} \left(\frac{E\left[\sigma^2_{i,t}\right]}{\sigma^2_{i,t}} - 1 \right), \tag{10}$$

and the process $\Pi_t(\gamma^\sigma_t)$ *is a valid Stochastic Discount Factor (SDF) for* $d\widetilde{R}_t$ *and volatility-managed strategies* $w(\Sigma_t)$, *that is,* $E[d(\Pi_t(\gamma^\sigma_t) w(\Sigma_t) \widetilde{R}_t)] = 0$.[20]

Equation (10) shows how volatility-managed portfolio alphas allow us to reconstruct the variation in the price of risk due to volatility. The volatility-implied price of risk has two terms. The term γ^u is the unconditional price of risk, the price of risk that prices static portfolios of returns dR_t. It is the term typically recovered in cross-sectional tests. The second is due to volatility. It increases the price of risk when volatility is low, with this sensitivity increasing in our strategy alpha. Thus, volatility-managed alphas allow us to answer the question of how much compensation for risk moves as volatility moves.

Tracking variation in the price of risk due to volatility can be important for pricing. Specifically, $\Pi(\gamma_t^\sigma)$ can price not only the original assets unconditionally, but also volatility-based strategies of these assets.[21] Thus, volatility-managed portfolios allow us to get closer to the true price of risk process γ_t^*, and as a result, closer to the unconditional mean-variance frontier, a first-order economic object. In the Internet Appendix we show how one can implement the risk adjustment embedded in model $\Pi(\gamma_t^\sigma)$ by adding our volatility-managed portfolios as a factor.

We finish this section by providing a measure of how "close" $\Pi(\gamma_t^{\sigma})$ gets to $\Pi(\gamma_t^{*})$ relative to the constant price of risk model $\Pi(\gamma^{u})$. Recognizing that $E[(d\Pi(\gamma_t^{a}) - d\Pi(\gamma_t^{b}))dR_t]$ is the pricing error associated with using model b when prices are consistent with a, it follows that the volatility of the difference between models, $\mathcal{D}_{b-a} \equiv Var\,(d\Pi(\gamma_t^{a}) - d\Pi(\gamma_t^{b}))$, provides an upper bound on pricing error Sharpe ratios (see Hansen and Jagannathan (1991)). It is thus a natural measure of distance. For the single-factor case, we obtain

$$SR_{new} = \sqrt{SR_{old}^2 + (\tfrac{\alpha}{\sigma_\varepsilon})^2},$$

appraisal ratio

$$\mathcal{D}_{u-\sigma} = \left(\frac{\alpha}{c}\right)^2 E\left[\sigma_t^2\right] J_\sigma^{-1}, \tag{11}$$

$$\mathcal{D}_{u-\zeta} = \frac{Var(\zeta_t)}{E\left[\sigma_t^2\right]}, \tag{12}$$

$$\mu_t = b\Sigma_t + \zeta_t$$

$$\mathcal{D}_{u-*} = \left(\frac{\alpha}{c}\right)^2 E\left[\sigma_t^2\right] J_\sigma^{-1} + \frac{Var(\zeta_t)}{E[\sigma_t^2]}(J_\sigma + 1). \tag{13}$$

Equation (11) shows that the distance between models u and σ grows with alpha. In particular, it implies that the maximum excess Sharpe ratio decreases proportionally with the strategy alpha when you move from the constant price of risk model u to the model σ that incorporates variation in the price of risk driven by volatility. This is similar in spirit to Nagel and Singleton (2011), who derive general optimal managed portfolios based on conditioning information to test unconditional models against. Analogously, equation (12) accounts for

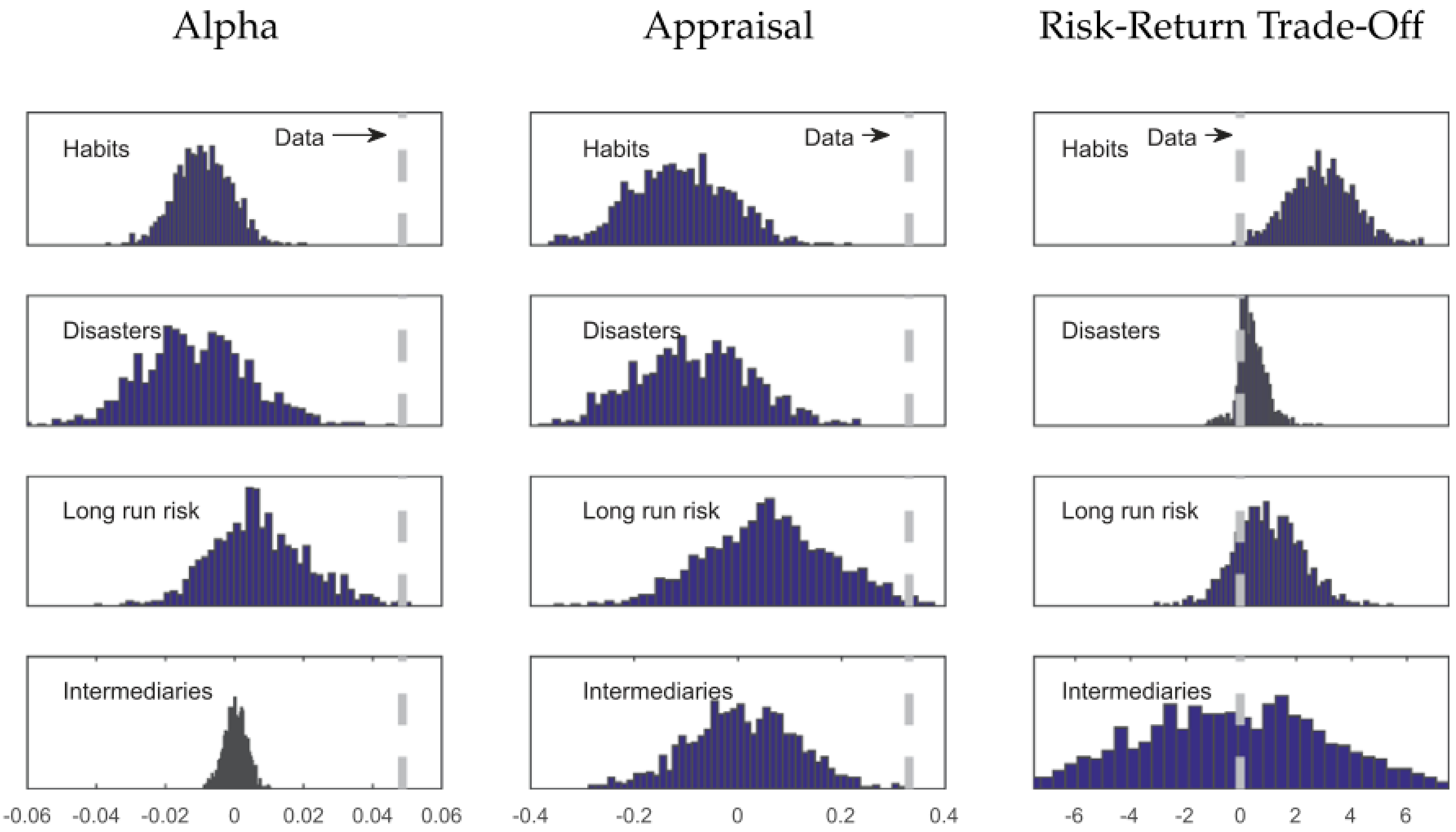
variation in the expected return signal ζ_t but ignores volatility information. Equation (13) shows the total difference between the constant price of risk model (u) and the true ($*$) model.

