A factor model for option returns

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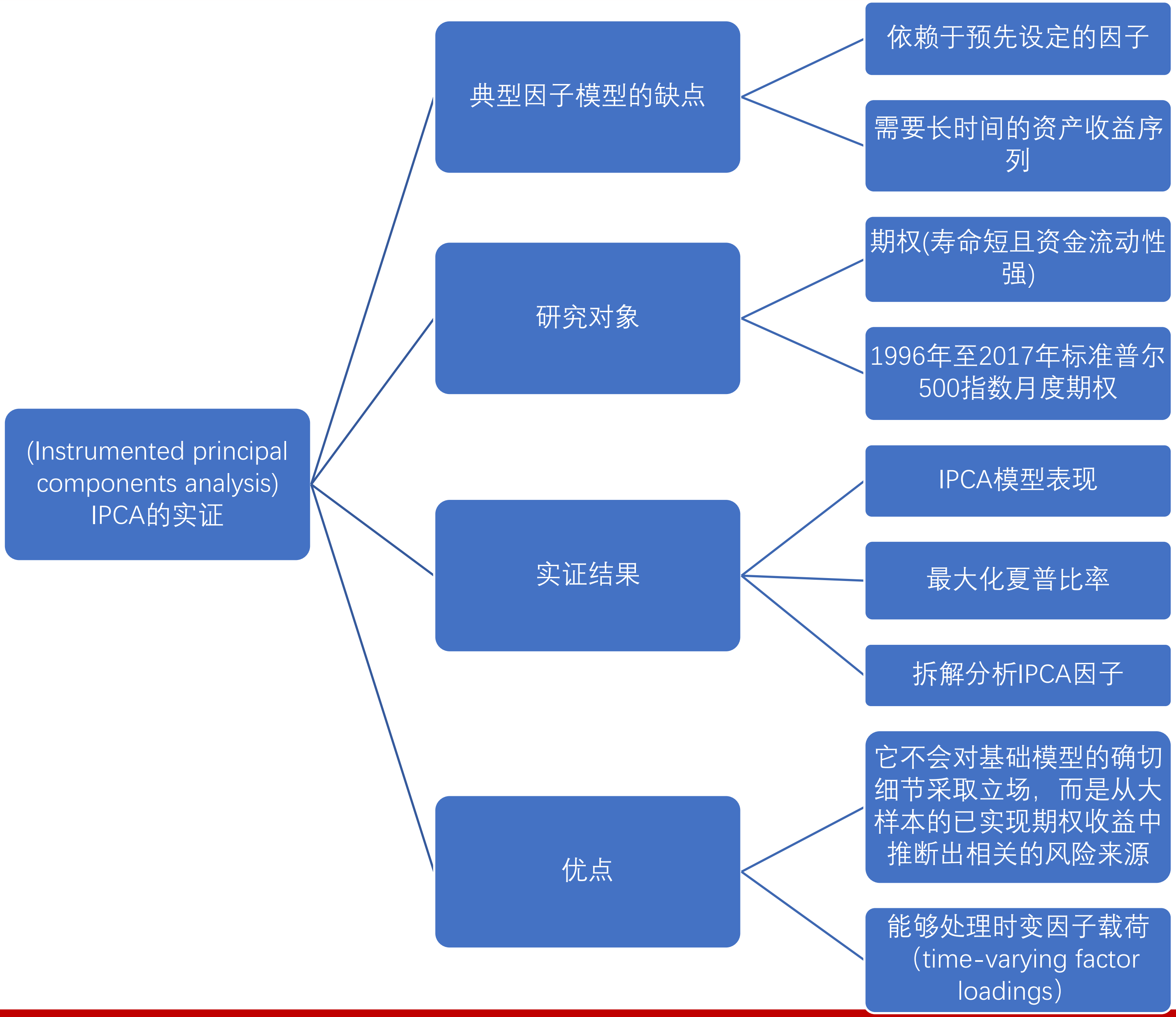
Framework of this paper



1. Introduction

- In practice, the no-arbitrage model cannot explain a large part of the empirical observation changes of option returns. (Israelov and Kelly, 2017)
- According to the Euler equation of asset pricing and the assumption of no arbitrage, there is a random discount factor, m_{t+1} ,which satisfies

$$E_t\left[m_{t+1}, r_{i,t+1}\right] = 0$$

, therefore

$$E_t[r_{i,t+1}] = -\underbrace{\frac{\mathrm{Cov}_t(m_{t+1}, r_{i,t+1})}{\mathrm{Var}_t(m_{t+1})}}_{\beta_{i,t}} \underbrace{\frac{\mathrm{Var}_t(m_{t+1})}{\mathrm{E}_t[m_{t+1}]}}_{\lambda_t}. \qquad (1)$$

$\beta_{i,t}$ 资产i对系统风险因子的条件暴露

λ_t 与因子相关的风险条件价格

1. Introduction

- When m_{t+1} is linear in factors f_{t+1} , as assumed in many asset pricing studies, the cross section of excess returns satisfies a linear factor model:

$$r_{i,t+1} = \alpha_{i,t} + \beta'_{i,t} f_{t+1} + \epsilon_{i,t+1}$$

- for all i and t:

$$\mathrm{E}_t[\epsilon_{i,t+1}] = \mathrm{E}[\epsilon_{i,t+1} f_{t+1}] = 0$$

$$\mathrm{E}_t[f_{t+1}] = \lambda_t, \text{ and } \alpha_{i,t} = 0$$

1. Introduction

- This approach is difficult with options data for a few reasons：

（1） Their short lives make it hard to estimate option betas with time series regression.

（2） Rapid migration of option attributes (such as moneyness and maturity) means that option risk exposures likewise migrate rapidly over time. Thus, it is important to incorporate conditional betas in option factor models, which further limits the viability of time series regression .

（3） The factors are difficult to ascertain a priori.

What is IPCA?