To have a sense of magnitudes, we assume that the market portfolio satisfies Assumption 1 and plug in numbers for the market portfolio. Notice that $\mathcal{D}_{u-\sigma}$ is the volatility-managed market's appraisal ratio squared, which measures the expansion of the MVE frontier for the managed strategy. We measure all the quantities in (11) to (13) but $Var(\zeta_t)$, which is tightly related to return predictability R^2. We use the estimate from Campbell and Thompson (2008), who obtain a number around 0.06.[22] We obtain $\mathcal{D}_{u-\sigma} = 0.33^2 =$ variation in volatility can reduce squared pricing error Sharpe ratios by approximately 0.11/0.29=38%, compared with 0.06/0.29=21% for time-variation in expected returns, with the large residual being due to the multiplicative interaction between them.

The above results show that accounting for time-variation in the price of risk driven by volatility seems at least as important as, and perhaps even more important than, accounting for variation in the price of risk driven by expected returns.

IV. General Equilibrium Implications

A. Macrofinance Models



four leading equilibrium asset pricing models

虚线是根据历史数据算的，柱状图是模拟数据算的。

- In these models, alphas are either near zero or negative on average,
- The positive alphas we document empirically imply that this covariance is negative.

All models frequently generate return histories consistent with the weak risk-return trade-off estimated in the data. However, no model comes close to reproducing our findings in terms of alphas or appraisal ratios.

$$R_{mkt,t+1} - R_{f,t+1} = a + \gamma \hat{\sigma}^2_{mkt,t} + \epsilon_{t+1}$$

$$\alpha = -cov\left(\frac{\mu_t}{\sigma_t^2}, \sigma_t^2\right)\frac{c}{E\left[\sigma_t^2\right]}$$

B. What Could Explain Our Results?

The easiest, but least plausible, explanation is that investors' willing
risk is **negatively** related to volatility . That is, investors choose
because they are less risk-averse during high-volatility perio
representative agent models, a plausible explanation is that vo
about structural parameters might be priced differently than vo
standard forms of risk (e.g, Veronesi(2000)).

sophisticated investors seem to sell more quickly in response to increases in volatility in the 2008 crisis.

One intuitive explanation for our results is that some investors are slow to trade. This could explain why a sharp increase in realized volatility does not immediately lead to a higher expected return in the data. This explanation is also consistent with our impulse responses where expected returns rise slowly but the true expected volatility process rises and mean-reverts quickly in response to a variance shock.

A final possibility is that the composition of shocks changes with volatility. Quantitatively , Moreira and Muir (2016) show that, because discount rate shocks seem to be very persistent in the data, even in the extreme case in which volatility is completely driven by discount rate volatility , investors with plausible investment horizons can still benefit somewhat from volatility timing.

V . Conclusion

- volatility-managed portfolios offer large risk-adjusted returns and are easy to implement in real time.
- Because volatility does not strongly forecast future returns, factor Sharpe ratios are improved by lowering risk exposure when volatility is high and increasing risk exposure when volatility is low. Our strategy runs contrary to conventional wisdom because it takes relatively less risk in recessions and crises yet still earns high average returns.
- We analyze both portfolio choice and general equilibrium implications of our findings. We find utility gains from volatility timing for mean-variance investors of around 65%, which is much larger than utility gains from timing expected returns.
- Furthermore, we show that our strategy performance is informative about the dynamics of effective risk-aversion, a key object for theories of time-varying risk premia.

2. On the performance of volatility-managed portfolios

Scott Cederburg[a], Michael S. O'Doherty[b,*], Feifei Wang[c], Xuemin (Sterling) Yan[d]

[a] Eller College of Management, University of Arizona, McClelland Hall, Room 315R, Tucson, AZ 85721, USA
[b] University of Missouri, Trulaske College of Business, 401C Cornell Hall, Columbia, MO 65211, United States
[c] Farmer School of Business, Miami University, 2064 FSB, Oxford, OH 45056, USA
[d] College of Business, Lehigh University, 471 Rauch Business Center, Bethlehem, PA 18015, USA

Abstract

- Using a comprehensive set of 103 equity strategies, we analyze the value of volatility-managed portfolios for real-time investors.
- Volatility-managed portfolios do not systematically outperform their corresponding unmanaged portfolios in direct comparisons.
- Consistent with Moreira and Muir (2017), volatility-managed portfolios tend to exhibit significantly positive alphas in spanning regressions.
- However, the trading strategies implied by these regressions are not implementable in real time, and reasonable out-of-sample versions generally earn lower certainty equivalent returns and Sharpe ratios than do simple investments in the original, unmanaged portfolios.
- This poor out-of-sample performance for volatility-managed portfolios stems primarily from structural instability in the underlying spanning regressions.