- Instrumented Principal Components Analysis (IPCA), allows for latent factors and time-varying loadings by introducing observable characteristics that instrument for the unobservable dynamic loadings.
- The general IPCA model specification for an excess return $r_{i,t+1}$:

$$r_{i,t+1} = \alpha_{i,t} + \beta_{i,t} f_{t+1} + \epsilon_{i,t+1},$$

$$\alpha_{i,t} = z_{i,t}' \Gamma_\alpha + \nu_{\alpha,i,t}, \quad \beta_{i,t} = z_{i,t}' \Gamma_\beta + \nu_{\beta,i,t}.$$

- $\beta_{i,t}$:dynamic factor loadings

What is IPCA？

- $t=1,\ldots,t$代表时间，$i=1,\ldots,N$代表个体。IPCA允许载荷在i和t维度上变化。等式（1）是因子结构，其中$x_{i,t}$是标量面板数据，因子分析的目标，即f_t和$\beta_{i,t}$分别是K因子和因子载荷，$\mu_{i,t}$为误差。等式（2）将工具变量$c_{i,t}$的L向量的信息与动态因子负载$\beta_{i,t}$联系起来。其中Γ是L×K 的参数矩阵。

$$x_{i,t}=\beta_{i,t}^{\top}f_t+\mu_{i,t}$$
$$\beta_{i,t}^{\top}=c_{i,t}\Gamma+\eta_{i,t}.$$

- PCA被广泛使用，一个原因是其易于通过主成分分析（PCA）进行估计。PCA对β_i没有限制，但依赖于它是静态的，不随时间变化。

$$x_{i,t}=\beta_i^{\top}f_t+\mu_{i,t}.$$

stochastic discount factor m_{t+1}

- x_{t+1} :the investor can buy or sell the payoff x_{t+1};
- p_t : the asset's price;
- e:the original consumption level (if the investor bought none of the asset);
- ξ :the amount of the asset he chooses to buy.
- (1) $\max_{\{\xi\}} u(c_t) + E_t \beta u(c_{t+1}) \quad s.t.$ (2) $p_t u'(c_t) = E_t \left[\beta u'(c_{t+1}) x_{t+1}\right]$

$$
\begin{aligned}
c_t &= e_t - p_t \xi \\
c_{t+1} &= e_{t+1} + x_{t+1} \xi
\end{aligned}
$$

$$p_t = E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1}\right].$$

- (3) $p = E(mx)$

$$m = \beta \frac{u'(c_{t+1})}{u'(c_t)}$$

2. Data

- Data:The options on the S&P 500 index.

- From: Daily option data is obtained from OptionMetrics for the period of January 1996 to December 2017. VIX index is obtained through CBOE.

- The information includes:Contract specifications (exercise date, strike, etc.) as well as underlying index values, historical dividend yields, and option sensitivity measures such as the BMS delta, gamma, vega, and theta. Data for the VIX index is obtained through CBOE.

2. Data

- The delta-hedged profit-and-loss (P&L) for a contract with value F over a period t = 1,...,T is given by

$$\Pi_{[1,T]} = \sum_{t=1}^{T-1} (F_{t+1} - F_t) - \sum_{t=1}^{T-1} \Delta_t (S_{t+1} - S_t) - \sum_{t=1}^{T-1} \frac{a_{t,t+1} r_t}{365} (F_t - \Delta_t S_t),$$

- where the first term is the raw P&L, the second term captures the adjustment from delta-hedging the position, and the last term adjusts for the cost of funding the delta-hedged portfolio at the risk-free rate where $a_{t,t+1}$ is the number of days between trading dates t and t + 1 .

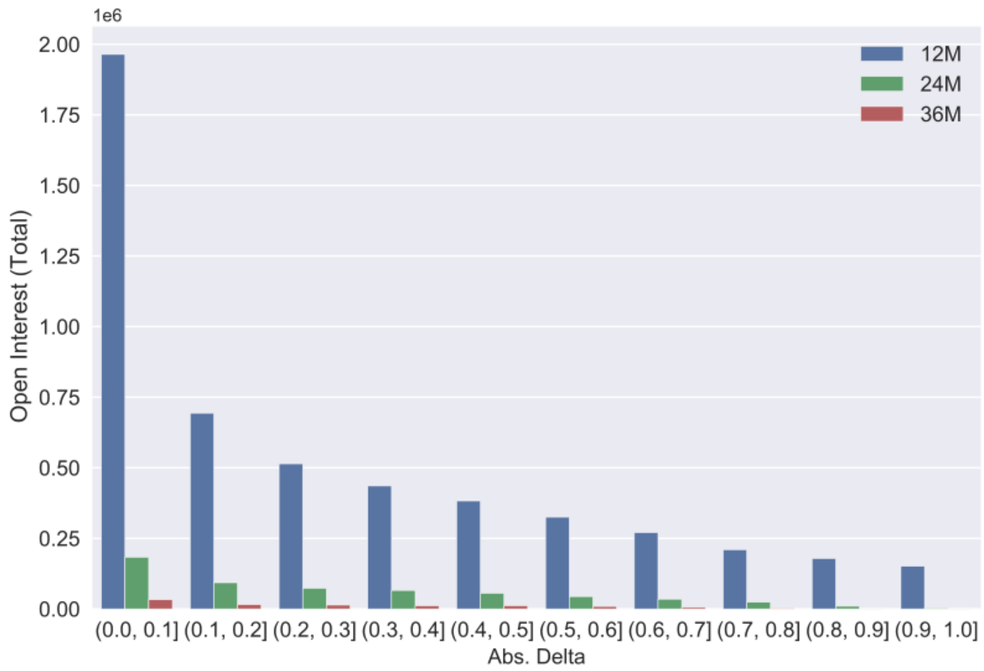
2. Data

- OptionMetrics data filters : The data excludes observations in which i) the bid price is negative, ii) the bid exceeds the ask, iii) no-arbitrage conditions are violated, or iv) the OptionMetrics implied volatility is missing.
- Only study observations for contracts with positive open interest. We exclude observations with extreme embedded leverage by trimming data below (above) the 1st (99th) percentile of the embedded leverage distribution, where embedded leverage is defined as

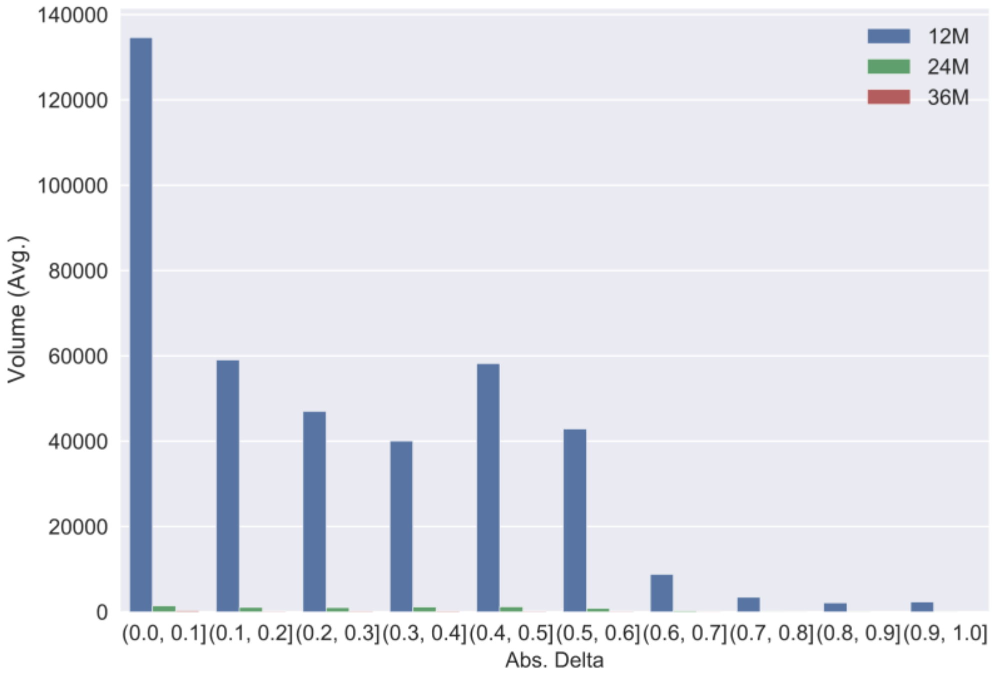
$$\Omega = |\Delta \cdot S/F|.$$

- Finally, we restrict our sample to call options with forward delta of 0.01 to 0.5 and -0.5 to -0.01 for put options, and require time-to-maturity (TTM) of 1 to 12 months.

2. Data



未平仓利率
(a) Total Open Interest



平均交易量
(b) Traded Volume

Fig1. Open Interest and Volume of S&P 500 Options

Panel A: Call Option Contracts

	ttm 到期时间	mness 货币价值	embed_lev	iv	delta	gamma	vega	theta	r^{Δ}_{Spot}
Mean	129	1.05	32.02	0.16	0.20	0.002	196.85	-55.10	-3.14%
Median	91	0.97	25.99	0.15	0.18	0.002	155.54	-44.19	-2.31%
Std. Dev.	99	0.67	21.00	0.07	0.15	0.002	159.82	45.86	1.33%
No. Obs.	24,749	24,749	24,749	24,749	24,749	24,749	24,749	24,749	24,749

Panel B: Put Option Contracts

	ttm	mness	embed_lev	iv	delta	gamma	vega	theta	r^{Δ}_{Spot}
Mean	123	-1.16	19.10	0.26	-0.15	0.001	168.03	-70.60	-5.18%
Median	91	-1.20	16.73	0.24	-0.10	0.001	123.02	-59.06	-4.26%
Std. Dev.	99	0.66	11.08	0.10	0.14	0.002	148.34	48.94	1.48%
No. Obs.	52,341	52,341	52,341	52,341	52,341	52,341	52,341	52,341	52,341

Table1. Summary Statistics of Option Level Variables

$$\text{mness} = ln(K/S)/(IV \cdot \sqrt{ttm})$$

3. Methodology

Instrumented principal components model(IPCA) (Kelly et al2019, 2020)。
The model is specified for a general excess return $r_{i,t+1}$:

$$r_{i,t+1} = \alpha_{i,t} + \beta_{i,t} f_{t+1} + \epsilon_{i,t+1}$$
$$\alpha_{i,t} = z'_{i,t}\Gamma_\alpha + \nu_{\alpha,i,t}, \quad \beta_{i,t} = z'_{i,t}\Gamma_\beta + \nu_{\beta,i,t}$$

Where $f_{i,t+1}$:K × 1; $z_{i,t+1}$:L × 1.

By restricting the IPCA model such that Γ_α = 0 , we can test whether risk compensation in option returns solely arises from exposure to systematic factors, ft , or whether returns partially line up with characteristics directly (i.e. Γ_α ≠0), hence constituting compensation without risk.

3. Methodology

Asset pricing performance: (Kelly, Pruitt and Su (2019)):

$$R^2_{total} = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z'_{i,t}(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{f}_{t+1})\right)^2}{\sum_{i,t} r^2_{i,t+1}}. \qquad (6)$$

R^2_{total} measures how well the set of factors and loadings captures realized returns.

$$R^2_{pred} = 1 - \frac{\sum_{i,t}\left(r_{i,t+1} - z'_{i,t}(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{\lambda})\right)^2}{\sum_{i,t} r^2_{i,t+1}}, \qquad (7)$$

$\hat{\lambda}$: The unconditional time-series mean of the factors. R^2_{pred} captures how well differences in average returns are explained through the model's description of conditional expected returns, i.e. the models ability to describe risk.

3. Methodology

Let Z_t be an N×L matrix of characteristics at time t. Then managed portfolios can be constructed via

$$x_{t+1} = \frac{Z_t' r_{t+1}}{N_{t+1}}, \tag{8}$$

N_{t+1} is the number of outstanding options at time t+1 . The managed portfolios, x_{t+1}, are a weighted average of option returns where the weights are determined by the characteristics in Z_t . We can define performance measures for managed portfolios:

$$R^2_{total,x} = 1 - \frac{\sum_{l,t}\left(x_{l,t+1} - z'_{l,t} z_{l,t}(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{f}_{t+1})\right)^2}{\sum_{l,t} x^2_{l,t+1}}, \tag{9}$$

$$R^2_{pred,x} = 1 - \frac{\sum_{l,t}\left(x_{l,t+1} - z'_{l,t} z_{l,t}(\hat{\Gamma}_\alpha + \hat{\Gamma}_\beta \hat{\lambda})\right)^2}{\sum_{l,t} x^2_{l,t+1}}, \tag{10}$$

where $l = 1, \ldots, L$.

4.2. IPCA Performance

	r^{Δ}_{Spot}	
	(1)	(2)
delta	-0.0088	0.0560
	(-0.7169)	(6.4211)
ttm	0.0244	0.0327
	(5.9405)	(8.3471)
embed_lev	-2.011e-05	0.0063
	(-0.0069)	(2.8575)
gamma	-0.0151	-0.0140
	(-3.4482)	(-4.1134)
vega	-0.0091	-0.0254
	(-1.8856)	(-6.3281)
theta	-0.0167	0.0118
	(-2.1788)	(1.8536)
impvol	-0.0821	-0.1467
	(-13.383)	(-14.711)
delta:put	0.0635	0.0291
	(3.7247)	(2.6428)
ttm:put	-0.0141	-0.0295
	(-3.4176)	(-7.2445)
embed_lev:put	0.0151	0.0074
	(4.4433)	(2.1717)
gamma:put	-0.0056	0.0080
	(-0.8119)	(1.7138)
vega:put	-0.0015	0.0311
	(-0.2155)	(6.2716)
theta:put	0.0524	0.0424
	(5.7418)	(6.0961)
impvol:put	0.0543	0.1109
	(11.800)	(15.992)
Effects		Time
R^2	3.15%	6.23%
No. Obs.	77,090	77,090