1. Introduction

Background： Moreira and Muir (2017) find that the empirical success of volatility management is a pervasive phenomenon. existing studies leave readers with the impression that volatility-managed equity strategies routinely lead to improved performance.

In this paper, we assess whether volatility management is systematically advantageous for investors and place specific emphasis on real-time implementation.

contribution: ①based on a substantially broader sample of 103 equity trading strategies, we find no statistical or economic evidence that volatility-managed portfolios systematically earn higher Sharpe ratios than their unmanaged counterparts do.
②we confirm that Moreira and Muir's (2017)) finding of systematically positive spanning regressions alphas for volatility-managed portfolios also holds in our extended sample. We explore an array of reasonable out-of-sample versions of these "combination" strategies and find that they typically underperform simple investments in the original, unscaled portfolios.
③We provide evidence that this result is driven by substantial structural instability in the underlying spanning regressions for these strategies.

frame

Volatility-managed portfolios

1.波动管理后单因素组合
volatility-managed Single-Factor Portfolios

2.波动管理后有效前沿组合
volatility-managed mean-variance efficient MVE frontier Portfolios

3.波动管理后组合
volatility-managed “combination” strategies

- Section 2 describes the data and introduces volatility-managed portfolios.
- Section 3 compares volatility-managed and original strategies. (single-factor)
- Section 4 contains our empirical tests on real-time strategies that combine volatility-managed portfolios with their unscaled versions.
- Section 5 concludes.

2.1. Data description

- 9 factors and 94 anomaly portfolios：

daily and monthly data on factor excess returns for nine equity factors

Identifies 94 anomaly variables, data from Center for Research in Security Prices (CRSP) Monthly and Daily Stock Files, the Compustat Fundamentals Annual and Quarterly Files, and the Institutional Brokers Estimate System (IBES) database.（we group them into the following eight categories based on the classification scheme in Hou et al. (2015) : accruals, intangibles, investment, market, momentum, profitability, trading, and value.）

2.2. Construction of volatility-managed portfolios

$$f_{\sigma,t} = \frac{c^*}{\hat{\sigma}_{t-1}^2} f_t, \quad (3)$$

$$\hat{\sigma}_t^2 = \frac{22}{J_t} \sum_{j=1}^{J_t} \left(f_t^j\right)^2. \quad (4)$$

$$f_{\sigma,t} = \alpha + \beta f_t + \varepsilon_t. \quad (5)$$

3. Direct comparisons

table1

	Factor								
	MKT (1)	*SMB* (2)	*HM* (3)						
Panel A: Performance measu									
Mean	7.80	2.57	4.8						
Standard deviation	18.61	11.12	12.1						
Sharpe ratio	0.42	0.23	0.4						
Panel B: Performance measures f									
Mean	9.55	0.86	4.6						
Standard deviation	18.61	11.12	12.14	16.39	7.71	6.97	8.83	6.48	10.71
Sharpe ratio	0.51	0.08	0.38	0.99	0.51	0.40	1.06	0.72	1.01
Panel C: Performance comparisons									
Sharpe ratio difference	0.09	−0.15	−0.02	0.50	0.13	−0.13	0.32	−0.05	0.24
	[0.30]	[0.09]	[0.86]	[0.00]	[0.29]	[0.23]	[0.01]	[0.68]	[0.01]
Panel D: Properties of volatility-managed factors									
Correlation with original factor	0.63	0.63	0.57	0.48	0.59	0.68	0.68	0.70	0.62
$P_{01}(c^*/\hat{\sigma}^2_{t-1})$	0.04	0.03	0.04	0.04	0.04	0.06	0.06	0.06	0.04
$P_{50}(c^*/\hat{\sigma}^2_{t-1})$	0.96	0.81	1.02	1.01	1.11	0.97	1.08	0.96	1.00
$P_{99}(c^*/\hat{\sigma}^2_{t-1})$	6.47	5.07	5.89	8.64	5.02	4.56	4.73	4.45	5.09

covariance between excess returns for the two portfolios. To test the null hypothesis of equal Sharpe ratios for portfolios i and j, we compute the following Jobson and Korkie (1981) test statistic, which is asymptotically distributed as a standard normal: $\hat{z}_{JK} = \frac{\hat{\sigma}_j\hat{\mu}_i - \hat{\sigma}_i\hat{\mu}_j}{\sqrt{\hat{\theta}}}$, where $\hat{\theta} = \frac{1}{T}\left(2\hat{\sigma}_i^2\hat{\sigma}_j^2 - 2\hat{\sigma}_i\hat{\sigma}_j\hat{\sigma}_{i,j} + \frac{1}{2}\hat{\mu}_i^2\hat{\sigma}_j^2 + \frac{1}{2}\hat{\mu}_j^2\hat{\sigma}_i^2 - \frac{\hat{\mu}_i\hat{\mu}_j}{\hat{\sigma}_i\hat{\sigma}_j}\hat{\sigma}_{i,j}^2\right)$. The test incorporates the correction noted by Memmel (2003).

In five cases the volatility-managed factor earns a higher average return and Sharpe ratio than the original strategy does, whereas the original factor outperforms in the remaining four cases. Three of the nine differences are significantly positive, as the volatility-managed versions of MOM, ROE , and BAB achieve Sharpe ratio gains by outperforming the original factors by 8.23%, 2.86%, and 2.58% per year.

The correlation coefficient between *ROE* and *MOM* (*BAB*) is 0.50 (0.26).

Although the median investment position for each of the dynamic portfolios is around one, the 99th percentile of required leverage exceeds 400% in each case and reaches as high as 864% for the momentum strategy.

table2

Sample (1)	Total (2)	Sharpe ratio difference: $\Delta SR > 0$ [Signif.] (3)	Sharpe ratio difference: $\Delta SR < 0$ [Signif.] (4)
Panel A: Combined sample			
All trading strategies	103	53 [8]	50 [4]
Panel B: By category			
Factors	9	5 [3]	4 [0]
Anomaly portfolios	94	48 [5]	46 [4]
Panel C: By trading strategy type			
Accruals	10	4 [0]	6 [0]
Intangibles	10	3 [0]	7 [0]
Investment	11	3 [0]	8 [1]
Market	1	1 [0]	0 [0]
Momentum	9	9 [5]	0 [0]
Profitability	22	15 [1]	7 [1]
Trading	21	11 [1]	10 [1]
Value	19	7 [1]	12 [1]

The results in Table 2 suggest that volatility-managed portfolios do not systematically outperform their original counterparts.

In Panel A, volatility management leads to improved and worsened performance at roughly the same frequency.

Panel C reveals that the majority of the significantly positive Sharpe ratio differences are attributable to them nine momentum strategies. Volatility management improves performance for every momentum strategy, and five of the nine performance differences are statistically significant at the 5% level.

This group includes growth in book equity ,change in sales less change in inventory,1/share price, and long-term reversal（4个显著下降）

three strategies exhibit statistically significant outperformance: ROE, BAB , and Loughran and Wellman ’s (2012) enterprise multiple.

$$\max_{a} U(a) = a^{\top}\hat{\mu} - \frac{\gamma}{2} a^{\top}\hat{\Sigma} a, \quad (6)$$

$$a = \begin{bmatrix} x_{\sigma}^{*} \\ x^{*} \end{bmatrix} = \frac{1}{\gamma}\hat{\Sigma}^{-1}\hat{\mu}, \quad (7)$$

$$\begin{bmatrix} w_{\sigma}^{*} \\ w^{*} \end{bmatrix} = \frac{\hat{\Sigma}^{-1}\hat{\mu}}{|1_{2}^{\top}\hat{\Sigma}^{-1}\hat{\mu}|}, \quad (8)$$

$$\hat{\Sigma} = \hat{\sigma}_{f}^{2}\begin{bmatrix} 1 & \hat{\rho} \\ \hat{\rho} & 1 \end{bmatrix}, \quad (9)$$

$$x_{\sigma}^{*} = \frac{\hat{\alpha}}{\gamma\hat{\sigma}_{f}^{2}(1-\hat{\rho}^{2})}. \quad (10)$$

$$y_{t}^{*} = x_{\sigma}^{*}\left(\frac{c^{*}}{\hat{\sigma}_{t-1}^{2}}\right) + x^{*}. \quad (11)$$

$$AR = \frac{\hat{\alpha}}{\hat{\sigma}_{\varepsilon}}, \quad (12)$$

$$AR^{2} = SR(y_{t}^{*})^{2} - SR(z^{*})^{2}, \quad (13)$$

$$\Delta CER = \frac{SR(y_{t}^{*})^{2} - SR(z^{*})^{2}}{2\gamma}. \quad (14)$$

$$\begin{bmatrix} x_{\sigma,t} \\ x_{t} \end{bmatrix} = \frac{1}{\gamma}\hat{\Sigma}_{t}^{-1}\hat{\mu}_{t}. \quad (15)$$

$$y_{t} = x_{\sigma,t}\left(\frac{c_{t}}{\hat{\sigma}_{t-1}^{2}}\right) + x_{t}. \quad (16)$$

$$\Delta SR = \frac{\hat{\mu}_{i}}{\hat{\sigma}_{i}} - \frac{\hat{\mu}_{j}}{\hat{\sigma}_{j}} \quad (17)$$

$$\Delta CER = \left(\hat{\mu}_{i} - \frac{\gamma}{2}\hat{\sigma}_{i}^{2}\right) - \left(\hat{\mu}_{j} - \frac{\gamma}{2}\hat{\sigma}_{j}^{2}\right). \quad (18)$$

interpretation of a positive alpha from this test is that an investor who holds the ex
optimal combination of the benchmark factors increases her Sharpe ratio by adding a

4. Combination strategies---*4.2 In-sample tests*

Shanxi University of Finance and Economics

table3

	Factor								
	MKT (1)	*SMB* (2)	*HML* (3)	*MOM* (4)	*RMW* (5)	*CMA* (6)	*ROE* (7)	*IA* (8)	*BAB* (9)
Panel A: Univariate regressions									
Panel A.1: Regression results									
Alpha, α (%)	4.63	−0.76	1.87	12.39	2.23	0.26	4.97	1.18	5.74
	(3.08)	(−0.87)	(1.88)	(7.31)	(2.57)	(0.39)	(5.10)	(1.83)	(5.97)
Beta, β	0.63	0.63	0.57	0.48	0.59	0.68	0.68	0.70	0.62
	(11.32)	(7.75)	(7.65)	(7.13)	(7.10)	(13.82)	(11.12)	(13.59)	(12.97)
R^2	0.40	0.40	0.33	0.23	0.34	0.46	0.46	0.50	0.38
Appraisal ratio, *AR*	0.32	−0.09	0.19	0.86	0.36	0.05	0.77	0.26	0.68
Panel A.2: Ex post optimization parameters									
Scaling parameter, c^*	10.33	2.63	2.95	4.60	1.48	1.53	2.06	1.64	3.20
Risky allocation, $x^*_\sigma + x^*$	0.61	0.34	0.82	1.22	1.45	1.60	2.44	0.70	2.05
Relative factor weights									
Vol-managed factor, w^*_σ	0.72	−0.60	0.46	0.98	0.79	0.12	0.97	0.41	0.78
Original factor, w^*	0.28	1.60	0.54	0.02	0.21	0.88	0.03	0.59	0.22
Panel A.3: Portfolio performance measures									
Sharpe ratio									
Original factor	0.42	0.23	0.40	0.48	0.38	0.53	0.74	0.77	0.77
Combination strategy	0.53	0.25	0.44	0.99	0.52	0.54	1.06	0.81	1.03
Difference	0.11	0.02	0.04	0.50	0.14	0.00	0.32	0.04	0.26
CER (%)									
Original factor	1.76	0.53	1.59	2.35	1.44	2.85	5.46	5.92	5.90
Combination strategy	2.79	0.61	1.94	9.74	2.71	2.88	11.32	6.57	10.52
Difference	1.03	0.08	0.35	7.39	1.27	0.03	5.86	0.65	4.63
Panel B: Additional controls for Fama and French (1993) three factors									
Alpha, α (%)	5.24	−0.56	2.52	10.28	3.02	−0.19	5.51	0.66	5.45
	(3.49)	(−0.65)	(2.52)	(6.56)	(3.49)	(−0.28)	(5.52)	(1.01)	(5.72)
R^2	0.41	0.40	0.35	0.26	0.43	0.47	0.49	0.51	0.39
Appraisal ratio, *AR*	0.37	−0.07	0.26	0.73	0.72	−0.04	0.88	0.15	0.65