Table 2 Panel Regression of Option Returns on Option Characteristics

4.2. IPCA Performance

Panel A: Abs. Delta Bin						
Range	K=1	K=2	K=3	K=4	K=5	No. Obs
0.01 to 0.1	57.44%	75.41%	76.85%	80.96%	83.12%	34,957
0.1 to 0.2	71.11%	81.95%	85.66%	88.48%	89.58%	14,307
0.2 to 0.3	75.56%	81.05%	86.39%	91.64%	92.42%	10,393
0.3 to 0.4	75.43%	79.99%	87.11%	91.49%	92.56%	8920
0.4 to 0.5	74.24%	78.27%	85.35%	87.83%	89.69%	8513
Panel B: Time-to-Maturity Bin						
Range	K=1	K=2	K=3	K=4	K=5	No. Obs
1 Month	41.11%	52.16%	65.59%	77.27%	79.49%	16,598
2 Months	80.38%	88.61%	90.77%	92.03%	92.70%	19,087
3 to 6 Months	88.23%	92.67%	93.80%	94.40%	95.41%	22,723
6 to 12 Months	82.57%	88.31%	92.28%	93.12%	94.34%	18,682
Panel C: VIX						
Range	K=1	K=2	K=3	K=4	K=5	No. Obs
0% to 10%	25.31%	63.86%	65.96%	69.91%	78.96%	2783
10% to 20%	69.65%	77.56%	83.67%	87.63%	89.52%	47,061
20% to 30%	65.77%	73.93%	81.85%	86.66%	88.40%	20,736
30% to 90%	78.83%	84.98%	88.34%	91.47%	91.99%	6510

Table3 IPCA Performance by Bins of Option Delta, Maturity, and VIX

4.2. IPCA Performance

		No. Factors				
		1	2	3	4	5
		Panel A: Individual Options				
R^2_{total}	$\Gamma_\alpha = 0$	72.28%	79.65%	85.07%	88.90%	90.22%
	$\Gamma_\alpha \neq 0$	74.03%	81.08%	85.57%	89.32%	90.46%
R^2_{pred}	$\Gamma_\alpha = 0$	5.47%	5.54%	6.39%	6.59%	6.77%
	$\Gamma_\alpha \neq 0$	7.59%	7.58%	7.42%	7.21%	7.13%
		Panel B: Managed Portfolios				
R^2_{total}	$\Gamma_\alpha = 0$	94.41%	96.64%	98.86%	99.39%	99.61%
	$\Gamma_\alpha \neq 0$	95.48%	97.00%	98.74%	99.41%	99.59%
R^2_{pred}	$\Gamma_\alpha = 0$	7.20%	7.44%	7.90%	7.99%	8.06%
	$\Gamma_\alpha \neq 0$	8.27%	8.27%	8.23%	8.18%	8.18%
		Panel C: Bootstrap Test ($H_0 : \Gamma_\alpha = 0$)				
W_α p-value		7.4%	2.6%	47.2%	22.6%	3.6%

Table8 IPCA Performance.

4.2.1. Comparison with extant factor models

Panel A: IPCA

	K				
	1	2	3	4	5
R^2_{tot}	72.28%	79.65%	85.07%	88.90%	90.22%
R^2_{pred}	5.47%	5.54%	6.39%	6.59%	6.77%
$R^2_{tot,x}$	94.41%	96.64%	98.86%	99.39%	99.61%
$R^2_{pred,x}$	7.20%	7.44%	7.90%	7.99%	8.06%

Panel B: Observable Factors - With Instruments

	CAPM	FF3	FFC4	FFCB5	FFCBS6
R^2_{tot}	23.81%	25.64%	26.08%	33.15%	49.94%
R^2_{pred}	2.47%	2.38%	2.71%	4.45%	6.24%
$R^2_{tot,x}$	22.55%	25.83%	26.34%	32.68%	56.79%
$R^2_{pred,x}$	3.37%	3.32%	3.58%	5.60%	7.57%

Panel C: Principal Components Analysis

	K				
	1	2	3	4	5
R^2_{tot}	18.08%	32.71%	41.70%	47.55%	52.11%
R^2_{pred}	-0.07%	-0.07%	-0.02%	-0.02%	0.15%
$R^2_{tot,x}$	94.09%	97.46%	98.74%	99.44%	99.72%
$R^2_{pred,x}$	7.29%	7.83%	7.87%	7.88%	7.90%

Table4 IPCA versus Observable Factor Models

4.3. Unconditional and conditional alphas

	IPCA		FFCBS6	
	alpha	t-stat	alpha	t-stat
delta	**4.6%**	5.01	**6.9%**	3.69
delta:put	**2.4%**	2.92	2.3%	1.23
ttm	0.4%	0.58	**5.8%**	3.11
ttm:put	1.0%	1.21	**6.0%**	3.25
embed_lev	0.0%	0.02	**3.5%**	2.04
embed_lev:put	0.2%	0.30	**5.1%**	2.74
theta	0.7%	1.21	2.7%	1.91
theta:put	**3.3%**	3.46	**7.9%**	5.36
impvol	-0.9%	-1.27	-3.0%	-1.31
impvol:put	0.5%	0.64	0.3%	0.16
gamma	**-0.6%**	-2.42	-0.4%	-0.27
gamma:put	**2.2%**	2.26	**4.3%**	3.08
vega	0.0%	0.10	-3.4%	-1.91
vega:put	1.0%	1.45	-1.4%	-0.70
const	-0.1%	-0.50	-1.8%	-1.15
Avg. Abs. Alpha	1.2%		3.7%	

Table 5 Managed Portfolio Alphas-IPCA vs. Observable Factors.