Panel A.3 of Table 3 confirms that almost all combination strategies exhibit strong in-sample performance gains relative to the original factors.

The “original factor” results correspond to the ex post optimal combination of original factor and risk-free asset, and the “combination strategy” results correspond to the ex post optimal combination of original factor, volatility-managed factor, and risk-free asset.

Table 4

Summary of spanning regressions: broad sample.

The table summarizes results from spanning regressions for 103 trading strategies. The spanning regressions are given by $f_{\sigma,t} = \alpha + \beta f_t + \varepsilon_t$, where $f_{\sigma,t}$ (f_t) is the monthly return for the volatility-managed (original) anomaly portfolio. The results in columns (3) and (4) correspond to univariate spanning regressions, and those in columns (5) and (6) are for regressions that add the Fama and French (1993) three factors as controls. Panel A reports results for the full set of 103 trading strategies. Panel B presents separate results for the 9 factors and the 94 anomaly portfolios. Panel C breaks the results down by trading strategy type. For each set of regressions, the table reports the number of alphas that are positive, positive and significant at the 5% level, negative, and negative and significant at the 5% level. We assess statistical significance of the alpha estimates using White (1980) standard errors.

		Univariate regressions		Additional controls for Fama and French (1993) factors	
Sample (1)	Total (2)	$\alpha > 0$ [Signif.] (3)	$\alpha < 0$ [Signif.] (4)	$\alpha > 0$ [Signif.] (5)	$\alpha < 0$ [Signif.] (6)
Panel A: Combined sample					
All trading strategies	103	77 [23]	26 [3]	70 [21]	33 [3]
Panel B: By category					
Factors	9	8 [5]	1 [0]	7 [6]	2 [0]
Anomaly portfolios	94	69 [18]	25 [3]	63 [15]	31 [3]
Panel C: By trading strategy type					
Accruals	10	8 [3]	2 [0]	6 [0]	4 [0]
Intangibles	10	6 [1]	4 [0]	5 [0]	5 [0]
Investment	11	7 [1]	4 [1]	5 [1]	6 [1]
Market	1	1 [1]	0 [0]	1 [1]	0 [0]
Momentum	9	9 [9]	0 [0]	9 [9]	0 [0]
Profitability	22	19 [2]	3 [0]	19 [4]	3 [0]
Trading	21	14 [4]	7 [1]	14 [4]	7 [2]
Value	19	13 [2]	6 [1]	11 [2]	8 [0]

We find that 77 of the 103 scaled portfolios earn positive alphas in univariate spanning tests and, accordingly, are assigned positive weights in the ex post optimal combination portfolios. Twenty-three of the positive estimates are statistically significant at the 5% level

table5

	MKT (1)	*SMB* (2)	*HML* (3)	*MOM* (4)	*RMW* (5)	*CMA* (6)	*ROE* (7)	*IA* (8)	*BAB* (9)
Panel A: Real-time combination strategies									
Sharpe ratio									
[S1] Combination strategy (real time)	0.42	0.14	0.38	0.92	0.44	0.52	1.13	0.70	1.09
[S2] Original factor (real time)	0.46	0.19	0.43	0.49	0.31	0.56	0.78	0.68	0.79
Difference, [S1]–[S2]	−0.04	−0.06	−0.06	0.44	0.13	−0.03	0.36	0.02	0.30
	[0.64]	[0.37]	[0.41]	[0.00]	[0.53]	[0.20]	[0.00]	[0.74]	[0.00]
[S3] Combination strategy (ex post optimal)	0.53	0.26	0.50	0.99	0.58	0.64	1.21	0.73	1.11
Difference, [S1]–[S3]	−0.11	−0.12	−0.12	−0.07	−0.14	−0.11	−0.07	−0.03	−0.02
	[0.01]	[0.14]	[0.08]	[0.07]	[0.37]	[0.00]	[0.20]	[0.41]	[0.78]
CER (%)									
[S1] Combination strategy (real time)	1.56	0.00	1.41	8.47	1.96	2.74	12.25	4.19	10.88
[S2] Original factor (real time)	1.75	0.38	1.61	2.29	0.91	3.09	5.44	3.68	6.23
Difference, [S1]–[S2]	−0.19	−0.37	−0.20	6.18	1.04	−0.35	6.81	0.51	4.65
	[0.83]	[0.27]	[0.73]	[0.00]	[0.57]	[0.21]	[0.00]	[0.60]	[0.00]
[S3] Combination strategy (ex post optimal)	2.79	0.67	2.47	9.87	3.42	4.04	14.55	5.36	12.34
Difference, [S1]–[S3]	−1.23	−0.66	−1.06	−1.40	−1.46	−1.30	−2.30	−1.17	−1.46
	[0.01]	[0.13]	[0.10]	[0.07]	[0.39]	[0.03]	[0.15]	[0.25]	[0.30]
Panel B: Real-time combination strategies including Fama and French (1993) three factors									
Sharpe ratio									
[S1] Combination strategy (real time)	0.51	0.50	0.53	1.14	0.83	0.77	1.30	0.94	1.19
[S2] Original factor + FF3 (real time)	0.61	0.61	0.61	0.94	0.85	0.80	1.23	0.97	0.98
Difference, [S1] and [S2]	−0.11	−0.11	−0.08	0.20	−0.02	−0.03	0.07	−0.03	0.20
	[0.22]	[0.03]	[0.31]	[0.00]	[0.85]	[0.12]	[0.23]	[0.10]	[0.00]
[S3] Combination strategy (ex post optimal)	0.72	0.71	0.71	1.28	1.11	0.98	1.63	1.09	1.38
Difference, [S1]–[S3]	−0.22	−0.21	−0.18	−0.14	−0.28	−0.21	−0.33	−0.15	−0.20
	[0.00]	[0.01]	[0.03]	[0.01]	[0.00]	[0.01]	[0.00]	[0.01]	[0.00]
CER (%)									
[S1] Combination strategy (real time)	2.51	2.13	2.72	12.88	6.43	5.54	16.25	8.73	13.70
[S2] Original factor + FF3 (real time)	2.52	2.52	2.52	8.75	6.63	6.07	14.88	9.33	9.66
Difference, [S1] and [S2]	−0.02	−0.40	0.20	4.13	−0.19	−0.53	1.38	−0.60	4.04
	[0.99]	[0.21]	[0.77]	[0.00]	[0.92]	[0.13]	[0.25]	[0.11]	[0.00]
[S3] Combination strategy (ex post optimal)	5.21	5.05	5.09	16.39	12.33	9.60	26.53	11.79	19.11
Difference, [S1]–[S3]	−2.71	−2.93	−2.37	−3.51	−5.89	−4.06	−10.28	−3.06	−5.42
	[0.00]	[0.02]	[0.04]	[0.02]	[0.00]	[0.01]	[0.00]	[0.01]	[0.00]

- The improvements for the MOM, ROE , and BAB factors are statistically significant at the 1% level. Across the remaining six strategies, the Sharpe ratio and CER differences are insignificant.
- All nine of the Sharpe ratio and CER differences are negative（real-time combination strategies and the ex post optimal combination strategies.）
- In summary, Table 5 shows that volatility management has potential benefits for real-time investors in some factors, but the gains are not systematic and are much less impressive than the corresponding in-sample results. These initial results indicate that real-time implementation issues degrade portfolio performance in the volatility-managed portfolios setting.

table6

broad sample Out-of-sample tests

Panel A: Real-time combination strategies

Sample (1)	Total (2)	Sharpe ratio difference: Combination strategy (real time) versus Original factor (real time) ΔSR +/− (3)	Combination strategy (ex post optimal) ΔSR +/− (4)	CER difference: Combination strategy (real time) versus Original factor (real time) ΔCER +/− (5)	Combination strategy (ex post optimal) ΔCER +/− (6)
Panel A.1: Combined sample					
All trading strategies	103	45 [8] / 58 [2]	1 [0] / 102 [39]	31 [7] / 72 [7]	0 [0] / 103 [41]
Panel A.2: By category					
Factors	9	5 [3] / 4 [0]	0 [0] / 9 [2]	5 [3] / 4 [0]	0 [0] / 9 [2]
Anomaly portfolios	94	40 [5] / 54 [2]	1 [0] / 93 [37]	26 [4] / 68 [7]	0 [0] / 94 [39]
Panel A.3: By trading strategy type					
Accruals	10	3 [0] / 7 [1]	0 [0] / 10 [5]	3 [0] / 7 [2]	0 [0] / 10 [4]
Intangibles	10	4 [0] / 6 [0]	0 [0] / 10 [0]	1 [0] / 9 [1]	0 [0] / 10 [4]
Investment	11	5 [0] / 6 [0]	0 [0] / 11 [6]	5 [0] / 6 [0]	0 [0] / 11 [5]
Market	1	0 [0] / 1 [0]	0 [0] / 1 [1]	0 [0] / 1 [0]	0 [0] / 1 [1]
Momentum	9	8 [4] / 1 [0]	0 [0] / 9 [5]	8 [5] / 1 [0]	0 [0] / 9 [5]
Profitability	22	10 [1] / 12 [0]	1 [0] / 21 [7]	6 [1] / 16 [1]	0 [0] / 22 [5]
Trading	21	10 [1] / 11 [1]	0 [0] / 21 [6]	6 [1] / 15 [1]	0 [0] / 21 [8]
Value	19	5 [2] / 14 [0]	0 [0] / 19 [9]	2 [0] / 17 [2]	0 [0] / 19 [9]

Panel B: Real-time combination strategies including Fama and French (1993) three factors

Sample (1)	Total (2)	Sharpe ratio difference: Combination strategy (real time) versus Original factor+FF3 (real time) ΔSR +/− (3)	Combination strategy (ex post optimal) ΔSR +/− (4)	CER difference: Combination strategy (real time) versus Original factor+FF3 (real time) ΔCER +/− (5)	Combination strategy (ex post optimal) ΔCER +/− (6)
Panel B.1: Combined sample					
All trading strategies	103	32 [2] / 71 [13]	0 [0] / 103 [77]	32 [3] / 71 [10]	0 [0] / 103 [92]
Panel B.2: By category					
Factors	9	3 [2] / 6 [1]	0 [0] / 9 [9]	4 [2] / 5 [0]	0 [0] / 9 [9]
Anomaly portfolios	94	29 [0] / 65 [12]	0 [0] / 94 [68]	28 [1] / 66 [10]	0 [0] / 94 [83]
Panel B.3: By trading strategy type					
Accruals	10	3 [0] / 7 [1]	0 [0] / 10 [8]	3 [0] / 7 [1]	0 [0] / 10 [8]
Intangibles	10	1 [0] / 9 [1]	0 [0] / 10 [5]	1 [0] / 9 [1]	0 [0] / 10 [9]
Investment	11	3 [0] / 8 [1]	0 [0] / 11 [10]	3 [0] / 8 [1]	0 [0] / 11 [10]
Market	1	0 [0] / 1 [0]	0 [0] / 1 [1]	0 [0] / 1 [0]	0 [0] / 1 [1]
Momentum	9	8 [1] / 1 [0]	0 [0] / 9 [9]	7 [2] / 2 [0]	0 [0] / 9 [9]
Profitability	22	8 [0] / 14 [2]	0 [0] / 22 [14]	9 [0] / 13 [2]	0 [0] / 22 [18]
Trading	21	6 [1] / 15 [5]	0 [0] / 21 [21]	5 [1] / 16 [3]	0 [0] / 21 [21]
Value	19	3 [0] / 16 [3]	0 [0] / 19 [9]	4 [0] / 15 [2]	0 [0] / 19 [16]