4.3. Unconditional and conditional alphas

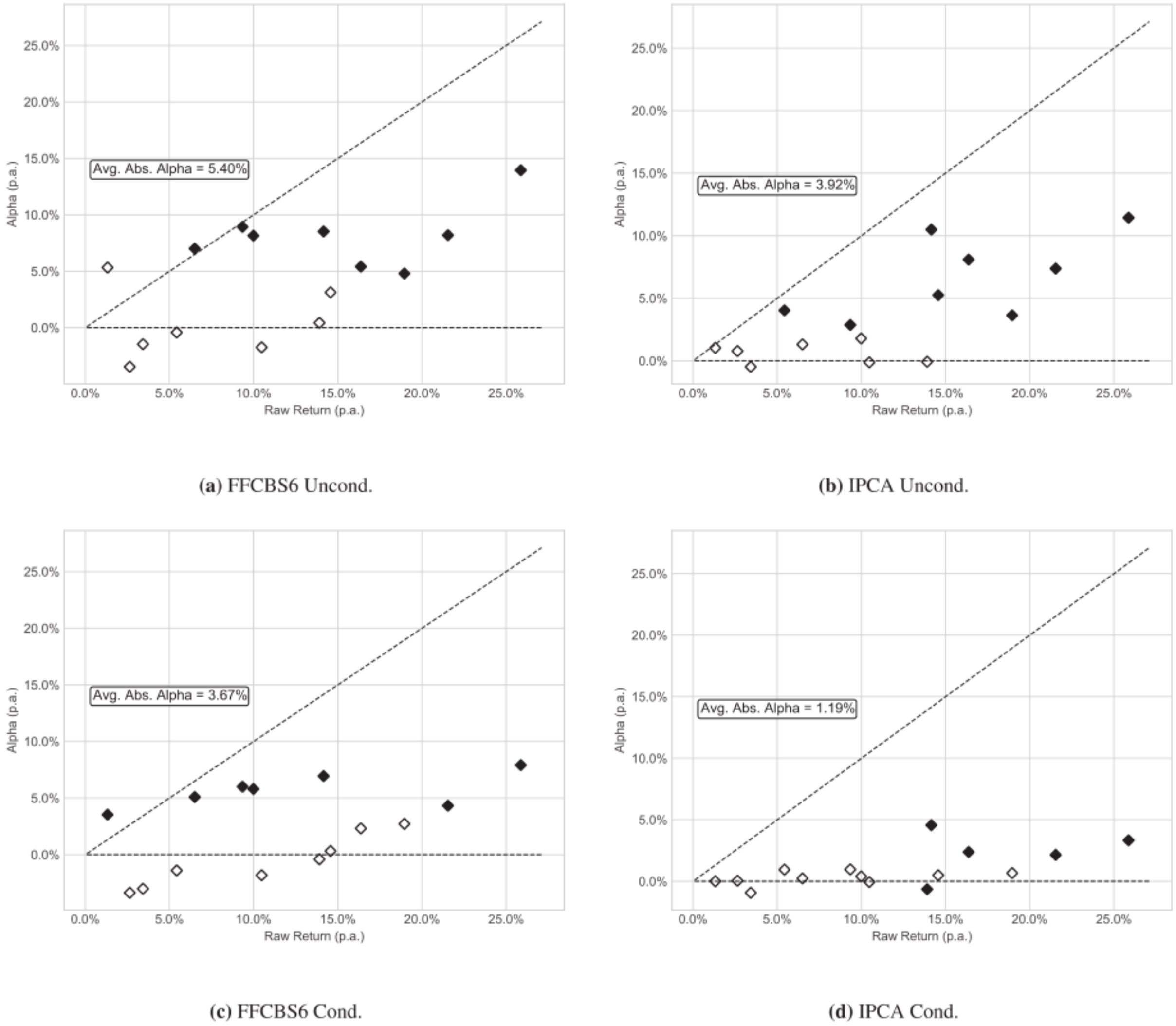
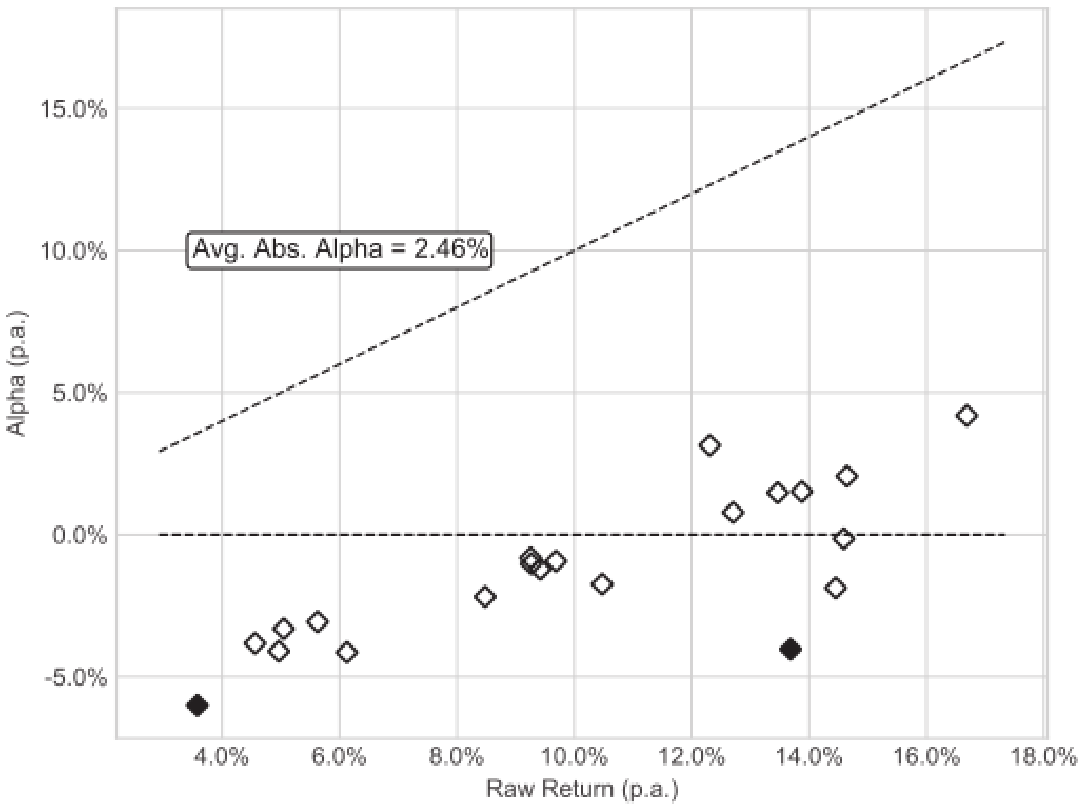
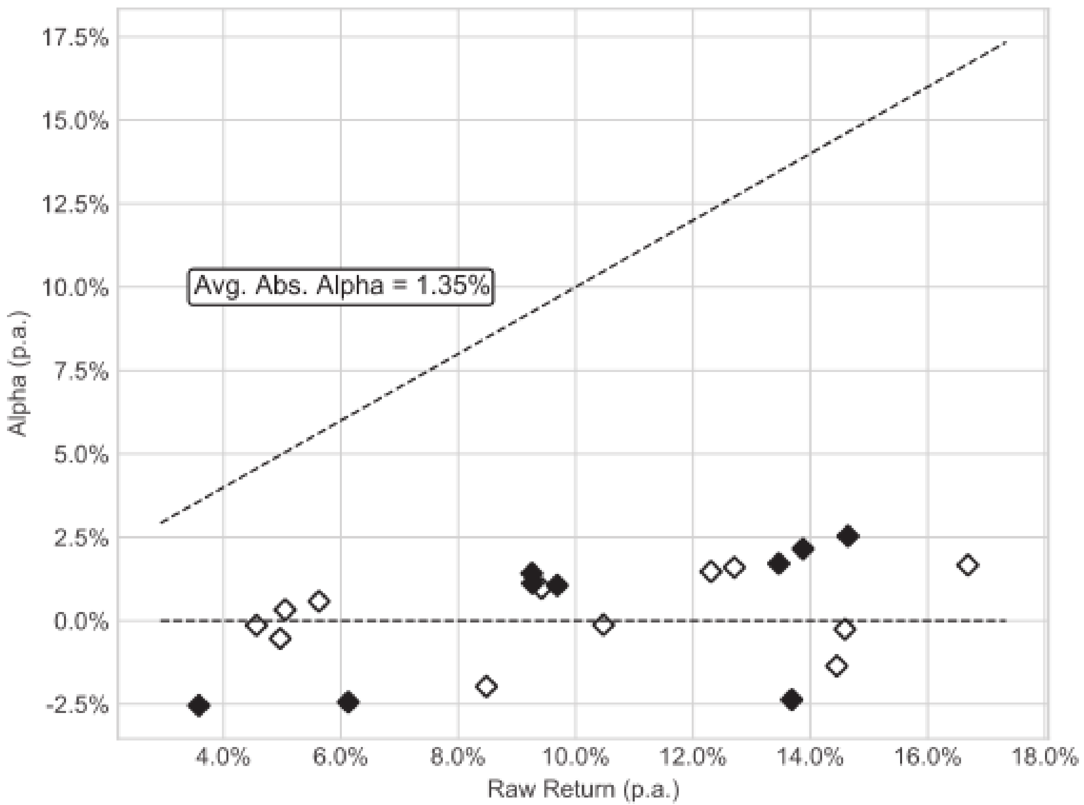


Fig2 Test of Mean-Variance Efficieny using Managed Portfolios.

4.3. Unconditional and conditional alphas



(a) FFCBS6



(b) IPCA

Fig3 Test of Unconditional Mean-Variance Efficieny using Double Sorted Portfolios

4.3. Unconditional and conditional alphas

	K=1	K=2	K=3	K=4	K=5
Unconditional	9.32%	8.26%	3.92%	3.02%	2.26%
Conditional	7.85%	6.33%	1.19%	1.04%	0.57%

Table 6 IPCA Portfolio Alphas

4.4. Out-of-sample performance

	No. Factors				
	1	2	3	4	5
	Panel A: Individual Options				
R^2_{total}	71.47%	76.18%	82.98%	86.53%	88.88%
R^2_{pred}	4.41%	3.31%	3.67%	4.34%	4.46%
	Panel B: Managed Portfolios				
R^2_{total}	95.57%	97.01%	98.23%	98.89%	99.21%
R^2_{pred}	3.31%	3.00%	3.30%	3.43%	3.47%

Table 7 Out-of-Sample Performance

4.5. Out-of-sample trading strategies

- In this section we present results from a trading strategy that aims to optimally combine the IPCA factors in a maximum Sharpe ratio sense. This provides a description of IPCA model performance in economic terms.

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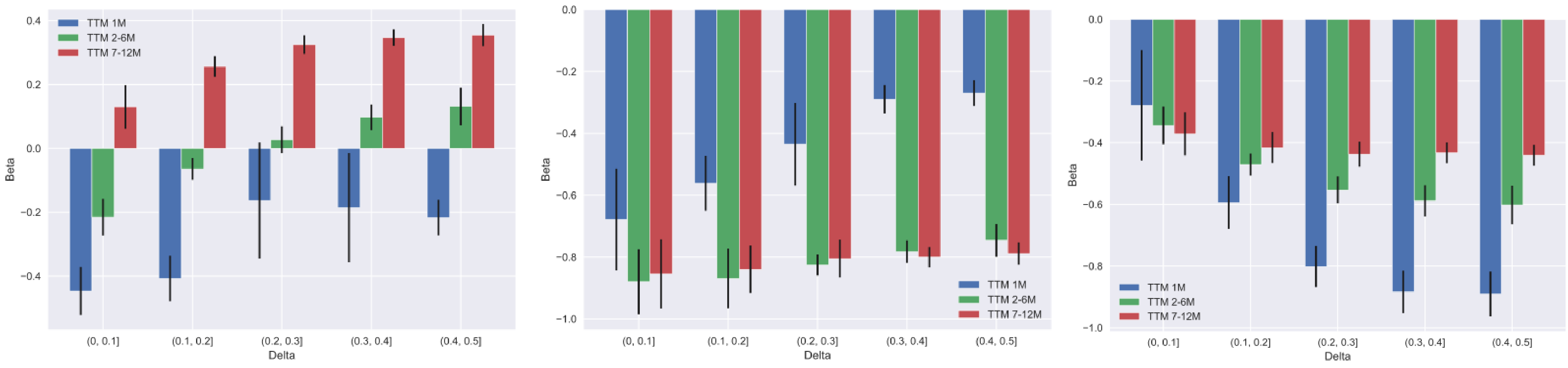
	ER	Vol	Sharpe	Skew	Kurtosis	$\alpha(BAB)$	$\alpha(Straddle)$	$\alpha(BAB+Straddle)$
IPCA K=1	0.094	0.096	0.986	-0.977	1.678	0.067 (2.218)	-0.01 (-0.125)	-0.010 (-0.768)
IPCA K=2	0.137	0.091	1.508	-0.598	0.899	0.109 (4.315)	0.041 (1.303)	0.037 (1.545)
IPCA K=3	0.166	0.099	1.673	-0.343	0.587	0.139 (4.920)	0.043 (1.438)	0.040 (1.669)
IPCA K=4	0.179	0.098	1.833	-0.326	0.520	0.151 (6.922)	0.066 (2.530)	0.063 (3.192)
IPCA K=5	0.197	0.109	1.802	-0.783	1.457	0.157 (5.689)	0.070 (1.827)	0.065 (2.310)

Table 9 Out-of-Sample Factor Portfolio Sharpe Ratios.

4.6 Interpreting the IPCA factors



(a) Factor 1 **(b)** Factor 2 **(c)** Factor 3

Fig. 4. IPCA Factor Regression Betas for Double Sorted Option Portfolios.

4.6 Interpreting the IPCA factors

	F1	F2	F3
Level	-0.10	**0.32**	**1.13**
	(-1.58)	(6.35)	(27.16)
Maturity Slope	**0.71**	**-0.38**	**0.59**
	(10.59)	(-9.44)	(12.82)
Moneyness Skew	**0.53**	**0.58**	**-0.53**
	(8.80)	(7.63)	(-8.61)
R^2_{adj}	72.3%	88.0%	89.3%
No. Obs.	261	261	261
		Shapley-Owen R^2	
Level	9.1%	28.4%	49.4%
Maturity Slope	38.6%	26.2%	20.2%
Moneyness Skew	24.7%	33.4%	19.8%

Table 10 IPCA Factors versus Option Return Factors.