Table 6 summarizes the results for out-of-sample tests using our base case design with an expanding, ten-year training sample, a leverage constraint of $|y_t| \leq 5$, and a risk aversion parameter of $\gamma = 5$.

table7

Description (1)	Sharpe ratio difference: (N = 103) ΔSR +/− (2)	Binomial p-value (3)	CER difference: (N = 103) ΔCER +/− (4)	Binomial p-value (5)
Panel A: Real-time combination strategies				
Base case design	45 [8] / 58 [2]	0.237	31 [7] / 72 [7]	0.000
Rolling-window training sample	49 [2] / 54 [1]	0.694	17 [1] / 86 [19]	0.000
Risk aversion, $\gamma = 2$	48 [9] / 55 [2]	0.555	35 [8] / 68 [7]	0.001
Risk aversion, $\gamma = 10$	45 [8] / 58 [2]	0.237	31 [7] / 72 [7]	0.000
Initial training sample length, $K = 240$	45 [9] / 58 [10]	0.237	36 [8] / 67 [11]	0.003
Initial training sample length, $K = 360$	40 [9] / 63 [6]	0.030	31 [8] / 72 [8]	0.000
Leverage constraint, $L \leq 1.0$	49 [10] / 54 [2]	0.694	38 [4] / 65 [5]	0.010
Leverage constraint, $L \leq 1.5$	47 [10] / 56 [3]	0.431	38 [7] / 65 [7]	0.010
Leverage constraint, $L \leq \infty$	45 [8] / 58 [2]	0.237	31 [7] / 72 [7]	0.000
Panel B: Real-time combination strategies including Fama and French (1993) three factors				
Base case design	32 [2] / 71 [13]	0.000	32 [3] / 71 [10]	0.000
Rolling-window training sample	32 [0] / 71 [8]	0.000	20 [0] / 83 [16]	0.000
Risk aversion, $\gamma = 2$	22 [3] / 81 [13]	0.000	24 [3] / 79 [10]	0.000
Risk aversion, $\gamma = 10$	31 [3] / 72 [12]	0.000	32 [3] / 71 [10]	0.000
Initial training sample length, $K = 240$	31 [6] / 72 [11]	0.000	35 [9] / 68 [10]	0.001
Initial training sample length, $K = 360$	30 [6] / 73 [9]	0.000	28 [7] / 75 [9]	0.000
Leverage constraint, $L \leq 1.0$	22 [2] / 81 [11]	0.000	27 [3] / 76 [12]	0.000
Leverage constraint, $L \leq 1.5$	21 [3] / 82 [13]	0.000	26 [2] / 77 [9]	0.000
Leverage constraint, $L \leq \infty$	31 [3] / 72 [12]	0.000	32 [3] / 71 [11]	0.000

p -value from a binomial test of the null hypothesis that positive and negative performance differences are equally likely：A 9个， B 18个。

- The robustness design with rolling-window parameter estimation leads to a slightly larger number of positive Sharpe ratio differences but a substantially smaller number of positive CER differences.
- Imposing a leverage constraint could either improve performance if real-time investors avoid taking extreme positions or hurt performance if the constraint prevents investors from capitalizing on the information content in lagged volatility.
- In all, more than half of the Sharpe ratio and CER differences are negative under each specification.

table8

Description (1)	Total (2)	Alpha: α +/− (3)	Alpha: Binomial p-value (4)	Sharpe ratio difference: ΔSR +/− (5)	Sharpe ratio difference: Binomial p-value (6)	CER difference: ΔCER +/− (7)	CER difference: Binomial p-value (8)
Panel A: Spanning regressions							
Spanning regressions	103	77 [23] / 26 [3]	0.000	45 [8] / 58 [2]	0.237	31 [7] / 72 [7]	0.000
Spanning regressions with FF3 controls	103	70 [21] / 33 [3]	0.000	32 [2] / 71 [13]	0.000	32 [3] / 71 [10]	0.000
Panel B: Anomaly regressions							
CAPM regressions	102	93 [73] / 9 [3]	0.000	75 [19] / 27 [1]	0.000	68 [18] / 34 [1]	0.001
FF3 regressions	100	81 [60] / 19 [5]	0.000	66 [18] / 34 [1]	0.002	55 [19] / 45 [1]	0.368

with real-time volatility-managed strategies

real-time anomaly strategies

- Studies showing cross-sectional anomalies routinely emphasize the alphas earned by these strategies relative to popular asset pricing models such as the Capital Asset Pricing Model (CAPM) or Fama and French (1993) three-factor model.
- A large proportion of these out-sample alpha estimates are statistically significant.
- Adding an anomaly portfolio to the CAPM market factor in real time, for example, leads to a Sharpe ratio improvement in 75 out of 102 cases and a CER improvement in 68 out of 102 cases. Real-time performance relative to the Fama and French (1993) three-factor benchmark is less impressive, with positive Sharpe ratio differences for 66 out of 100 strategies and positive CER differences for 55 out of 100 strategies. These results indicate that real-time anomaly strategies fare substantially better compared with real-time volatility-managed strategies.
- The results in Table 8 provide a useful backdrop to examine why the statistical support for out-of-sample combination strategies is particularly weak in the volatility-managed portfolios setting. We consider three potential explanations: (i) estimation risk in the out-of-sample portfolio choice exercise, (ii) low power in the out-of-sample tests, and (iii) structural instability in the conditional risk-return trade-off for the various factors and anomaly portfolios.