4.6 Interpreting the IPCA factors

Panel A: IPCA Factors

Period	N. Obs	F1 Mean	F1 StdDev	F1 Sharpe	F2 Mean	F2 StdDev	F2 Sharpe	F3 Mean	F3 StdDev	F3 Sharpe
Full Sample	261	0.06	0.06	**1.04**	0.01	0.04	0.26	0.02	0.02	**1.42**
Price Jump	25	0.14	0.15	**0.93**	-0.11	0.09	**-1.19**	0.01	0.02	0.48
Non Price Jump	236	0.05	0.04	**1.32**	0.02	0.02	**0.98**	0.02	0.01	**1.60**
Vol Jump	50	0.10	0.11	**0.94**	-0.06	0.07	-0.86	-0.01	0.02	-0.52
Non Vol Jump	211	0.05	0.04	**1.33**	0.03	0.02	**1.28**	0.03	0.01	**2.10**
Recession	26	0.06	0.13	0.50	-0.04	0.08	-0.46	0.03	0.02	**1.44**
Non Recession	235	0.06	0.05	**1.32**	0.02	0.03	**0.52**	0.02	0.01	**1.42**
High VIX	65	0.14	0.10	**1.48**	-0.02	0.06	-0.38	0.02	0.02	**1.21**
Medium VIX	130	0.04	0.04	**0.94**	0.02	0.03	**0.85**	0.02	0.01	**1.49**
Low VIX	66	0.02	0.02	**1.22**	0.02	0.01	**1.39**	0.02	0.01	**1.96**

Panel B: Option Level, Slope & Skew

Period	N. Obs.	Level Mean	Level StdDev	Level Sharpe	Maturity Slope Mean	Maturity Slope StdDev	Maturity Slope Sharpe	Moneyness Skew Mean	Moneyness Skew StdDev	Moneyness Skew Sharpe
Full Sample	261	0.02	0.02	**1.03**	0.01	0.01	**0.91**	0.01	0.01	**0.58**
Price Jump	25	-0.03	0.03	-0.82	0.03	0.02	**1.63**	-0.03	0.03	**-0.95**
Non Price Jump	236	0.02	0.01	**1.66**	0.01	0.01	**0.81**	0.01	0.01	**1.29**
Vol Jump	50	-0.03	0.03	-1.09	0.02	0.01	**1.58**	-0.01	0.02	-0.39
Non Vol Jump	211	0.03	0.01	**2.46**	0.01	0.01	**0.70**	0.01	0.01	**1.39**
Recession	26	0.01	0.03	0.32	0.01	0.01	0.98	-0.01	0.02	-0.52
Non Recession	235	0.02	0.01	**1.25**	0.01	0.01	**0.90**	0.01	0.01	**0.98**
High VIX	65	0.01	0.03	0.25	0.02	0.01	**1.63**	0.00	0.02	0.17
Medium VIX	130	0.02	0.01	**1.75**	0.01	0.01	0.62	0.01	0.01	**0.81**
Low VIX	66	0.02	0.01	**2.23**	0.00	0.00	0.53	0.01	0.00	**1.70**

Table 11 IPCA Factor Summary Statistics.

4.6 Interpreting the IPCA factors

	F1	F2	F3
VIX	-0.17	**-0.46**	-0.02
	(-0.88)	(-3.17)	(-0.22)
Realized Variance	**-0.23**	-0.08	**-0.40**
	(-4.19)	(-0.74)	(-3.27)
Realized Return	-0.13	0.14	**-0.18**
	(-1.29)	(1.04)	(-2.53)
Intermed. Cap. Risk	**-0.48**	-0.08	**0.25**
	(-2.45)	(-1.07)	(3.02)
R^2_{adj}	22.1%	34.5%	19.4%
No. Obs.	215	215	215
		Shapley-Owen R^2	
VIX	2.3%	16.2%	2.8%
Realized Variance	4.5%	6.3%	9.5%
Realized Return	4.7%	9.0%	1.4%
Intermed. Cap. Risk	10.6%	3.0%	5.8%

Table12 IPCA Factors and the Dynamics & Liquidity of the Underlying.

	Reduction R^2_{total} (abs.)	W_β p-value
impvol:put	-26.80%	0.00
vega	-23.85%	0.00
impvol	-22.46%	0.00
gamma	-14.88%	0.00
theta	-14.78%	0.00
ttm	-7.03%	0.00
delta:put	-4.41%	0.00
delta	-2.90%	0.00
theta:put	-2.85%	0.00
vega:put	-2.43%	0.00
embed_lev	-2.24%	0.00
ttm:put	-2.17%	0.01
gamma:put	-1.64%	0.00
embed_lev:put	-1.51%	0.00

Table 13 IPCA Instrument Significance.

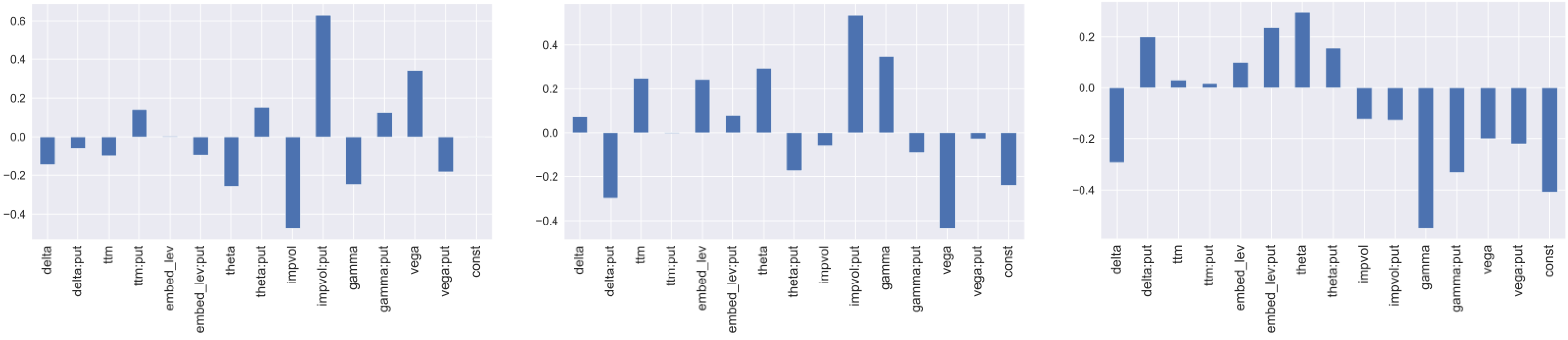
The second column contains p-values for the bootstrap test Wβ with 1000 draws that tests

$$H_0 : \Gamma_\beta = \left[\gamma_{\beta,1}, \ldots, \gamma_{\beta,l-1}, \mathbf{0}_{K\times 1}, \gamma_{\beta,l+1}, \ldots, \gamma_{\beta,L}\right]$$

against the alternative

$$H_1 : [\gamma_{\beta,1}, \ldots, \gamma_{\beta,L}].$$

4.6 Interpreting the IPCA factors



(a) Factor 1 **(b)** Factor 2 **(c)** Factor 3

Fig. 5. Plots of loadings Γβ

4.6 Interpreting the IPCA factors

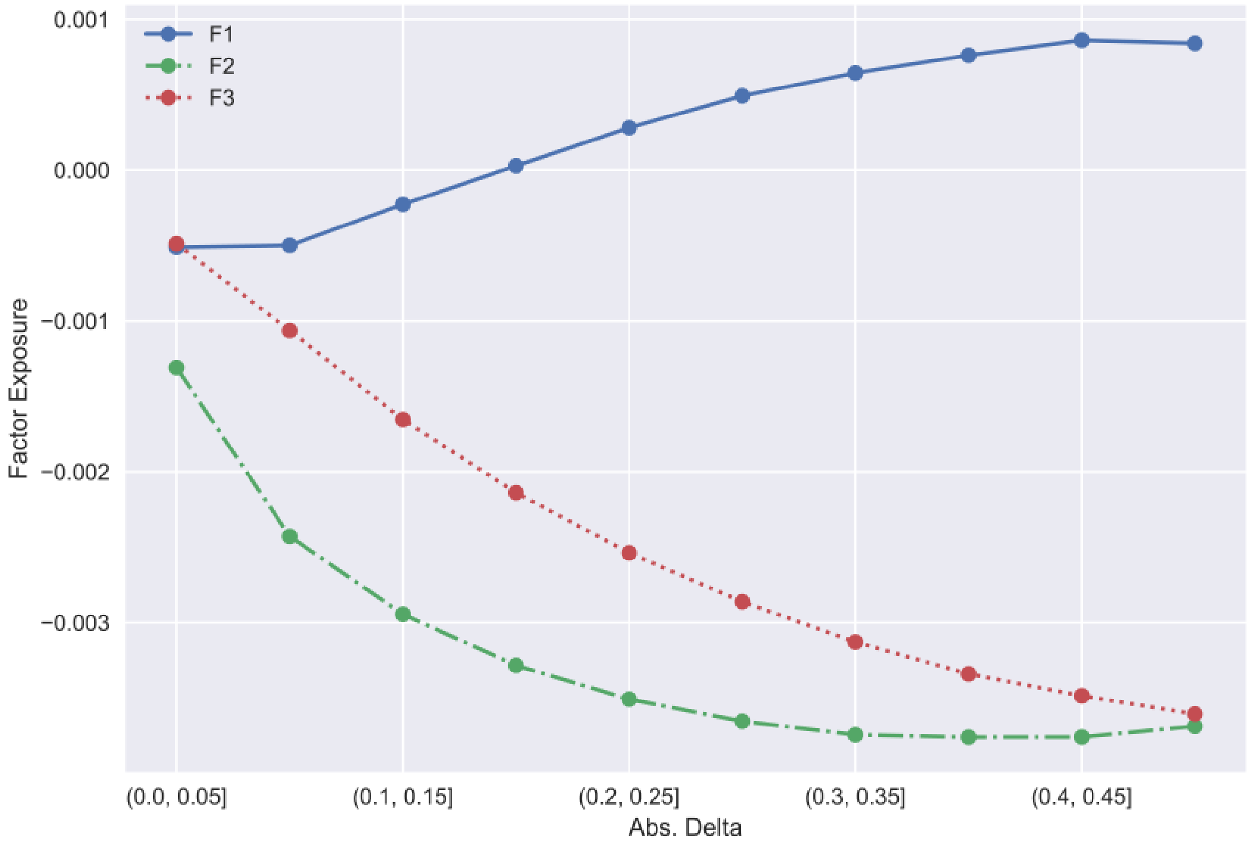


Fig. 6. IPCA Factor Exposure by Level of Moneyness.

For each bucket Δ_j compute the average factor exposure $\beta^{\Delta_j} = \tilde{z}_j \Gamma_\beta$ where $\tilde{z}_j$ is the mean of the characteristics in bucket Δ_j.

4.8. Model performance at the daily frequency

	Carr & Wu - R^2_{total}				IPCA - R^2_{total}				
Panel A: Average Performance									
	R_1	R_2	R_3	R_4	K=1	K=2	K=3	K=4	K=5
All Options	72.22%	85.29%	82.94%	85.20%	69.98%	85.89%	91.53%	92.92%	93.76%
Panel B: Average Performance by Moneyness Bin									
abs. delta	R_1	R_2	R_3	R_4	K=1	K=2	K=3	K=4	K=5
0 to 0.1	65.78%	70.96%	58.38%	69.32%	64.5%	67.7%	73.1%	81.5%	85.3%
0.1 to 0.2	72.65%	83.54%	81.54%	85.26%	77.3%	84.8%	90.6%	92.0%	93.0%
0.2 to 0.3	73.46%	87.30%	86.64%	88.08%	75.6%	88.0%	93.8%	94.2%	94.7%
0.3 to 0.4	73.05%	89.08%	88.89%	89.35%	70.4%	88.5%	94.3%	94.8%	95.2%
0.4 to 0.5	73.50%	90.12%	90.09%	90.18%	62.9%	88.0%	93.4%	94.2%	94.7%
Panel C: Average Performance by Time-to-Maturity Bin									
ttm	R_1	R_2	R_3	R_4	K=1	K=2	K=3	K=4	K=5
1 Month	51.9%	72.5%	67.7%	72.0%	69.0%	80.2%	91.7%	94.1%	95.1%
2 Months	76.8%	92.1%	91.2%	92.6%	77.9%	91.9%	93.1%	94.7%	95.6%
3 to 6 Months	87.4%	94.6%	94.0%	94.7%	72.8%	90.8%	92.8%	93.7%	94.2%
6 to 12 Months	93.2%	96.4%	96.3%	96.5%	58.0%	78.8%	87.8%	88.4%	89.4%
Panel D: Average Performance by VIX bin									
VIX	R_1	R_2	R_3	R_4	K=1	K=2	K=3	K=4	K=5
0% to 10%	65.90%	74.41%	69.07%	69.52%	56.2%	76.1%	85.7%	90.1%	92.3%
10% to 20%	70.36%	86.33%	84.64%	86.91%	71.5%	85.4%	91.4%	93.4%	94.2%
20% to 30%	76.79%	82.85%	79.13%	81.36%	63.9%	85.6%	90.8%	91.9%	92.6%
30% to 90%	71.95%	95.33%	93.85%	97.32%	74.5%	86.5%	92.3%	93.6%	94.5%

Table14 Comparison of IPCA against a No-Arbitrage Model at Daily Frequency

For the no-arbitrage model the total R2 is computed as follows: for a series R_i,1…4 the R-Squared is computed as $R^2_{total,i} = 1 - Var\,(R_i)\,/\,Var\,(R_0)$ where R_0 is the series of delta-hedged daily returns.

5. Conclusion

- We demonstrate that a coherent factor- based description of option returns is possible in a model with time-varying factor loadings.
- We find that a low dimensional latent factor model is successful in capturing variation in option returns and describing differences in risk across a wide range of options. The model also provides an accurate description of the risk-return trade-off in options markets.

5. Conclusion

- We also find that a trading strategy designed to efficiently capture the risk-return trade-off (as estimated from IPCA) earns an annualized Sharpe ratio as high as 1.8 and has positive alpha versus previously proposed investment strategies using index options.
- The risk factors recovered by IPCA can be interpreted as capturing fluctuations in the level of the volatility surface, in the maturity slope, and in the short-dated moneyness skew. While most of our analysis focuses on monthly data, we also find that the IPCA model matches the behavior of options returns at the daily frequency across a wide range of option contracts.

Thanks for listening

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