(i) estimation risk

- Although estimation error is always a challenge with real-time portfolio choice applications, we are skeptical that it fully accounts for our results for a variety of reasons. First, our empirical design incorporates several features intended to mitigate estimation risk, including a leverage constraint on portfolio positions, a risk-free asset in the investment opportunity set (Kirby and Ostdiek, 2012), and expanding-window parameter estimation.
- Second, DeMiguel et al. (2009b) emphasize that estimation risk is less problematic in applications, like ours, in which the number of test assets is small.
- Third, our main results are based on comparisons of real-time strategies that include volatility-managed portfolios in the investment opportunity set with those that exclude volatility-managed portfolios from the investment opportunity set. Thus, both the combination strategy and the benchmark suffer from estimation risk, and it is not obvious why one of the two would be more adversely impacted.
- Fourth, if estimation risk is the primary explanation of the poor performance of the combination strategies, then we should see more favorable results under specifications with longer training samples. Table 7 reveals, however, that lengthening the initial training sample has little impact on our conclusions.
- Finally, Panel B of Table 8 provides direct evidence that in-sample alphas do translate into improved real-time performance measures much more frequently outside of the volatility-managed portfolios setting.

(ii) low power

Another common concern with out-of-sample tests is that they lack power relative to in-sample tests because the evaluation period is shorter (e.g., Inoue and Kilian, 2004). Our focus on assessing the value of volatility management for real-time investors necessitates the use of out-of-sample tests.

Low power also does not seem to be a satisfactory explanation for our results. If volatility management is systematically beneficial to investors, then we should see a majority of performance differences that are positive in Tables 6 and 7. Low power might be an explanation for why an individual result is statistically insignificant, but it does not account for why **most of the performance differences have the wrong sign**.

(iii) structural instability

A more plausible economic explanation for the poor out-of-sample performance for the combination strategies is structural instability in the spanning regression parameters from Eq. (5) and the implied optimal weights.

table9

Description (1)	Total (2)	Frequency distribution for breaks: $N_b = 0$ (3)	$N_b = 1$ (4)	$N_b = 2$ (5)	$N_b = 3$ (6)	$N_b \geq 4$ (7)	$\bar{N}_b$ (8)
Panel A: Spanning regressions							
Spanning regressions	103	0	10	52	34	7	2.37
Spanning regressions with FF3 controls	103	1	8	53	35	6	2.37
Panel B: Anomaly regressions							
CAPM regressions	102	15	38	39	9	1	1.44
FF3 regressions	100	10	25	36	21	8	1.92

- In Panel A, we find strong statistical evidence of structural breaks in the spanning tests for the 103 volatility-managed portfolios. 41 out of 103 tests identify three or more breaks. The average number of breaks is 2.37 for both the univariate spanning regressions and the spanning regressions that control for the Fama and French (1993) factors.
-
- In contrast, structural breaks are less common in the standard time-series anomaly regressions in Panel B. In the CAPM regressions, for example, 53 out of 102 strategies have one break or less, and the average number of breaks is 1.44.

- Recent literature suggests that investors can enhance Sharpe ratios and lifetime utility by adopting simple trading rules that scale positions in popular equity portfolios by lagged variance. Two forms: direct investments in **volatility-managed portfolios** or **combination portfolios** that invest in both the volatility-managed version and the original version of an underlying strategy.
- Across a broad sample of 103 equity portfolios, volatility management degrades and improves performance at about the same frequency. From a practical perspective, the results suggest that direct investments in volatility-managed portfolios are not a panacea of improved performance.
- Combination strategies that incorporate volatility management, in contrast, exhibit systematically strong in-sample performance. On this point, we extend Moreira and Muir 's (2017) spanning regression analysis to our broader set of 103 equity strategies and show that these portfolios tend to exhibit positive alphas.
- The Sharpe ratios and CERs for the out-of-sample combination portfolios are dramatically less impressive than those earned by their in-sample versions. Moreover, the real-time combination strategies routinely underperform simpler strategies constrained to invest in the original, unscaled portfolios.

covariance between excess returns for the two portfolios. To test the null hypothesis of equal Sharpe ratios for portfolios i and j, we compute the following Jobson and Korkie (1981) test statistic, which is asymptotically distributed as a standard normal: $\hat{z}_{JK} = \frac{\hat{\sigma}_j \hat{\mu}_i - \hat{\sigma}_i \hat{\mu}_j}{\sqrt{\hat{\theta}}}$, where $\hat{\theta} = \frac{1}{T}\left(2\hat{\sigma}_i^2\hat{\sigma}_j^2 - 2\hat{\sigma}_i\hat{\sigma}_j\hat{\sigma}_{i,j} + \frac{1}{2}\hat{\mu}_i^2\hat{\sigma}_j^2 + \frac{1}{2}\hat{\mu}_j^2\hat{\sigma}_i^2 - \frac{\hat{\mu}_i\hat{\mu}_j}{\hat{\sigma}_i\hat{\sigma}_j}\hat{\sigma}_{i,j}^2\right)$. The test incorporates the correction noted by Memmel (2003).

In contrast, when expected returns are constant or independent of volatility, equation (8) implies $\alpha = c\frac{E[\mu_t]}{E[\sigma_t^2]}J_\sigma$, where $J_\sigma = (E[\sigma_t^2]E[\frac{1}{\sigma_t^2}] - 1) > 0$ is a Jensen's inequality term that is increasing in the volatility of volatility. This is because the more volatility varies, the more variation there is in the price of risk that the portfolio can capture. Thus, the alpha of our strategy is increasing in the volatility of volatility and the unconditional compensation for risk.